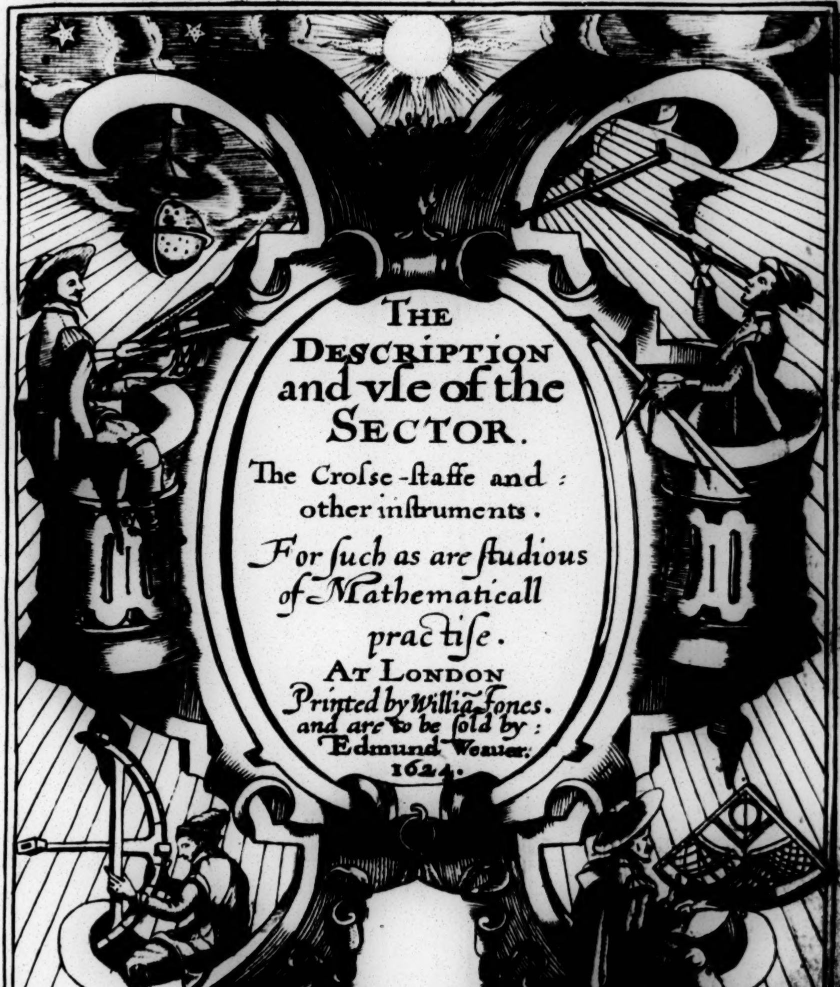

THE DESCRIPTION and vſe of the SECTOR.

The Croſse-ſtaffe and :
other inſtruments .

For ſuch as are ſtudious
of Mathematicall
practiſe .

AT LONDON
Printed by Williā Jones .
and are to be ſold by :
Edmund Weauer.
1624.

Mr Sutton

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THE CROSSE-STAFFE, and other Inſtruments,

For ſuch as are ſtudious of
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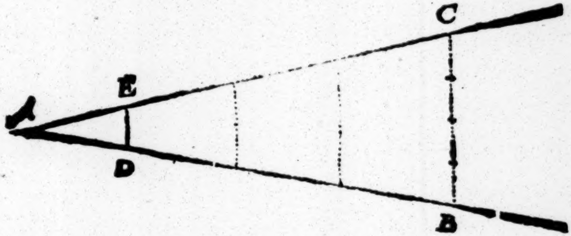

LONDON,
Printed by WILLIAM IONES.
and are to be ſold by *Edmund Weauer.*
1623.

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HONORATISSIMO

DOMINO *Dn. IOHANNI*
COMITI de BRIDGEWATER,
VICECOMITI de BRACKLEY,
BARONI de ELLESMERE,

Hæc organa M.

Sectorem,
Radium,
Arcum,
Quadrantem,

D. D. D.

EDM. GVNTER.

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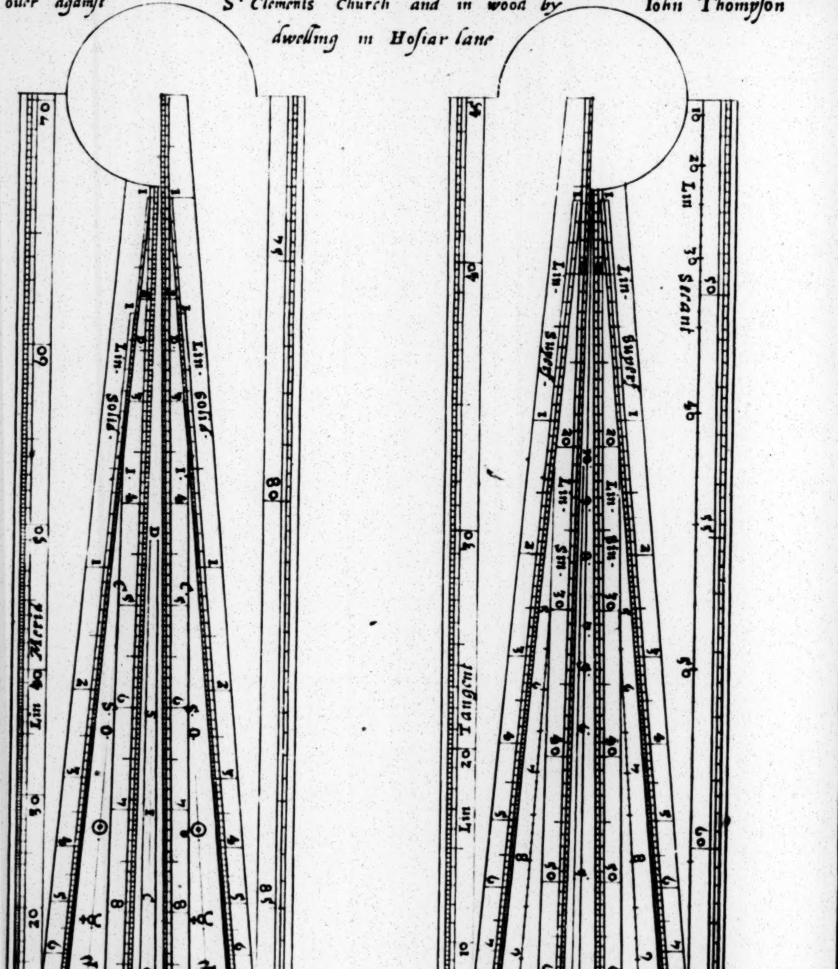
Theſe inſtruments are wrought in braſſe by Elias Allen dwelling without Tempel barre ouer againſt St Clements Church and in wood by Iohn Thompſon dwelling in Hoſiar lane



THE FIRST BOOKE OF THE SECTOR.

CHAP. I.

The Deſcription, the making, and the generall vſe of the Sector.

A *Sector* in Geometrie, is a figure comprehended of two right lines containing an angle at the center, and of the circumference aſſumed by them. This *Geometricall inſtrument* hauing two legs containing all varietie of angles, & the diſtance of the feete, repreſenting the ſubtenſes of the circumference, is therefore called by the ſame name.

It containeth 12 ſeuerall lines or ſcales, of which 7 are generall, the other 5 more particular. The firſt is the ſcale of *Lines* diuided into 100 equall parts, and numbred by 1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

The ſecond, the lines of *Superficies* diuided into 100

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vnequall parts, and numbred by 1. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

The third, the lines of *Solids*, diuided into 1000 vnequal parts, & numbred by 1. 1. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

The fourth, the lines of *Sines* and *Chords* diuided into 90 degrees, and numbred with 10. 20. 30. vnto 90.

Theſe foure lines of *Lines*, of *Superficies*, of *Solids*, and of *Sines*, are all drawne from the center of the *Sector* almoſt to the end of the legs. They are drawne on both the legs, that euery line may haue his fellow. All of them are of one length, that they may anſwere one to the other. And euery one hath his parallels, that the eye may the better diſtinguiſh the diuiſions. But of the parallels thoſe onely which are inward moſt containe the true diuiſions.

There are three other generall lines, which becauſe they are infinite are placed on the ſide of the *Sector*. The firſt a line of *Tangents*, numbred with 10. 20. 30. 40. 50. 60. ſignifying ſo many degrees from the beginning of the line, of which 45 are equall to the whole line of *Sines*, the reſt follow as the length of the *Sector* will beare.

The ſecond, a line of *Secants*, diuided by pricks into 60 degrees, whoſe beginning is the ſame, with that of the line of *Tangents*, to which it is ioyned.

The third, is the *Meridian* line, or line of *Rumbs*, diuided vnequally into degrees, of which the firſt 70 are almoſt equall to the whole line of *Sines*, the reſt follow vnto 84 according to the length of the *Sector*.

Of the particular lines inſerted among the generall, becauſe there was voyd ſpace, the firſt are the lines of *Quadrature* placed betweene the lines of *Sines*, and noted with 10. 9. 8. 7. *S*. 6. 5. 90. *Q*.

The ſecond, the lines of *Segments* placed betweene the lines of *Sines* and *Superficies*, diuided into 50 parts, and numbered with 5. 6. 7. 8. 9. 10.

The third, the lines of *Inſcribed bodies in the ſame Sphere*, placed betweene the ſcales of *Lines*, and noted with **D. S. I. C. O. T**

The fourth, the lines of *Equated bodies*, placed between the lines of *Lines* and *Solids*, and noted with *D.I.C.S.O.T.*

The fift, are the lines of *Mettalls*, inſerted with the lines of *Equated bodies* (there being roome ſufficient) and noted with theſe Characters. ☉.☿.♄ ☽.♀.♂ ♃.

There remaine the edges of the *Sector*, and on the one I haue ſet a line of *Inches*, which are the twelth parts of a foote Engliſh: on the other a leſſer line of *Tangents*, to which the *Gnomon* is *Radius*.

2 *Of the making of the Sector.*

Let a *Ruler* be firſt made either of braſſe or of wood, like vnto the former figure, which may open and ſhut vpon his center. The head of it may be about the twelth part of the whole length, that it may beare the moueable foote, and yet the moſt part of the diuiſions may fall without it. Then let a moueable *Gnomon* be ſet at the end of the moueable foote, and there turne vpon an *Axis*, ſo as it may ſometime ſtand at a right angle with the feete, and ſometimes be incloſed within the feet. But this is well knowne to the workeman.

For drawing of the lines. Vpon the center of the *Sector*, and ſemidiameter ſomewhat ſhorter then one of the feet, draw an occult arke of a circle, croſſing the cloſure of the inward edges of the *Sector* about the letter *T*. In this arke, at one degree on either ſide from the edge, draw right lines from the Center, fitting them with Parallels and diuide them into an hundred equall parts, with ſubdiviſions into 2.5. or 10. as the line will beare, but let the numbers ſet to them, be onely 1.2.3.4.&c. vnto 10. as in the example. Theſe lines ſo divided, I call the lines or ſcales of *Lines*; and they are the ground of all the reſt.

In this Arke at 5 degrees on either ſide, from the edge neere *T*, drawe other right lines from the Center, and fit them with Parallells. Theſe ſhall ſerue for the lines of *Solids*.

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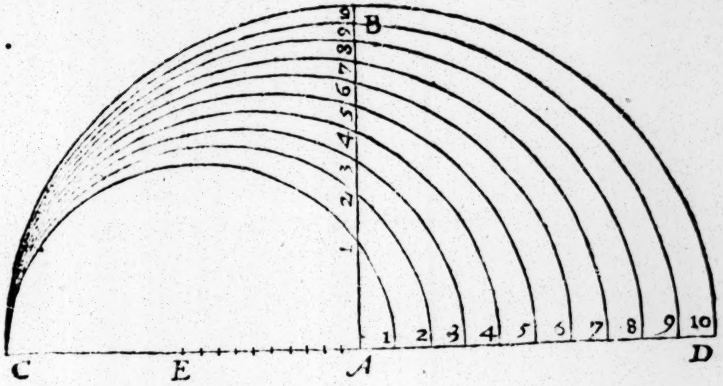
Then on the other ſide of the *Sector* in like manner, vpon the Center & equall Semidiameter, drawe another like Arke of a circle: & heere againe at one neere degree on either ſide frō the edge neere the letter *Q* draw right lines from the Center, and fit them with parallells. Theſe ſhall ſerue for the lines of *Sines*.

At 5 Degrees on either ſide from the edge neere *Q* drawe other right lines from the center, and fit them with parallels: theſe ſhall ſerue for the lines of *Superficies*.

Theſe foure principall lines being drawne, and fitted with parallels, we may drawe other lines in the middle betweene the edges and the lines of *Lines*, which ſhall ſerue for the lines of *inſcribed bodies*, and others betweene the edges and the *Sines* for the lines of *quadrature*. And ſo the reſt as in the example.

3 *To diuide the lines of* Superficies.

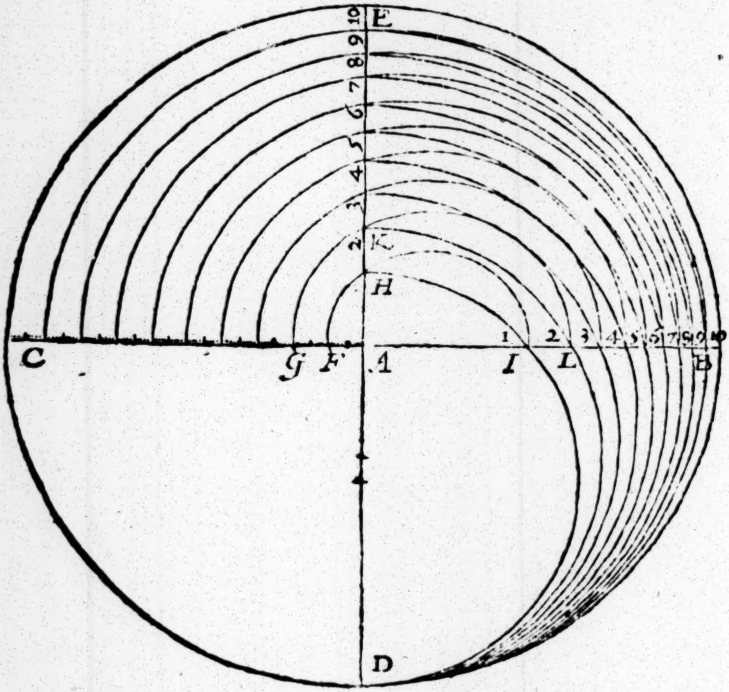
SEeing like *Superficies* doe hold in the proportion of their *homologall* ſides duplicated, by the 29 Pro. 6 lib. *Euclid*. If you ſhall find meane proportionals between the whole ſide, and each hundred part of the like ſide, by the 13 Pro. 6 lib. *Euclid*. all of them cutting the ſame line, that line ſo cut ſhal conteine the diviſions required. wherefore vpon the center *A* and Semidiamiter equall to the line of *Lines*, deſcribe a Semicircle *A C B D*, with *A B* perpendicular to the diameter *C D*. And let the Semidiameter *A*D be divided as the line of *Lines* into an hundred parts, & *A E* the one halfe of *AC* diuided alſo into an hundred parts, ſo ſhall the diuiſions in *A E* be the centers from whence you ſhall deſcribe the Semicircles *C* 10. *C* 20. *C* 30. &c. diuiding the lin *A B* into an hundred vnequall parts: and this line *A* B ſo diuided ſhall be the line of *Superficies*, and muſt be transferred into the *Sector*. But let the numbers ſet to them be onely 1. 1. 2. 3. vnto 10. as in the example.



4 *To diuide the lines of* Solids.

SEing like Solids do hold in the proportion of their *homologall* ſides triplicated, if you ſhall finde two meane proportionalls between the whole ſide & each thouſand part of the like ſide : all of them cutting the ſame two right lines, the former of thoſe lines ſo cut, ſhall containe the diuiſions required.

Wherefore vpon the center *A* & Semidiameter equall to the line of *Lines*. deſcribe a circle and diuide it into 4 equall parts *C E B D*, drawing the croſſe diameters *C B*, *E D*. Then diuide the ſemidiameter *A C*, firſt into 10 equall parts, and betweene the whole line *A D* & *A F* the tenth part of *A C*, ſeeke out two meane proportionall lines *A I* and *A H*. againe betweene *A D* and *A G* being two tenth parts of *A C*, ſeeke out two meane proportionals *A L* and *A K*, and ſo forward in the reſt. So ſhall the line *A B* be diuided into 10 vnequall parts.

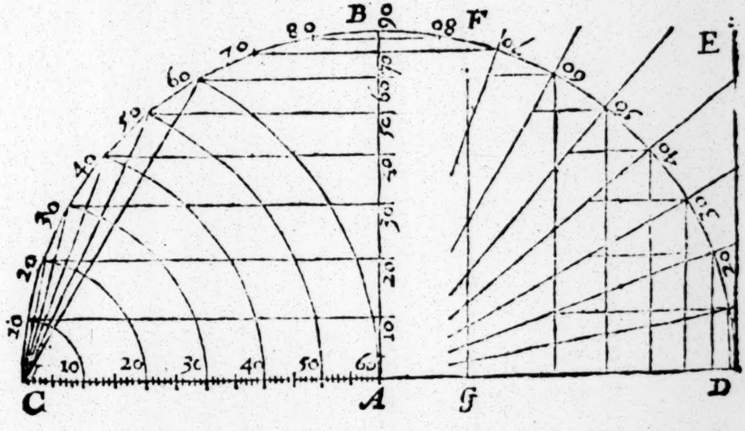


Secondly, diuide each tenth part of the line ***AC*** into 10 more, and betweene the whole line ***AD***, and each of them, ſeeke out two meane proportionalls as before: So ſhall the line ***AB*** be diuided now into an hundred vnequall parts.

Thirdly, If the length will beare it, ſubdiuide the line ***AC*** once againe, each part into ten more: and betweene the whole line ***AD*** and each ſubdiuiſion, ſeeke two meane proportionalls as before. So ſhould the line *A* B be now diuided into 1000 parts. But the ruler being ſhort, it ſhall ſuffice, if thoſe 10 which are neareſt the center be expreſſed, the reſt be vnderſtood to be ſo diuided, though actually they be diuided into no more then 5 or 2. and this line *A B* ſo diuided ſhall be the line of *Solids*, and muſt be transferred into the *Sector*: But let the numbers ſet to them be onely 1. 1. 1. 2. 3. &c. vnto 10. as in the example.

5 *To diuide the lines of* Sines *and* Tangents *on the ſide of the Sector.*

VPon the center *A*, and ſemidiameter equall to the line of *Lines*, deſcribe a ſemicircle *A B C* D, with *A B*, perpendicular to the diameter *C* D. Then diuide the quadrants *C B*, *B* D, each of them into 90. and ſubdiuide each degree into 2 parts: For ſo, if ſtreight lines be drawne parallell to the diameter *C* D, through theſe 90, and their ſubdiuiſions they ſhall diuide the perpendicular *A* B vnequally into 90.



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And this line A B ſo diuided ſhall be the line of *Sines*; and muſt be transferred into the *Sector*. The numbers ſet to them are to be 10. 20. 30. &c. vnto 90 as in the example.

If now in the poynt *D*, vnto the diameter *C D*, we ſhall raiſe a perpendicular *D E*, and to it drawe ſtreight lines from the center *A*, through each degree of the quadrant DB. This perpendicular ſo diuided by them ſhall be the line of *Tangents*, & muſt be transferred vnto the ſide of the *Sector*. The numbers ſet to them, are to be 10. 20 30. &c. as in the example.

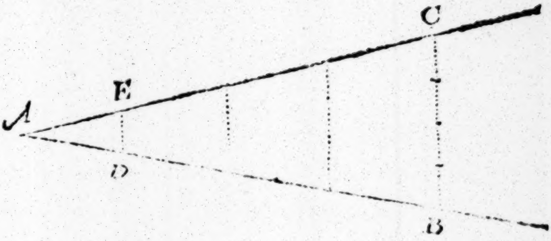
If betweene A and D, another ſtreight line *G F*, be drawne parallell to D E, it will be diuided by thoſe lines from the center in like ſort as D E is diuided, and it may ſerue for a leſſer line of *Tangents*, to be ſet on the edge of the *Sector*.

Theſe lines of *Sines* and *Tangents*, may yet otherwiſe be transferred into the *Sector* out of the line of *Lines*, (or rather out of a diagonall Scale equall to the line of *Lines*) by tables of Sines and Tangents. In like manner may the lines of *Superficies*, be transferred by tables of ſquare rootes; and the line of *Solids*, by tables of cubique rootes: which I leaue to others to extract at leaſure.

6 *To ſhew the ground of the Sector.*

LEt A B, A C, repreſent the leggs of the *Sector*: then ſeuering theſe two A B, A C, are equall, and their ſections A D, AE, alſo equall, they ſhall be cut proportionally: and if we draw the lines B C, D E, they will be parallell by the ſecond Pro. 6 lib. of *Euclid*, and ſo the Triangles A B C, A D E, ſhalbe equiangle; by reaſon of the common angle at A, and the equall angles at the baſe, and therefore ſhall haue the ſides proportionall about thoſe equall angles, by the 4 Pro. 6 lib. of *Euclid*.

The



The ſide A D, ſhalbe to the ſide A B, as the baſis D E, vnto the parallell baſis BC, and by conuerſion A B, ſhall be vnto A D, as B C, vnto D E: and by permutation A D, ſhall be vnto D E, as A B, to B C.&c. So that if A D, be the fourth part of the ſide A B, then D E, ſhall alſo be the fourth part of his parallell baſis B C. The like reaſon holdeth in all other ſections.

7 *To ſhew the generall vſe of the Sector.*

THere may ſome cõcluſions be wrought by the *Sector*, euen then when it is ſhut, by reaſon that the lines are all of one length: but generally the vſe hereof conſiſts in the ſolution of the *Golden rule*, where three lines being giuen of a known denominaton, a fourth proportionall is to be found. And this ſolution is diuerſe in regard both of the *lines*, and of the *entrance* into the worke.

The ſolution in regard of the *lines* is ſometimes *ſimple*, as when the worke is begun and ended vpon the ſame *lines*. Sometimes it is compound, as when it is begun on one kind of *lines*, and ended on another. It may be begun vpon the lines of *Lines*; & finiſhed vpon the lines of *Superficies*. It may begin on the *Sines*, and end on the *Tangents*.

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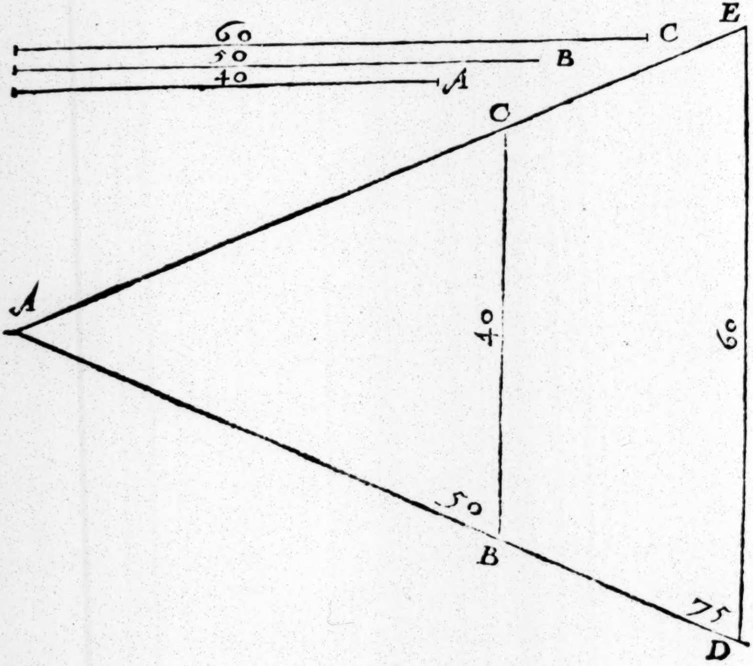
The ſolution in regard of the ***entrance*** into the worke, may be either with a ***parallell*** or elſe ***laterall*** on the ſide of the Sector, I cal it ***parallell entrance***, or entring with a parallell, when the two lines of the firſt denomination are applied in the parallells, and the third line, and that which is ſought for, are on the ſide of the *Sector*. I call it ***laterall entrance,*** or entring on the ſide of the *Sector*, when the two lines of the firſt denomination are one the ſide of the *Sector*, and the third line and that which is to be found out, doe ſtand in the parallells.



As for example, let there be giuen three lines A,B,C, to which I am to find a fourth proportionall. let A, meaſured in the line of *lines*, be 40, B 50, and C 60, and ſuppoſe the queſtion be this. If 40 *Monthes* giue 50 *pounds*, what ſhal 60? Here are lines of two denominatiõs, one of *months*, another of *pounds*, and the firſt with which I am to enter muſt be that of 40 *monthes*. If then I would *enter with a parallell*, firſt I take A, the line of 40, and put it ouer as a *parallell* in 50, reckoned in the line of *lines*, on either ſide of the *Sector* from the center, ſo as it may be the Baſe of an *Iſoſcheles* triangle B A C, whoſe ſides A B, A C are equal to B, the line of the ſecond denomination.

Then the *Sector* being thus opened, I take C the line of 60, betweene the feete of the compaſſes, and carrying them *parallell* to B C, I finde them to croſſe the lines A B, A C, on the ſide of the *Sector* in D and E, numbred with 75, wherefore I conclude the line A D, or A E, is the fourth proportionall and the correſpondent number 75 which was required.

But if I would *enter on the ſide* of the *Sector*, then would I diſpoſe the lines of the firſt denomination A and C, in the line of *Lines*, on both ſides of the *Sector*, in A B, A C, & in A D, A E, ſo as they ſhould all meete in the center A, and then taking C the line of the ſecond denominationput it ouer as a *parallell* in B C, that it may be the Baſis of the Iſocheles triangle B A C, whoſe ſides A B, A C, are equall to A, the firſt line of the firſt denomination, for ſo the *Sector* being thus opened, the other *parallell* from D to E, ſhall be the fourth proportionall which was required, and if it be meaſured with the other lines, it ſhal be 75, as before.

In both this manner of operations, the two firſt lines do ſerue to opẽ the *Sector* to his due angle, the difference betweene them is eſpecially this, that in *parallell entrance*, the two lines of the firſt denomination, are placed in the parallells B,C,D,E, & in *latterall entrance* they are placed on both ſides of the *Sector*, in A B, A D and in A C, A E.

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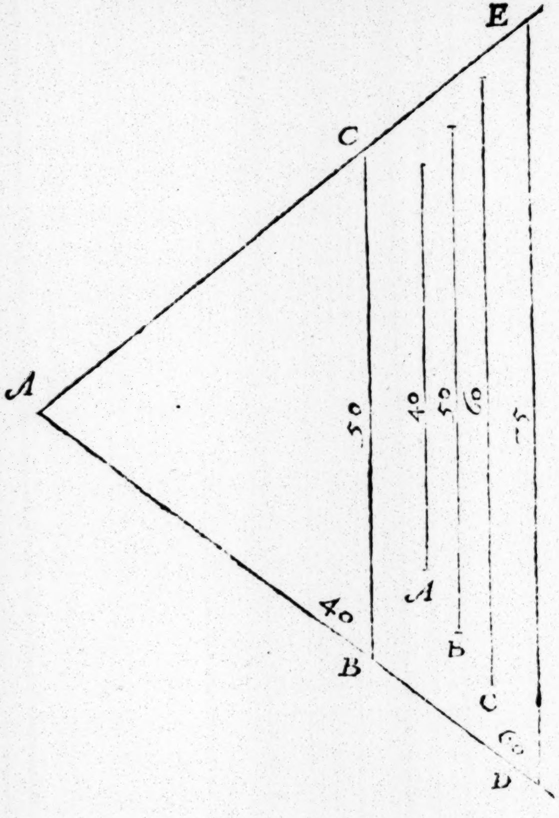


Now in *ſimple ſolution* which is begun and ended, vpon the ſame kinde of lines, it is allone which of the two latter lines be put in the ſecōd or third places. As in our exāple we may ſay, as 40 are to 50, ſo 60 vnto 75, or elſe as 40 are to 60, ſo 50 vnto 75. And hence it cōmeth that we may enter both with a *parrallell*, & on the ſides two manner of wayes at either entrance, and ſo the moſt part of queſtions may be wrought 4 ſeuerall wayes, though in the propoſitions following, I mention onely that which is moſt conuenient. Thus much for the generall vſe of the *Sector*, which being conſidered and well vnderſtood, there is nothing hard in that which followeth.

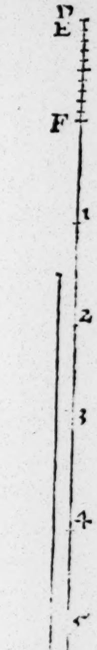
CHAP. II.

The vſe of the Scale of Lines.

1. *To ſet downe a Line, reſembling any giuen parts or fraction of parts.*

THe lines of *Lines* are diuided actually into 100 parts, but we haue put onely 10 numbers to them. Theſe we would haue to ſignifie either themſelues alone, or ten times themſelues, or an hundred times themſelues, or a thouſand times themſelues, as the matter ſhall require. As if the numbers giuen be no more then 10, then we may thinke the lines onely diuided into 10 parts according to the numbers ſet to them. If they be more then 10, and not more then 100, then either line ſhall containe 100 parts, and the numbers ſet by them ſhall be in value 10. 20. 30. &c. as they are diuided actually. If yet they be more then 100, then euery part muſt be thought to be diuided into 10, and either line ſhall be 1000 parts, and the numbers ſet to them ſhall be in value 100. 200. 300, and ſo forward ſtill increaſing themſelues by 10. This being preſuppoſed, we may number the parts and fraction of parts giuen in the line of *lines*; and taking out the diſtance with a paire of compaſſes, ſet it by, for the line ſo taken ſhall reſemble the number giuen.

In this manner may we ſet downe a line reſembling 75, if either we take 75 out of the hundred parts, into which one of the line of *lines* is actually diuided, and note it in A, or $7\frac{1}{2}$. of the firſt 10 parts, and note it in B, or onely $\frac{3}{4}$. of one of thoſe hundred parts, and note it in C. Or



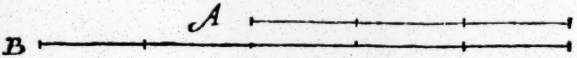
if this be either to great or to ſmall, we may run a Scale at pleaſure, by opening the compaſſe to ſome ſmall diſtance, and running it ten times ouer, then opening the compaſſe to theſe ten, run them ouer nine times more, & ſet figures to them as in this example, and out of this we may take what parts we will as before.

To this end I haue diuided the line of inches on the edge of the *Sector*, ſo as one inch containeth 8 parts, another 9, another 10, &c. according as they are figured, and as they are diſtant from the other end of the *Sector*, that ſo we might haue the better eſtimate.

2 *To encreaſe a line in a giuen proportion.*

3 *To diminiſh a line in a giuen proportion.*

TAke the line giuen with a paire of compaſſes, and open the **Sector**, ſo as the feete of the compaſſes may ſtand in the points of the number giuen, then keeping the **Sector** at this angle, the parallell diſtance of the points of the number required, ſhall giue the line required.



Let *A*, be a line giuen to be increaſed in the proportion of 3 to 5. Firſt I take the line *A*, with the compaſſes, and open the ***Sector*** till I may put it ouer in the poynts of 3 and 3, ſo the parallell betweene the poynts of 5 & 5, doth giue me the line *B*, which was required.

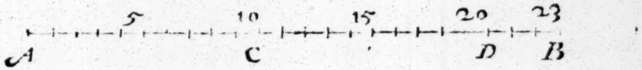
In like manner, if *B*, be a line giuen to be diminiſhed in the proportiõ of 5 to 3, I take the line *B* & to it open the ***Sector*** in the poynts of 5, ſo the parallell betweene the points of 3, doth giue me the line *A*, which was required.

If this manner of worke doth not ſuffice, we may multiplie or diuide the numbers giuen by 1, or 2, or 3, &c. And ſo worke by their numbers ***equimultiplices***, as for 3 and 5, wee may open the *Sector* in 6 and 10, or elſe in 9 and 15, or elſe in 12 and 20, or in 15 and 25, or in 18 and 30. &c.

4 *To diuide a line into parts giuen.*

TAke the line giuen, and open the *Sector* according to the length of the ſaid line in the points of the parts, wherevnto the line ſhould be diuided, then keeping the *Sector* at this angle, the parallell diſtance betweene the points of 1 and 1 ſhall diuide the line giuen into the parts required.

Let A B, be the line giuen to be diuided into fiue parts, firſt I take this line A B, and to it open the *Sector* in the points of 5 and 5, ſo the parallell betweene the points of 1 and 1, doth giue me the line A C, which doth diuide it into the parts required.

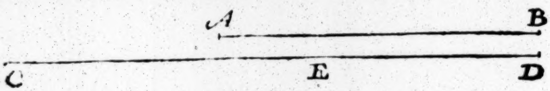


Or let the like line A B, be to be diuided into twenty three parts. Firſt I take out the line and put it vpon the *Sector* in the points of 23, then may I by the former propoſition diminiſh it in A C, C D, in the proportion of 23, to 10, and after that diuide the line A C into 10, &c. As before.

5 *To finde a proportion betweene two or more right lines giuen.*

TAke the greater line giuen, and according to it open the

the *Sector* in the points of 100 and 100, then take the leſſer lines ſeuerally, & carry them parallell to the greater, till they ſtay in like points, ſo the number of points wherein they ſtay, ſhall ſhew their proportion vnto 100.



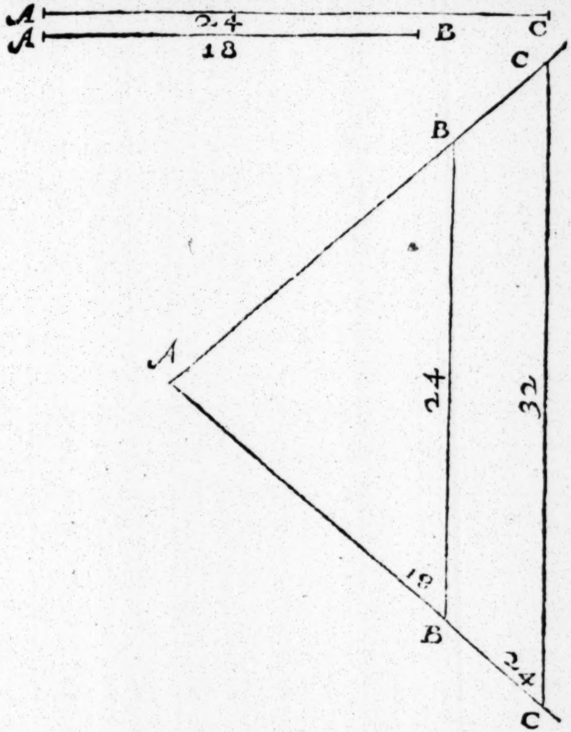
Let the lines giuen be *AB*, *CD*, firſt I take the line *CD*, & to it open the *Sector* in the points of 100, and 100, then keeping the *Sector* at this angle, I enter the leſſer line *AB*, parallell to the former, and find it to croſſe the lines of *Lines* in the poynts of 60. Wherefore the proportion of *AB* to *CD*, is as 60 to 100.

Or if the line *C* D, be greater then can be put ouer in the poynts of 100, then I admit the leſſer line *AB*, to be 100, & cutting off *CE* equal to *AB*, I find the proportion of *CE*, vnto *E* D, to be as 100, almoſt to 67; wherefore this way ẏ proportiõ of *AB* vnto *C*D, is as 100 vnto almoſt 167.

This propoſition may alſo not vnfitly be wrought by any other number, that admits ſeuerall diuiſions, and namely, by the numbers of 60. And ſo the leſſer line will be found to be 36, which is as before in leſſer numbers, as 3 vnto 5. It may alſo be wrought without opening the *Sector*. For if the lines between which we ſeek a proportion, be applyed to the lines of *Lines*, (or any other Scale of equall parts) there will be ſuch proportiõ found between them, as betweene the lines to which they are equall.

6 *Two lines being giuen to finde a third incontinuall proportion.*

First place both the lines giuen, on both ſides of the *Sector* from the Center, and marke the termes of their extenſion, then take out the ſecond line againe, and to it open the *Sector*, in the terme of the firſt line, ſo keeping the *Sector* at this angle, the parallell diſtance betweene the termes of the ſecond line, ſhall be the third proportionall.

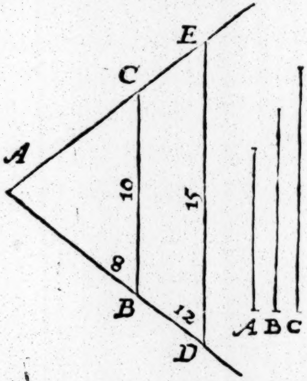


Let the two lines giuen be A B, A C, which I take out and place on both ſides of the *Sector*, ſo as they all meete in the center A, let the termes of the firſt line be B and B, the termes of the ſecond C and C. Then doe I take out A C the ſecond line againe, and to it open the *Sector* in the termes B B. So the parallell betweene C and C doth giue me the third line in continuall proportion. For as *A B* is vnto *A C*, ſo BB, equall to *A C*, is vnto *C C*.

7 *Three lines being giuen to finde the fourth in diſcontinuall proportion.*

HEre the firſt line and the third are to be placed on both ſides of the *Sector* from the center, then take out the ſecond line, and to it open the *Sector* in the termes of the firſt line. For ſo keeping the *Sector* at this angle, the parallell diſtance between the termes of the third line, ſhalbe the fourth proportionall.

Let the three lines giuen be A, B, C.



Firſt I take out A and C, and place them on both ſides of the *Sector*, in A B, A C, and A D, A E, laying the beginning of both lines at the center A, then do I take out B the ſecond line, according to it I open the *Sector* in B and C, the termes of the firſt line: ſo the parallell betweene D and E, doth giue me the fourth proportionall which was required.

As in *Arithmetique*, it ſufficeth if the firſt and third number giuen be of one denomination, the ſecond & the fourth which is required be of another. For one and the ſame denomination is not required neceſſarily in them all. So in *Geometrie*, it ſufficeth if the ſides A B, A D, reſembling the firſt and third lines giuen be meaſured in one Scale, and the parallells B C, D E be meaſured in another. Wherefore knowing the proportion of A the firſt line, and C the third line, by the fift *prop.* before. Which is here as 8 to 12, & deſcẽding in leſſer numbers is as 4 to 6, or as 2 to 3, or aſcending in greater numbers, as 16 vnto 24, or 18 to 27, or 20 to 30, or 30 to 45, or 40 to 60 &c. If the *Sector* be opened in the points of 8 and 8, to the quantity of *B*, the ſecond line giuen, then a parallell betweene 12 and 12, ſhall giue D E, the fourth line required. So likewiſe if it be opened in 4 and 4, then a parallell betweene 6 and 6, or if in 16 and 16, then a parallell betweene 24 and 24 ſhall giue the ſame D E. And ſo in the reſt.

8 *To diuide a line in ſuch ſort as another line is before diuided.*

FIrſt take out the line giuen, which is already diuided, and laying it on both ſides of the *Sector* from the center, marke how farre it extendeth. Then take out the ſecond line which is to be diuided, and to it open the *Sector* in the termes of the firſt line. This done, take out the parts of the firſt line, and place them alſo on the ſame ſide of the *Sector* from the center. For the parallells taken in the termes of theſe parts, ſhalbe the correſpondent parts in the line which is to be diuided.

Let *A B*, be a line diuided in *D* and *E*, and *B C*, the line which I am to diuide in ſuch ſort, as *A B* is diuided.

Firſt I take out the line *A B*, and place it on the line of *Lines* in *A B*, *A C*, both from the center *A*, then take I out the ſecond *B C*, and to it open the *Sector* in *B* and *C*, the termes of the firſt line. The *Sector* thus opened to his due angle, I take out *A D* and *A E*, the parts of the firſt line *A B*, and place them alſo on both the ſides of the *Sector* in *A D*, *A E*, ſo the parallell *D D*, giueth me *B F*, and the parallell *E E*, giueth me *B G*, and now the line *B C*, is diuided in *F* & *G*, as is the other line *A B*, in D and *E*, which is that which was

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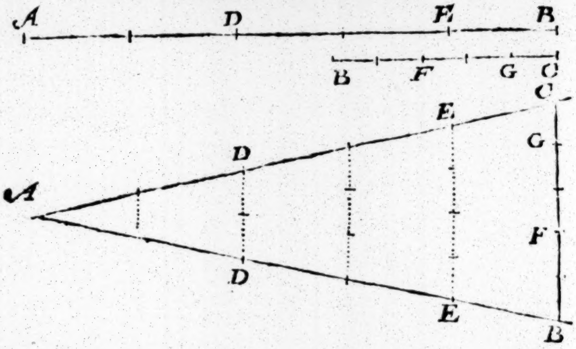

required.

If the line *AB*, were longer then one of the ſides of the Ruler, then ſhould I finde what proportion it hath to his parts *A* D, *A E*, and that knowne I may worke as before in the former propoſition.

9 *Two numbers being giuen to finde a third in continuall proportion.*

FIrſt reckon the two numbers giuen on both ſides of the lines of *Lines* from the center, and marke the termes to which either of them extendeth, then take out a line reſembling the ſecond number againe, and to it open the *Sector* in the termes of the firſt number, for ſo keeping the *Sector* at this angle, the parallell diſtance betweene the termes of the ſecond laterall number, being meaſured in the ſame Scale, from whence his parallell was taken, ſhall giue the third number proportionall.

Let the two numbers giuen be 18, 24, theſe being reſembled in lines, the worke will be in a manner all one, with that in the ſixt *Prop.* and ſo the third proportionall number will be found to to be 32.

10 *Three*

10. *Three numbers being giuen to finde a fourth in diſcontinuall proportion.*

THe ſolution of this propoſition, is in a manner all one with that before in the ſeuenth *Prop.* onely there may be ſome difficulty in placing of the numsers. To avoyd this, we muſt remember that three number being giuen, the queſtion is annexed but to one, and this muſt allwayes be placed in the third place, that which agrees with this third number in denomination, ſhalbe the firſt number, and that which remaineth the ſecond number. This being conſidered, reckon the firſt, and third numbers, which are of the firſt denomination on both ſides of the lines of *Lines* from the center, and marke the termes to which either of them extendeth, then take out a line reſembling the ſecond number, and to it open the *Sector* in the termes of the firſt number, for ſo keeping the *Sector* at this angle, the parallell diſtance betweene the termes of the third literall number, being meaſured in the ſame Scale from whence his parallell was taken, ſhall giue the fourth number proportionall.

As if a queſtion were propoſed in this manner, 10 yards coſt 8 l', how many yards may we buy for 12 l'? heere the queſtion is annexed to 12; and therefore it ſhall be the third number, and becauſe 8 is of the ſame denomination, it ſhall be the firſt number, then 10 remaining, it muſt be the ſecond number, ſo will they ſtand in this order, 8, 10, 12. Theſe being reſembled in lines, the worke will be in a manner the ſame, with that in the ſeuenth *Prop.* and the fourth proportionall number will be found to be 15. For as 8 are to 10, ſo 12 vnto 15.

And this holdeth in direct proportion, where, as the firſt number is to the ſecond, ſo the third to the fourth. So that if the third number be greater then the firſt, the fourth will be greater then the ſecond, or if the third number be leſſe then the firſt, the fourth will be leſſe then the ſecond, but in *reciprocall* proportion, commonly called the *Back rule*,

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where by how much the firſt number is greater then the third, ſo much the ſecond will be leſſe then the fourth, or by how much the firſt number is leſſe then the third, ſo much the ſecond will be greater then the fourth. The manner of working muſt be contrary, that is; the *Sector* is to be opened in the termes of the third number, and the parallell reſembling the number required, is to be found betweene the termes of the firſt number, the reſt may be obſerued as before, as for example.

If twelue men would raiſe a frame in ten dayes, in how many dayes would eight men raiſe the ſame frame? Here, becauſe the fewer men would require the longer time, though the numbers be 12, 10, 8; *yet the fourth proportionall will be found to be* 15.

So if 60 *yards, of three quarters of a yard in bredth, would hang round about a roome, and it were required to know how many yards of halfe a yard in bredth, would ſerue for the ſame roome. The fourth proportionall would be found to be* 90.

So if to make a foote ſuperficiall, 12 *inches in bredth doe require* 12 *inches in length, and the bredth being* 16 *inches, it were required to knowe the length. Here, becauſe the more bredth, the leſſe length, the fourth proportionall will be found to be* 9.

So if to make a Solid foote, a baſe of 144 *inches, require* 12 *inches in height, and a baſe giuen being* 216 *inches, it were required to knowe how many inches it ſhall haue in height. The fourth proportionall would be found to be* 8.

This laſt propoſition of finding a fourth proportionall number, may be wrought alſo by the lines of *Superficies*, and by the lines of *Solids*.

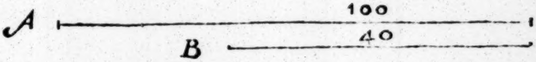
CHAP. III.

CHAP, III.

The vſe of the lines of Superficies.

1 *To finde a proportion betweene two or more like* Superficies.

TAke one of the ſides of the greater *Superficies* giuen,and according to it open the *Sector* in the points of 100 and 100, in the lines of *Superficies*, then take the like ſides of the leſſer *Superficies* ſeuerally, and carry them parallell to the former, till they ſtay in like points, ſo the number of points wherein they ſtay, ſhall ſhew their proportion vnto 100.



Let *A* and *B*, be the ſides of like *Superficies*, as the ſides of two ſquares, or the diameters of two circles, firſt I take the ſide *A*, and to it open the *Sector* in the points of 100, then keeping the *Sector* to this angle, I enter the leſſer ſide *B*, parallell to the former, and finde it to croſſe the lines of *Superficies* in the points of 40, wherefore the proportion of the *Superficies*, whoſe ſide is *A*, to that whoſe ſide is *B*, is as 100 vnto 40, which is in leſſer numbers, as 5 vnto 2.

This propoſition might haue beene wrought by *60*, or any other number that admits ſeuerall diuiſions. It may alſo be wrought without opening the *Sector*, for if the ſides of the *Superficies* giuen, be applied to the lines of *Superficies* beginning alwayes at the center of the *Sector*, there will be ſuch proportion found betweene them, as betweene the number of parts whereon they fall.

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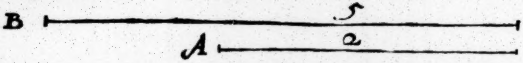
2 *To augment a Superficies in a giuen Proportion.*
3 *To diminiſh a Superficies in a giuen Proportion.*

TAke the ſide of the *Superficies*, and to it open the *Sector* in the points of the numbers giuen; then keeping the *Sector* at that angle, the parallell diſtance between the points of the number required, ſhall giue the like ſide of the *Superficies* required.



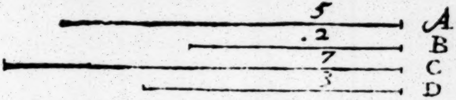
Let *A* be the ſide of a Square to be augmented in the proportion of 2 to 5. Firſt I take the ſide *A*, and put it ouer in the lines of *Superficies*, in 2 and 2; ſo the parallel between 5 and 5, doth giue me the ſide *B*, on which if I ſhould make a Square, it would haue ſuch proportion to the ſquare of *A*, as 5 vnto 2.

In like maner if *B* were the ſemidiameter of a circle to be diminiſhed in the proportion of 5 vnto 2, I would take out *B*, and put it ouer in the lines of *Superficies*, in 5 and 5; ſo the parallell betweene 2 and 2, would giue me *A*; on which Semidiameter if I ſhould make a circle, it would be leſſe then the circle made vpon the Semidiameter *B*, in ſuch proportion as 2 is leſſe then 5.

For varietie of worke the like caution may be here obſerued to that which we gaue in the third *Prop.* of *Lines*.

4 *To adde one like Superficies to another.*
5 *To ſubtract one like Superficies from another.*

FIrſt, the proportion betweene like ſides of the *Superficies* giuen, is to be found by the firſt *Prop.* of *Superficies*, then adde or ſubtract the numbers of thoſe proportions, and accordingly augment or diminiſh by the former *Prop.*

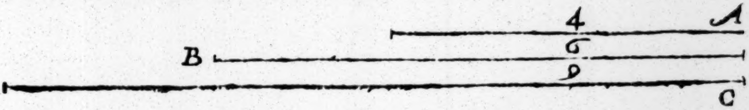


As if *A* and *B* were the ſide of two Squares, and it were required to make a third Square equall to them both. Firſt the proportion betweene the ſquares of *A* and *B*, would be found to be as 100 vnto 40, or in the leſſer numbers as 5 to 2; then becauſe 5 and 2 added do make 7, I augment the ſide *A* in the proportion of 5 to 7, and produce the ſide *C*, on which if I make a ſquare, it will be equall to both the ſquares of *A* and *B*, which was required.

In like maner *A* and *B* being the ſides of two Squares, if it were required to ſubtract the ſquare of *B* out of the ſquare of *A*, and to make a ſquare equall to the remainder, here the proportion being as 5 to 2, becauſe 2 taken out of 5, the remainder is 3, I would diminiſh the ſide *A* in the proportion of 5 to 3, and ſo I ſhould produce the ſide *D*, on which if I make a ſquare, it will be equall to the remainder when the ſquare of *B* is taken out of the ſquare of *A*, that is, the two ſquares made vpon *B* and *D*, ſhall be equall to the firſt ſquare made vpon the ſide *A*.

4 *To find a meane proportionall betweene two lines giuen.*

First find what proportion is betweene the lines giuen, as they are lines, by the fifth *Prop.* of *Lines*, then open the *Sector* in the lines of *Superficies*, according to his number, to the quantitie of the one, and a parallell taken betweene the points of the number belonging to the other line ſhall be the meane proportionall.



Let the lines giuen be *A* and *C*. The proportion between them as they are lines wil be found by the fifth *Prop.* of *Lines* to be as 4 to 9. Wherefore I take the line C, and put it ouer in the lines of *Superficies* betweene 9 and 9, and keeping the *Sector* at this angle, his parallell betweene 4 and 4 doth giue me *B* for the meane proportionall. Then for proofe of the operation I may take this line B, and put it ouer between 9 and 9: ſo his parallel between 4 and 4 ſhall giue me the firſt line A. Whereby it is plaine that theſe three lines do hold in continuall proportion; and therefore B is a meane proportionall betweene A and C the extremes giuen.

Vpon the finding out of this meane proportion depend many Corollaries, as

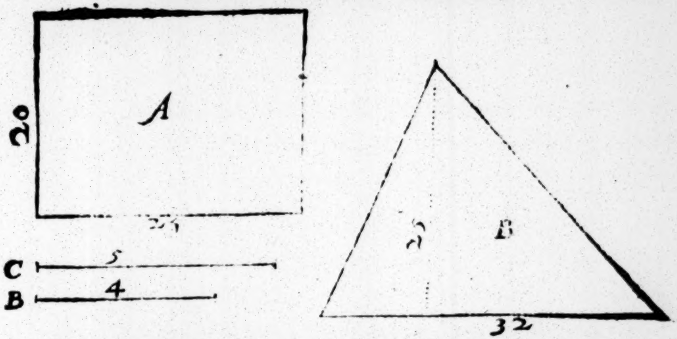
To make a Square equall to a Superficies giuen.

IF the *Superficies* giuen be a rectangle parallellogram, a meane proportionall betweene the two vnequall ſides ſhall be the ſide of his equall ſquare.

If it ſhall be a triangle, a meane proportion betweene the perpendicular and halfe the baſe ſhal be the ſide of his equal ſquare. If it ſhall be any other right-lined figure, it may be reſolued into triangles, and ſo a ſide of a ſquare found equall to euery triangle; and theſe being reduced into one equall ſquare, it ſhall be equall to the whole right-lined figure giuen.

To finde a proportion betweene Superficies, though they be vnlike one to the other.

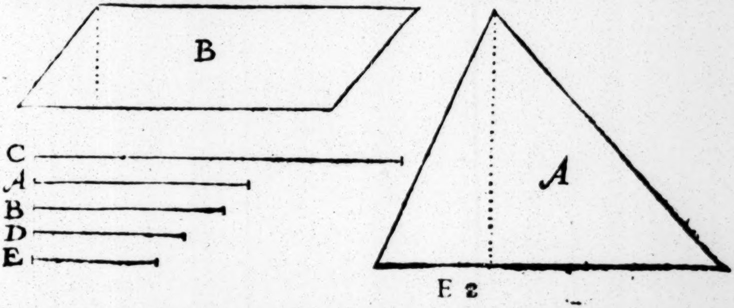
IF to euery *Superficies* we find the ſide of his equall ſquare, the proportion betweene theſe ſquares, ſhall be the proportion betweene the *Superficies* giuen.



Let the *Superficies* giuen, be the oblonge *A*, and the triangle *B*. Firſt between the vnequall ſides of *A*, I find a meane proportionall, and note it in *C*: this is the ſide of a ſquare equall vnto *A*. Then betweene the prependicular of B, and halfe his baſe, I finde a meane proportionall, and note it in B: this is the ſide of a Square equall to *B*: but the proportion between the ſquares of *C* and B, will be found by the firſt *Prop.* of *Superficies* to be as 5 to 4: and therefore this is the proportion betwene thoſe giuen *Superficies*.

To make a Superficies like to one Superficies and equall to another.

LEt the one *Superficies* giuen be the *triangle* A, and the other the *Rhomboides* B; and let it be required to make an-



other *Rhomboides* like to B, and equall to the *triangle* A.

Firſt between the perpendicular and the baſe of *B*, I find a meane proportionall, and note it in *B*, as the ſide of his equall ſquare: then betweene the perpendicular of the triangle *A*, and halfe his baſe, I find a meane proportionall, and note it in *A*, as the ſide of his equall ſquare. Wherefore now as the ſide *B* is to the ſide *A*, ſo ſhall the ſides of the Rhomboides giuen be to *C* and *D*, the ſides of the Romboides required, & his pendicular alſo to *E*, the perpēdicular required.

Hauing the ſides and the perpendicular, I may frame the *Rhomboides* vp, and it will be equall to the triangle *A*.

If the *Superficies* giuen had been any other right-lined figures, they might haue been reſolued into triangles, and then brought into ſquares as before.

Many ſuch Corollaries might haue been annexed, but the meanes of finding a meane proportionall being knowne, they all follow of themſelues.

7 *To finde a meane proportionall betweene two numbers giuen.*

First reckon the two numbers giuen on both ſides of the Lines of *Superficies*, from the center, and mark the termes whereunto they extend; then take a line out of the Line of *Lines*, or any other ſcale of equall parts reſembling one of thoſe numbers giuen, and put it ouer in the termes of his like number in the lines of *Superficies*; for ſo keeping the *Sector* at this angle, the parallell taken from the termes of the other number and meaſured in the ſame ſcale from which the other parallell was taken, ſhall here ſhew the meane proportionall which was required.

Let the numbers giuen be 4 and 9. If I ſhall take the line *A*, in the *Diagram* of the ſixt *Prop.* reſembling 4 in a ſcale of equall parts, and to it open the *Sector* in the termes of 4 and 4, in the lines of *Superficies*, his parallell betweene 9 and 9 doth giue me *B* for the meane proportionall. And this meaſured in the ſcale of equall parts doth extend to 6, which is the meane proportionall number between 4 and 9.

For as 4 to 6, ſo 6 to 9.

In like maner if I take the line *C*, reſembling 9 in a ſcale of equall parts, and to it open the *Sector* in the termes of 9 and 9, in the lines of *Superficies*, his parallell between 4 and 4 doth giue me the ſame line *B*, which will proue to be 6, as before, if it be meaſured in the ſame ſcale whence *D* was taken.

8 *To find the ſquare roote of a number.*

9 *The roote being giuen to find the ſquare number of that roote.*

IN the extraction of a ſquare roote it is vſuall to ſet pricks vnder the firſt figure, the third, the fifth, the ſeuenth, and ſo forward, beginning from the right hand toward the left, and as many pricks as fall to be vnder the ſquare number giuen, ſo many figures ſhall be in the roote: ſo that if the number giuen be leſſe then 100, the roote ſhall be only of one figure; if leſſe then 10000, it ſhall be but two figures; if leſſe then 1000000, it ſhall be three figures, &c.

Thereupon the lines of *Superficies* are diuided firſt into an hundred parts, and if the number giuen be greater then 100, the firſt diuiſion (which before did ſignifie only one) muſt ſignifie 100, and the whole line ſhall be 10000 parts: if yet the number giuen be greater then 10000, the firſt diuiſion muſt now ſignifie 10000, and the whole line be eſteemed at 1000000 parts: and if this be too little to expreſſe the number giuen, as oft as we haue recourſe to the beginning, the whole line ſhall increaſe it ſelfe an hundred times.

By this meanes if the laſt pricke to the left hand ſhall fall vnder the laſt figure, which will be as oft as there be odde figures, the number giuen ſhall fall out betweene the center of the *Sector* and the tenth diuiſion: but if the laſt prick ſhall fall vnder the laſt figure but one, which will be as oft as there be euen figures, then the number giuen ſhall fall out betweene the tenth diuiſion and the end of the *Sector*.

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This being conſidered, when a number is giuen and the ſquare roote is required, take a paire of compaſſes and ſetting one foote in the center, extend the other to the terme of the number giuen in one of the lines of *Superficies*; for this diſtance applied to one of the Lines of *Lines*, ſhall ſhew what the Square roote is, without opening the *Sector*.

Thus 64 doth giue a roote of 8, and 860 a roote of almoſt 19, and 1296 a roote of 36, and 7056 a roote of 84, and 62500 a roote of 250, and 714000 a roote of about 845, and ſo in the reſt.

On the contrary, a number giuen may be ſquared, if firſt we extend the compaſſes to the number giuen in the lines of *Lines*, and then apply the diſtance to the *Lines* of *Superficies*, as may appeare by the former examples.

10 *Three numbers being giuen to find the fourth in a duplicated proportion.*

IT is plaine by the 19 and 20 *Prop.6.Lib.* of *Euclid*, that like *Superficies* do hold in a duplicated proportion of their homologall ſides, whereupon a queſtion being moued concerning *Superficies* and their ſides. It is vſuall in Arithmeticke that the proportion be firſt duplicated before the queſtion be reſolued, which is not neceſſarie in the vſe of the *Sector*, only the numbers which do ſignifie *Superficies* muſt be reckoned in the lines of *Superficies*, and they which ſignifie the ſides of *Superficies*, in the lines of *Lines*, after this maner.

If a queſtion be made concerning a *Superficies*, the two numbers of the firſt denomination muſt be reckoned in the lines of *Lines*, and the *Sector* opened in the termes of the firſt number to the quantitie of a line out of the ſcale of *Superficies* reſembling the ſecond number; ſo his parallels taken betweene the termes of the third number, being meaſured in the ſame ſcale of *Superficies*, ſhall giue the Superficiall number which was required.

As if a Square, whoſe ſide is fortie perches in length, ſhall containe ten acres in the *Superficies*, and it be required to know how many acres the Square ſhould contain, whoſe ſide is ſixtie perches.

Here if I tooke 10 out of the line of *Superficies*, and put it ouer in 40 in the lines of *Lines*, his parallell between 60 and 60 meaſured in the line of *Superficies*, would be 22½; and ſuch is the number of acres required. For Squares do hold in a duplicated proportion of their ſides; wherefore when the proportion of their ſides is as 4 to 6, and 4 multiplied into 4 become 16, and 6 multiplied into 6 become 36, the proportion of their ſquares ſhall be as 16 to 36; and ſuch is the proportion of 10 to 22½.

If a field meaſured with a ſtatute perch of 16½ foote, ſhall containe 288 acres, and it be required to know how many acres it would containe if it were meaſured with a woodland perch of 18 foote.

Here becauſe the proportion is reciprocall, if I tooke 288 out of the line of *Superficies*, and put it ouer in 18, in the lines of *Lines*, his parallell betweene 16½ and 16½ meaſured in the line of *Superficies*, would be 242; and ſuch is the number of acres required.

For ſeeing the proportion of the ſides is as 16 to 18, or in leſſer numbers as 11 to 12, and that 11 multiplied into 11 become 121, and 12 into 12 become 144, the proportion of theſe *Superficies* ſhall be as 121 to 144, and ſo haue 288 to 242, in *reciprocall* proportion.

On the contrary, if a queſtion be propoſed concerning the ſide of a *Superficies*, the two numbers of the firſt denomination muſt be reckoned in the lines of *Superficies*, and the *Sector* opened in the termes of the firſt number, to the quantitie of a line, out of the line of *Lines*, or ſome Scale of equall parts, reſembling the ſecond number; ſo his parallell taken between the termes of the third number being meaſured in the ſame ſcale with the ſecond number, ſhal giue the fourth number required.

As if a field contained 288 acres when it was meaſured with a ſtatute perch of 16½, and being meaſured with another

ther perch, was found to containe 242 acres, it were required to know what was the length of the perch with which it was ſo meaſured.

Here becauſe the proportion is reciprocall, if I tooke 16½ out of the line of *Lines*, and put it ouer in 242 in the lines of *Superficies*, his parallell betweene 288 and 288, being meaſured in the line of *Lines*, would be 18, and ſuch is the length of the perch in foote whereby the field was laſt meaſured.

For ſeeing the proportion of the acres is as 288 vnto 242, or in the leaſt numbers as 144 to 121, and that the roote of 144 is 12, and the root of 121 is 11, the proportion of roots and conſequently of the perches ſhall be as 12 to 11, and ſo are 16½ to 18, in *reciprocall* proportion.

If 360 men were to be ſet in forme of a long ſquare, whoſe ſides ſhall haue the proportion of 5 to 8; and it were required to know the number of men to be placed in front and file: if the ſides were onely 5 and 8, there ſhould be but 40 men; but there are 360: therefore working as before, I find that

As 40 to the ſquare of 5,
ſo 360 to the ſquare of 15.

As 40 to the ſquare of 8,
ſo 360 to the ſquare of 24.

and ſo 15 and 24 are the ſides required.

If 1000 men were lodged in a ſquare ground, whoſe ſide were 60 paces, and it were required to know the ſide of the ſquare wherein 5000 might be ſo lodged, here working as before, I ſhould find that

As 1000 are to the ſquare of 60:
ſo 5000 to the ſquare of 134.

And ſuch very neare is the number of paces required.

CHAP.

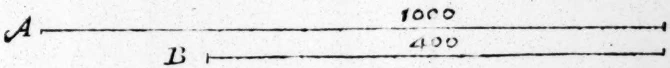
CHAP. IV.

The vſe of the lines of Solids.

To finde a proportion betweene two or more like Solids.

IN the Sphere, in regular, parallell, and other like bodies, whoſe ſides next the equall angles are proportionall, the worke is in a manner the ſame, with that in the firſt *Prop.* of *Superficies*, but that it is wrought on other lines.

Take one of the ſides of the greater *Solid*, & according to it open the *Sector* in the points of 1000 and 1000, in the lines of *Solids*, then take the like ſides of the leſſer *Solids* ſeuerally, and carry them parallell to the former, till they ſtay in like points, ſo the number of points wherein they ſtay, ſhall ſhew their proportion to 1000.



Let *A* and *B*, be the like ſides of like Solids, either the diameters, or ſemidiameters of two ſpheres, or the ſides of two cubes, or other like. Firſt I take the ſide *A*, and to it open the *Sector* in the points of 1000, then keeping the *Sector* at this angle, I enter the leſſer ſide B, parallell to the former, and finde it to croſſe the line of *Solids* in the points of 400, and ſuch is the proportion betweene the Solids required, which in leſſer number is as 5 to 2.

This propoſition might haue beene wrought by 60, or any other number that admits ſeuerall diuiſions.

It may alſo be wrought without opening the *Sector*, for if the ſides of the Solids giuen, be applied to the lines of *Solids*, begining allwayes at the center of the *Sector*, there will be ſuch proportion betweene them, as betweene the numbers of parts whereon they fall.

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2 *To augment a Solid in a giuen proportion.*
3 *To diminiſh a Solid in a giuen proportion.*

TAke the ſide of the Solid giuen, and to it open the *Sector*, in the points of the number giuen: then keeping the Sector at that angle, the parallell diſtance betweene the points of the number required, ſhall giue the like ſide of the Solid required.

If it be a *paralleleopipedon*, or ſome irregular Solid, the other like ſides may be found out in the ſame manner, and with them the Solids required, may be made vp with the ſame angles.

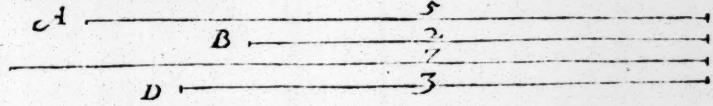
Let *A* be the ſide of a cube, to be augmented in the proportion of 2 to 3. Firſt I take the ſide *A*, and put it ouer in the lines of *Solids* in 2 and 2, ſo the parallell betweene 3 and 3, doth giue me the ſide B, on which if I make a cube, it will haue ſuch proportion to the cube of *A*, as 3 to 2.

In like manner, if B were the diameter of a Sphere, to be diminiſhed in the proportion of 3 to 2. I would take out B, and put it ouer in the lines of *Solids*, in 3 and 3, ſo the parallell betweene 2 and 2, would giue me *A*: to which diameter if I ſhould make a Sphere, it would be leſſe then the Sphere, whoſe diameter is B, in ſuch proportion as 2 is leſſe then 3.

Here alſo for variety of worke, may the like caution be obſerued to that which we gaue in the third *Prop.* of *Lines*.

4 *To adde one like Solid to another.*
5 *To ſubtract one like Solid from another.*

FIrſt the proportion betweene the ſides of the like Solids giuen, is to be found by the firſt *Prop.* of *Solids*: then adde or ſubtract thoſe proportions, and accordingly augment or diminiſh by the former *Prop.*



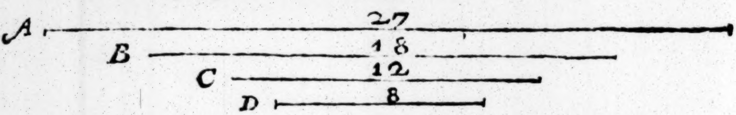
As if *A* and *B* were the ſides of two cubes, and it were required to make a third cube equall to them both: firſt the proportion betweene the ſides *A* and *B*, would be found to be as 100 to 40, or in leſſer termes as 5 to 2. Then becauſe 5 and 2 being added do make 7, I augment the ſide *A* in the proportion of 5 to 7, and produce the ſide *C*, on which if I make a cube, it will be equall to both the cubes of *A* and *B*, which was required.

In like maner *A* and *B* being the ſides of two cubes, if it were required to ſubtract the cube of B out of the cube of A, and to make a cube equal to the remainder. Here the proportion being as 5 to 2, becauſe 2 taken out of 5, the remainder is 3, I ſhould diminiſh the ſide A in the proportion of 5 to 3, and ſo I ſhould haue the ſide D, on which if I make a cube, it will be equall to the remainder when the cube of B is taken out of the cube of A, that is the two cubes made vpon B and D, ſhall be equall to the firſt cube made vpon the ſide A.

6 *To find two meane proportionall lines betweene two extreme lines giuen.*

FIrſt I find what proportion is betweene the two extreme lines giuen as they are lines, by the fifth *Prop.* of *Lines*, then open the *Sector* in the lines of *Solids*, to the quantitie of the former extreme, and a parallell betweene the points of the number belonging to the other extreme, ſhall be that meane proportionall which is next the former extreme. This done, open the *Sector* againe to this meane proportionall in the points of the former extreme, and the parallell diſtance be-

betweene the points of the latter extreme, ſhall be the other meane proportionall required.

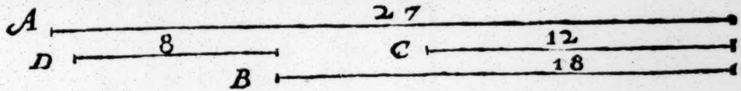


Let the two extreme lines giuen be A and D, the proportion betweene them, as they are lines, will be found to be as 27 to 8. Wherefore I take the line A, and put it ouer in the lines of *Solids* betweene 27 and 27, and keeping the *Sector* at this angle, his parallell betweene 8 and 8, doth giue me B, the meane proportionall next vnto A. Then put I ouer this line B, betweene the aforeſaid 27 and 27, and his parallell betweene 8 and 8 doth giue me the line C, the other meane proportionall which was required.

Againe, for proofe of the operation I put ouer this line C in the aforeſaid 27 and 27, and his parallell between 8 and 8 doth giue me the very line D: whereby it is plaine that theſe foure lines do hold in continuall proportion; and ſo B and C are found to be the meane proportionals betweene A and D the extremes giuen.

7 *To find two meane proportionall numbers between two extreme numbers giuen.*

FIrſt reckon the numbers giuen on both ſides of the lines of *Solids*, beginning from the center, and marking the termes whereto they extend: then take a line out of the line of *Lines*, or any other ſcale of equall parts reſembling the former of thoſe numbers, and put it ouer in the lines of *Solids*, betweene the points of his like number, and a parallell betweene the points belonging to the other extreme, meaſured in the ſcale from whence the other parallell was taken, ſhall giue that meane proportionall number which is next the former extreme. This done, open the *Sector* againe to this meane proportionall in the points of the former extreame, and the parallell diſtance betweene the points of the latter extreame, meaſured in the ſame ſcale as before, ſhall there ſhew the other meane proportionall required.



Let the two extreame numbers giuen be 27 and 8; if I ſhall take the line A, reſembling 27 in a ſcale of eqnall parts, and to it open the *Sector* in 27 and 27, in the line of *Solids*, his parallell betweene 8 and 8 doth giue me B for his next meane proportionall, and this meaſured in the former ſcale doth extend to 18. Then put I ouer this line B between the aforeſaid 27 and 27, and his parallell between 8 and 8 doth giue me C for the other meane proportionall, and this meaſured in the former ſcale doth extend to 12. Againe, for proofe of my worke, I put ouer this line C betweene 27 and 27, as before, and his parallell betweene 8 and 8 doth giue me D, which meaſured in the former ſcale doth extend to 8, which was the latter extreame number giuen; whereby it is plaine that theſe foure numbers do hold in continuall proportion: and therefore 18 and 12 are meane proportionals betweene 27 and 8, which was required.

8 *To finde the cubique roote of a number.*
9 *The roote being giuen to finde the cube number of that roote.*

IN the extraction of a cubique root, it is vſuall to ſet pricks vnder the firſt figure, the fourth, the ſeuenth, the tenth, and ſo forward, omitting two, and pricking the third from the righthand toward the left; and as many pricks as fall to be vnder the cubique number, ſo many figures ſhall be in the roote. So that if the number giuen be leſſe then 1000, the roote ſhall be only of one figure; if leſſe then 1000000, it ſhall be but of two figures; if aboue theſe, and leſſe then 1000000000, it ſhall be but three figures; &c. whereupon the lines of *Solids* are diuided, firſt into 1000 parts, and if the numbers giuen be greater thē 1000, the firſt diuiſion (wheih before did ſignifie onely one) muſt ſignifie 1000, and the whole line ſhall be 1000000: if yet the number giuen be greater then 1000000, the firſt diuiſion muſt now ſignifie 1000000, and the whole line be eſteemed at 1000000000 parts, and if theſe be to little to expreſſe the numbers giuen, as oft as wee haue recourſe to the begining, the whole line ſhall encreaſe it ſelfe a thouſand times.

By theſe meanes, if the laſt pricke, to the left hand, ſhall fall vnder the laſt figure, the number giuen ſhall be reckoned at the beginning of the lines of *Solids*, from 1 to 10, and the firſt figure of the roote ſhall be alwayes either 1, or 2. If the laſt pricke ſhall fall vnder the laſt figure but one, then the number giuen ſhall be reckoned in the middle of the line of *Solids*, betweene 10 and 100, and the firſt figure of the roote ſhall be alwayes either 2, or 3, or 4. But if the laſt pricke ſhall fall vnder the laſt figure but two, then the number giuen, ſhall be reckoned at the end of the line of Solids, betweene 100, and 1000.

This being cōſidered when a number is giuen, and the cubique roote required: Set one foote of the compaſſes in the center of the *Sector*, extend the other in the line of *Solids*, to the points of the number giuen: for this diſtance applied to one of the line of *Lines*, ſhall ſhew what the cubique roote is, without opening the *Sector*.

So the neareſt roote of 8490000, is about 204.
The neareſt roote of 84900000, is about 439.
The neareſt roote of 849000000, is about 947.

On the contrary, a number may be cubed, if firſt we extend the compaſſes to the number giuen, in the line of *Lines*, and then apply the diſtance to the lines of *Solids*; as may appeare by the former examples.

10 *Three numbers being giuen to finde a fourth in a triplicated proportion.*

AS like *Superficies* do hold in a duplicated proportion, ſo like ſolids in a triplicated proportion of their homologall ſides: and therefore the ſame worke is to be obſerued here on the lines of *Solids*, as before in the lines of *Superficies*; as may appeare by theſe two examples.

If a cube whoſe ſide is 4 inches, ſhall be 7 pound weight, and it be required to know the weight of a cube whoſe ſide is 7 inches; here the proportion would be,

As 4 are to a cube of 7:
ſo 7 to a cube of $37\frac{1}{2}$.

And if I tooke 7 out of the lines of *Solids*, and put it ouer in 4 and 4, in the lines of *Lines*, his parallell between 7 and 7 meaſured in the lines of *Solids*, would be $37\frac{1}{2}$; and ſuch is the weight required.

If a bullet of 27 pound weight haue a diameter of 6 inches, and it be required to know the diameter of the like bullet, whoſe weight is 125 pounds; here the proportion would be,

As the cubique root of 27 is vnto 6:
ſo the cubique root of 125 is vnto 10.

And if I tooke 6 out of the line of *Lines*, and put it ouer in 27 and 27 of the lines of *Solids*, his parallell betweene 125 and 125 meaſured in the line of *Lines*, would be 10; and ſuch is the length of the diameter required.

The end of the firſt booke.

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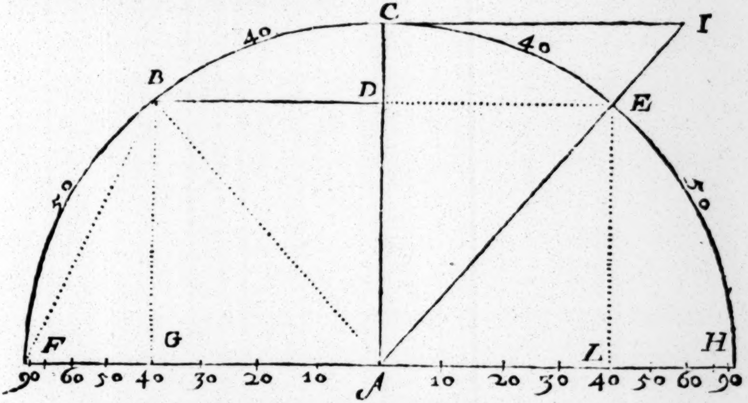
THE SECOND BOOKE OF THE *SECTOR*, Containing the vſe of the Circular *Lines*.

CHAP. I.

Of the nature of Sines, Chords, Tangents *and* Secants, *fit to be knowne before hand in reference to right-line Triangles.*

IN the ***Canon*** of ***Triangles***, a circle is commonly diuided into 360 ***degrees***, each ***degree*** into 60 ***minutes***, each ***minute*** into 60 ***ſeconds***.



A ſemicircle therefore is an arke of 180 gr.

A quadrant is an arke of 90 *gr.*

The meaſure of an angle is the arke of a circle, deſcribed out of the angular point, intercepted betweene the ſides ſufficiently produced.

So the meaſure of a right angle is alwayes an arke of 90 *gr.* and in this example the meaſure of the angle *B A D* is the arke *B C* of 40 *gr*; the meaſure of the angle *B A G*, is the arke *B F* of 50 *gr.*

The complement of an arke or of an angle doth commōly ſignifie that arke which the giuen arke doth want of 90 *gr*: and ſo the arke *B F* is the cōplement of the arke *B C*; & the angle *B A F*, whoſe meaſure is *B F*, is the complement of the angle *B A C*; and on the contrary.

The complement of an arke or angle in regard of a ſemicircle, is that arke which the giuen arke wanteth to make vp 180 *gr*: and ſo the angle *E A H* is the complement of the angle *E A F*, as the arke *E H* is the complement of the arke *F E*, in which the arke *C E* is the exceſſe aboue the quadrant.

The proportions which theſe arkes (being the meaſures of angles) haue to the ſides of a triangle, cannot be certaine, vnleſſe that which is crooked be brought to a ſtraight line; and that may be done by the application of *Chords*, *Right Sines*, *verſed Sines*, *Tangents* and *Secants*, to the ſemidiameter of a circle.

A *Chorde* is a right line ſubtending an arke: ſo *B E* is the chorde of the arke *B C E*, and *B F* a chorde of the arke *B F*.

A *right Sine* is halfe the chorde of the double arke, viz. the right line which falleth perpendicularly from the one extreme of the giuen arke, vpon the diameter drawne to the other extreme of the ſaid arke.

So if the giuen arke be *B C*, or the giuen angle be *B A C*, let the diameter be drawne through the center *A* vnto *C*; and a perpendicular *B D* be let downe from the extreme *B*, vpon *A C*; this perpendicular *B D* ſhall be the *right ſine* both of the arke *B C*, and alſo of the angle *B A C*: and it is alſo the halfe of the chord B E, ſubtending the arke B C E, which is double to the giuen arke B C. In like maner, the ſemidiameter F A, is the *right ſine* of the arke F C, and of the right angle F A C; for it falleth perpendicularly vpon A C, and it is the halfe of the chord F H.

This whole Sine of 90 *gr.* is hereafter called *Radius*; but the other *Sines* take their denomination from the degrees and minutes of their arks.

Sinus verſus, the *verſed ſine* is a ſegment of the diameter, intercepted betweene the *right ſine* of the ſame arke, and the circumference of the circle. So D C is the *verſed ſine* of the arke C B, and G F the *verſed ſine* of the arke B F, and G H the *verſed ſine* of the arke B H.

A *Tangent* is a right line perpendicular to the diameter, drawne by the one extreme of the giuen arke, and terminated by the *ſecant* drawne from the center through the other extreme of the ſaid arke.

A *Secant* is a right line drawne from the center, through one extreme of the giuen arke, till it meete with the *tangent* raiſed from the diameter at the other extreme of the ſaid arke.

So if the giuen arke be C E, or the giuen angle be C A E, let the diameter be drawne through the center A to C, and in C to A C, be raiſed a perpendicular C I. Then let another line be drawne from the center A through E, till it meet with the perpendicular C I in I; the line C I is a *Tangent*, and A I is the *Secant* both of the arke C E, and of the angle C A E.

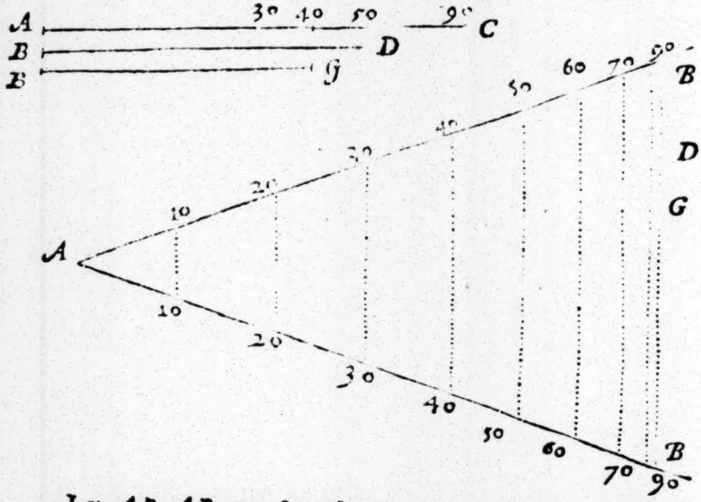
CHAP. II.

Of the generall vſe of Sines *and* Tangents.

1 *The Radius being knowne to find the right ſine of any arke or angle.*

IF the Radius of the circle giuen be equall to the laterall Radius, that is, to the whole line of *Sines* on the *Sector*, there needs no farther worke, but to take the other ſines alſo out of the ſide of the *Sector*. But if it be either greater or leſſer, then let it be made a parallell Radius, by applying it ouer in the lines of *Sines*, betweene 90 and 90; ſo the parallell taken from the like laterall ſines, ſhall be the *ſine* required.

As if the giuen Radius be *A C*, and it were required to find the ſine of 50 *Gr*. & his complement agreeable to that radius.



Let *A B, A B* repreſent the lines of *ſines* on the *Sector*, and let B B, the diſtance betweene 90 and 90, be equall to the giuen radius *A C*. Here the lines *A* 40, *A* 50, *A* 90, may be called the *laterall ſines* of 40,50,& 90; in regard of their place on the ſides of the *Sector*. The lines betweene 40 and 40, betweene 50 and 50, betweene 90 and 90, may be called the *parallell ſines* of 40,50,and 90; in regard they are parallell one to the other. The whole ſine of 90 *Gr.* here ſtanding for the ſemidiameter of the circle, may be called the Radius. And therefore if *A C* be put ouer in the line of *Sines* in 90 and 90, and ſo made a *parallell radius*, his parallell ſine betweene 50 and 50, ſhall be B D, the ſine of 50 required. And becauſe 50 taken out of 90, the complement is 40; his *parallell ſine* betweene 40 and 40, ſhall be B G, the ſine of the complement which was required.

2 *The right ſine of any arke being giuen to finde the Radius.*

TVrne the ſine giuen into a parallell ſine, and his parallell *Radius* ſhall be the *Radius* required.

As if B D were the giuen ſine of 50 *Gr.* and it were required to finde the Radius: let B D be made a parallell ſine of 50 *Gr.* by applying it ouer in the lines of *Sines*, betweene 50 and 50; ſo his parallell Radius betweene 90 and 90 ſhall be A C, the Radius required.

3 *The Radius of a circle, or the right Sine of any arke being giuen, and a ſtreight line reſembling a Sine, to find the quantitie of that vnknowne* Sine.

LEt the Radius or right ſine giuen be turned into his parallell; then take the right line giuen, and carrie it parallell to the former, till it ſtay in like *Sines*: ſo the number of degrees and minutes where it ſtayeth, ſhall giue the quantitie of the Sine required.

As if B D were the giuen ſine of 50 *Gr.* and B G the ſtreight line giuen: firſt I make B D a parallell ſine of 50 *Gr*; then keeping the *Sector* at this angle, I carie the line B G pa-

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parallell, and find it to ſtay in no other but 40 and 40; and therefore 40 *gr.* is his quantitie required.

4 *The Radius or any right Sine being giuen, to finde the verſed ſine of any arke.*

IF the arke, whoſe *verſed ſine* is required, be leſſe then the quadrant, take the ſine of the complement out of the radius, and the remainder ſhall be the *ſinus verſus*, the verſed ſine of that arke.

As if A B being the laterall *Radius*, it were required to find to find the verſed ſine of 40 *gr*; here the ſine of the complement is A 50, and therefore B 50 is the *verſed ſine* required. Or if I reckon from B, at the end of the Sector, toward the center, the diſtance from 90 to 80, is the verſed ſine of 10 *gr*; from 90 to 70, the verſed ſine of 20 *gr*; from 90 to 60, is the verſed ſine of 30 *gr*; and ſo in the reſt.

If A D be the giuen *ſine* of 50 *gr.* and it be required to find the *verſed ſine* of 50 *gr*; here becauſe A D is vnequall to the laterall ſine of 50 *gr*, I make it a parallell. And firſt I find the radius A C; then the ſine of the complement A 40, which being taken out of A C, leaueth C 40 for the verſed ſine of 50 *gr.* which was required.

But if the arke, whoſe verſed ſine is required, be greater then the quadrant, his verſed ſine alſo is greater then the *Radius*, by the right ſine of his exceſſe aboue 90 *gr*.

As if A C being the Radius giuen, it were required to find the verſed ſine of 130 *gr:* here the exceſſe aboue 90 *gr.* is 40 *gr*; and therefore the verſed ſine required is equall to the Radius A C and A 40, both being ſet together.

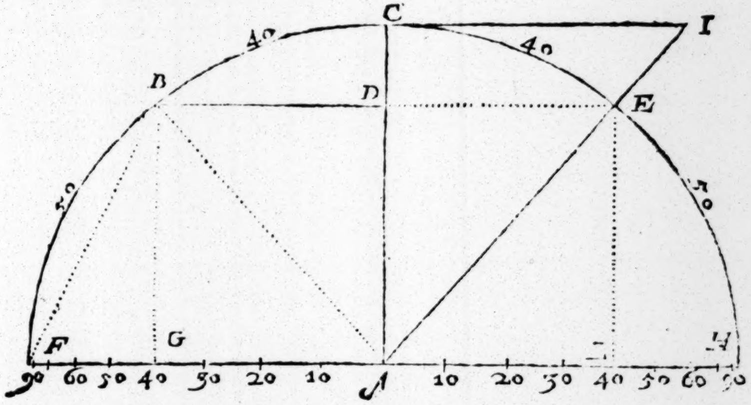
5 *The Diameter or* Radius *being giuen to finde the Chords of euery arke.*

The ſines may be fitted many wayes to ſerue for chords.

1 A *ſine* being the halfe of the *chord* of the double arke, if the *ſine* be doubled, it giueth the *chord* of the double ark,

a *Sine* of 10 *gr.* doubled giueth a *Chord* of 20 *gr*; and a *Sine* of 15 *gr.* being doubled. giueth a *Chord* of 30 *gr*; and ſo in the reſt. As here B D, the ſine of B C, an arke of 40 *gr.* being doubled giueth B E the chord of B C E, which is an arke of 80 *gr.* Wherefore if the Radius of the circle giuen be equall to the laterall Radius, let the *Sector* be opened neare vnto his length, ſo that both the lines of *Sines* may make but one direct line: ſo the diſtance on the ſines betweene 10 and 10, ſhall be a chord of 20; the diſtance betweene 20 and 20, ſhall be a chord of 40; and the diſtance betweene 30 and 30, ſhall be a chord of 60; and ſo in the reſt.

2 Becauſe a ſine is the halfe of the chord of the double arke, the proportion holdeth.



As the diameter F H vnto the radius A H, ſo the chord B E vnto the ſine D E, or the chord G L vnto the ſine A L: and then if the radius A H, be put for the diameter, which is a chord of 180 gr, the ſine D E or A L ſhall ſerue for a chord of 80 *gr*, and the ſemiradius which is the ſine of 30 *gr*, ſhall ſerue for a chord of 60 *gr*, and go for the ſemidiameter of a circle, and ſo in the reſt. So that by theſe meanes we ſhall not need to double the lines of *Sines* as before, but onely to double the numbers. And to this purpoſe I haue ſubdiuided

each

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each degree of the ſines into two, that ſo they might ſhew how far the halfe degrees do reach in the ſines, and yet ſtand for whole degrees when they are vſed as chords.

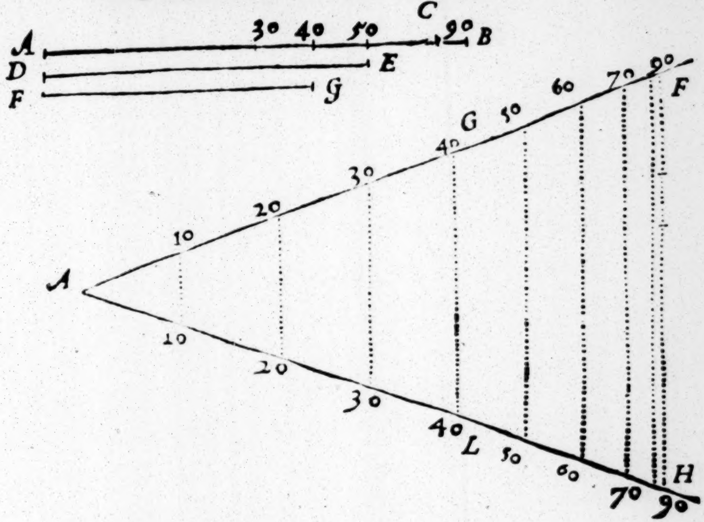
Wherefore if the Radius of the circle giuen be equall to the lateral ſemiradius (the ſine of 30 *Gr.* and chord of 60 *Gr.*) there needs no farther work then to take the ſine of 10 *Gr.* for a chord of 20 *Gr.* and a ſine of 15 *Gr.* for a chord of 30 *Gr.* &c.

But if the Radius of the circle giuen be either greater or leſſer then the laterall ſemiradius, take the diameter of it, and make it a parallell chord of 180 *Gr.* by applying it ouer the lines of *Sines* between 90 and 90: or take the Radius or Semidiameter which is equall to the chord of 60 *Gr.* and make it a parallell Radius of 60 *Gr.* by applying it ouer in the ſines of 30 and 30, and keepe the Sector at this angle. The parallels taken from the laterall *chords* ſhall be the *chords* required.

As if the diameter of a circle giuen were the line *A B*, and it were required to find the chord of 80 gr: firſt I make *A B* a parallell chord of 180 *Gr.* or the halfe of it a parallell chord of 60 *Gr*; ſo his parallell *L G* doth giue me *F G* the chord of 80 *Gr.* which was required.

3 Seeing that as the ſine of the complement of the halfe arke is vnto the *Radius*, ſo the ſine of the ſame whole arke is vnto the chord of it: if we ſeeke but for one ſingle chord, we may finde it without either doubling the ſines, or doubling the number. For applying ouer the Radius giuen in the ſine of the complement of halfe the arke required, his parallell ſine ſhall be the chord required.

As if the ſemidiameter of the circle giuen were *A C*, and it were required to find the chord of 40 *Gr*: the halfe of 40 *Gr.* is 20 *Gr.* the complement of 20 *Gr.* is 70 *Gr.* Wherefore I make *A C* a parallell ſine of 70 *Gr.* and his parallell ſine *G L* doth giue me *F G* the chord of 40 *Gr.* agreeable to the ſemidiameter *A C.*



6 The chord of any arke being giuen to find the diameter and Radius.

TVrne the chord giuen vnto a parallell chord, and his parallell ſemiradius ſhall be the ſemidiameter, and the parallell radius ſhall be the diameter.

As if *F G* be the chord of 80 *gr.* I put this ouer in *G* and *L*, the ſine of 40, and chord of 80 *gr.* and the parallell chord of 180 *gr.* giueth me *A B* the diameter required.

Or if I turne the chord giuen into a parallell ſine of the ſame quantitie, his parallell ſine of the complement of halfe the arke, doth giue me the ſemidiameter.

As if *F G* be the giuen chord of 40 *gr.* I put it ouer in *G* and *L*, the ſines of 40 *gr*; then becauſe the halfe of 40 *gr.* is 20 *gr.* and the complement of 20 *gr.* is 70 *gr.* I take out the parallell ſine of 70 *gr.* and it giueth me *A C* for the ſemidiameter, agreeable to that chord of 40 *gr.*

7 *To open the Sector to the quantitie of any angle giuen.*

8 *The Sector being opened, to find the quantitie of the angle.*

IT is one thing to open the edges of the Sector to an angle, and another thing to open the lines on the Sector to the ſame angle. For the lines of *lines* on the one ſide, & the lines of *ſines* on the other ſide, do make an angle of 2 *gr.* when the Sector is cloſe ſhut, and the edges doe make no angle at all. So likewiſe the lines of *Superficies* and the lines of *Solids* doe make an angle of 10 *gr*, which are to be allowed to the edges.

The lines of *lines* may be opened to a right angle, if the whole line of 100 parts be applied ouer in 80 and 60.

The lines of *ſines* may be opened to a right angle, if the large ſecant of 45 *gr.* be applied ouer in the ſines of 90 *gr.* or if the ſine of 90 *gr.* be applied ouer in the ſines of 45 *gr.* or if the ſine of 45 *gr.* be applied ouer in the ſines of 30 *gr.*

If it be required to open thoſe lines to any other angle, take out the chord thereof, and apply it ouer in the *ſemiradius*, and thoſe lines ſhall be opened to that angle.

As if it were required to open the Sector in the lines of *ſines* to an angle of 40 *gr*, take out the chord of 40 *gr*, and to it open the Sector in the chord of 60 *gr*; ſo ſhall the lines of *ſines* be opened to the angle required. Or if the ſame chord of 40 *Gr.* be applied ouer betweene 50 and 50, in the lines of *lines*, they ſhall alſo be opened to the ſame angle. If it be applied ouer in 25 of the lines of *Superficies*, or 125 in the lines of *Solids*, they alſo ſhall be opened to the ſame angle: becauſe the chord of 60 *Gr.* or ſine of 30 *Gr.* and 50 in the lines of *lines*, and 25 in the lines of *Superficies*, and 125 in the *Solids*, are all of the ſame length with the ſemiradius.

Or if the *Semiradius* be applied ouer betweene the ſine of 30 *Gr.* and the ſine of the complement of the angle required, it will open the lines of *Sines* to that angle.

As if the ſemiradius be applied ouer in the ſines of 30 *Gr.* and the ſine of 50 *Gr.* it ſhall open the lines of *Sines* to an angle of 40 *Gr.*

On the contrary, if the *Sector* be opened to an angle, and it be required to know the quantitie thereof, open the compaſſes to the ſemiradius, and ſetting one foote in the ſine of 30 *Gr.* turne the other toward the other line of *ſines*, and it ſhall fall there in the complement of the angle; if it fall on 50 *Gr.* the angle is 40 *Gr*; if on 60 *Gr.* the angle is 30 *Gr.* &c.

Or take ouer the parallell chord of 60 *Gr.* and meaſure it in the laterall chord, and it ſhall there ſhew the quantitie of the angle. As if the *Sector* being opened to an angle, I ſhould take ouer the parallell of 30 *Gr.* of the ſines, and 60 *Gr.* of the chords, and meaſure it in the laterall chords, find it to be 40 *Gr*; the angle comprehended betweene the lines of *Sines* is 40 *Gr.* but the angle betweene the edges of the *Sector* is 2 *Gr.* leſſe, and therefore but 38 *Gr.*

9 *To finde the quantitie of any angle giuen.*

IF out of the angular point, to the quantitie of the *Semiradius*, be deſcribed an occult arke that may cut both ſides of the angle, the chord of this arke meaſured in the laterall chord, ſhall giue the quantitie of the angle.

Let the angle giuen be *B A C*: firſt I take the *Semiradius* with the compaſſes, and ſetting one foote in *A*, I cut the ſides of the angle in *B* and *C*; then I take the chord *B C*, and meaſure it in the laterall chord, and I find it to be 11 *Gr.* and 15 *M.* and ſuch is the quantitie of the angle giuen.

Or if the arke be deſcribed out of the angular point at any other diſtance, let the ſemidiameter be turned into a pa-

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rallell chord of 60 *Gr.* then take the chord of this arke, and carrie it parallell till it croſſe in like chords: ſo the place where it ſtayeth ſhall giue the quantitie of the angle.

As in the former example, if I make the ſemidiameter *A B* a parallell chord of 60 *Gr.* and then keeping the *Sector* at that angle, carrie the chord *B C* parallell, till it ſtay in like chords; I ſhall finde it to ſtay in no other but 11 *Gr* 15 *M.* and ſuch is the angle *B A C.*

10 *Vpon a right line and a point giuen in it, to make an angle equall to any angle giuen.*

FIrſt out of the point giuen deſcribe an arke, cutting the ſame line: then by the 5. *Prop.* afore, find the chord of the angle giuen agreeable to the ſemidiameter, and inſcribe it into this arke: ſo a right line drawne through the point giuen, and the end of this chord, ſhall be the ſide that makes vp the angle.

Let the right line giuen be *A B*, and the point giuen in it be *A*, and let the angle giuen be 11 *gr.* 15 *m.* Here I open the compaſſes to any ſemidiameter *A B*, (but as oft as I may conueniently to the laterall ſemiradius) and ſetting one foot in *A*, I deſcribe an occult arke *B C*; then I ſeeke out the chord of 11 *gr.* 15 *m.* and taking it with the compaſſes, I ſet one foote in B, the other croſſeth the arke in C, by which I draw the line *A C*, and it makes vp the angle required.

11 *To diuide the circumference of a circle into any parts required.*

IF 360 the meaſure of the whole circumference be diuided by the number of parts required, the quotient giueth the chord, which being found will diuide the circumference.

So a chord of 120 *gr.* will diuide the circumference into 3 equall parts; a chord of 90 *gr.* into 4 parts; a chord of 72 *gr.* into 5 parts; a chord of 60 *gr.* into 6 parts; a chord of 51 *gr.* 26. into 7 parts; a chord of 45 *gr.* into 8 parts; a chord of 40 *gr.*

into 9 parts; a chord of 36 *gr.* into 10 parts; a chord of 32 *gr.* 44 *m.* into 11 parts; a chord of 30 *gr.* into 12 parts.

In like maner if it be required to diuide the circumference of the circle, whoſe ſemidiameter is *A B*, into 32: firſt I take the ſemidiameter *A B*, and make it a parallell chord of 60 *gr*; then becauſe 360 *gr.* being diuided by 32, the quotient will be 11 *gr.* 15 *m.* I find the parallell chord of 11 *gr.* 15 *m.* and this will diuide the circumference into 32.

But here the parts being many, it were better to diuide it firſt into fewer, and after to come ouer it againe. As firſt to diuide the circumference into 4, and then each 4 parts into 8, or otherwiſe, as the parts may be diuided.

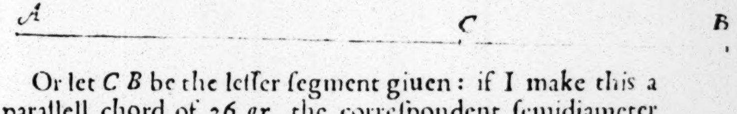
12 *To diuide a right line by extreme and meane proportion.*

THe line to be diuided by extreme and meane proportion, hath the ſame proportion to his greater ſegment, as in figures inſcribed in the ſame circle, the ſide of an *hexagon* a figure of ſix angles, hath to a ſide of a *decagon* a figure of ten angles: but the ſide of a *hexagon* is a chord of 60 *gr.* and the ſide of a *decagon* is a chord of 36 *gr.*

Let *A B* be the line to be diuided: if I make *A B* a parallell chord of 60 *gr.* and to this ſemidiameter find *A C* a chord of 36 *gr.* this *A C* ſhall be the greater ſegment, diuiding the whole line in *C*, by extreme and meane proportion. So that,

As *A B* the whole line is vnto *A C* the greater ſegment:
ſo *A C* the greater ſegment vnto *C B* the leſſer ſegment.

Or let *A C* be the greater ſegment giuen: if I make this a parallell chord of 36 *gr.* the correſpondent ſemidiameter ſhall be the whole line *A C*, and the difference *C B* the leſſer ſegment.



Or let *C B* be the leſſer ſegment giuen: if I make this a parallell chord of 36 *gr.* the correſpondent ſemidiameter ſhall be greater ſegment *A C*, which added to C B, giueth the whole line A B.

To auoid doubling of lines or numbers, you may put ouer the whole line in the *Sines* of 72 *gr.* and the parallell ſine of 36 *gr.* ſhall be the greater ſegment.

Or if you put ouer the whole line in the ſines of 54 *gr.* the parallell ſine of 30 *gr.* ſhall be the greater ſegment, and the parallell ſine of 18 *gr.* ſhall be the leſſer ſegment.

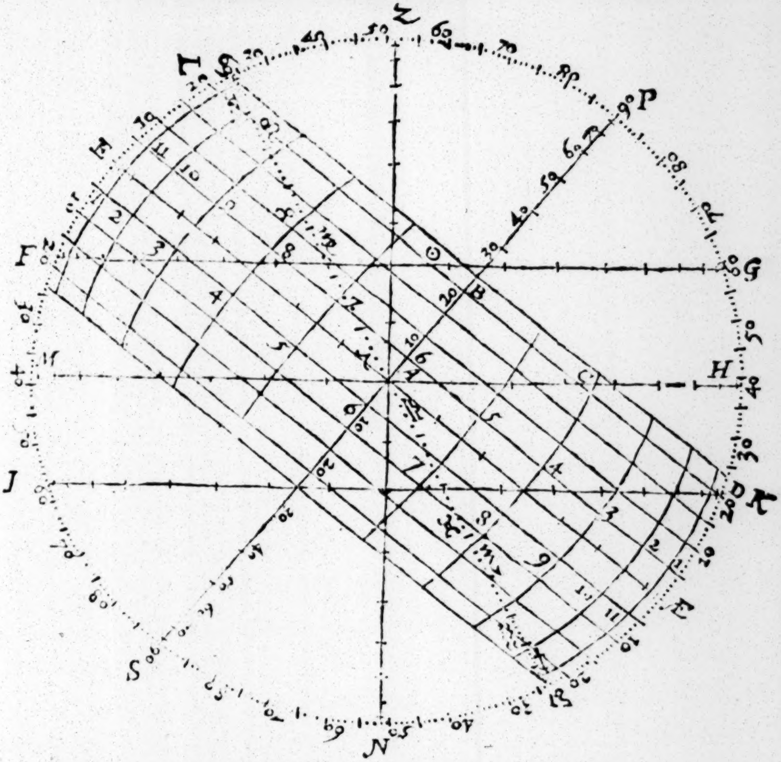
CHAP. III.

Of the proiection of the Sphere in Plano.

THe Sphere may be proiected in *Plano* in ſtreight lines, as in the *Analemma*, if the ſemidiameter of the circle giuen be diuided in ſuch ſort as the line of *Sines* on the Sector.

As if the Radius of the circle giuen were *A E*, the circle thereon deſcribed may repreſent the plane of the generall meridian, which diuided into foure equal parts in *E*, *P*, *Æ*, *S*, and croſſed at right angles with *E Æ* and *P S*, the diameter *E Æ* ſhall repreſent the equator, and *P S* the circle of the houre of *6*. And it is alſo the axis of the world, wherein *P* ſtands for the North pole, and *S* for the South pole. Then may each quarter of the meridian be diuided into *90 gr.* from the equator towards the poles. In which if we number 23 *gr.* 30 *m.* the greateſt declination of the Sun from *E* to *69* Northwards, from *Æ* to ♑ Southward, the line drawne from *69* to ♑ ſhal be the ecliptique, and the lines drawne parallell to the equator through ♋ and ♑ ſhall be the tropiques.

Hauing theſe common ſections with the plane of the meridian, if we ſhall diuide each diameter of the Ecliptique into *90 gr.* in ſuch ſort as the *Sines* are diuided on the Sector. The firſt 30 *gr.* from *A* toward *69*, ſhall ſtand for the ſine of ♈. The 30 *gr.* next following for ♉. The reſt for ♊. ♋. ♌. &c. in their order. So that by theſe meanes we haue the place of the Sun for all times of the yeare.



If againe we diuide *A P*, *A S*, in the like ſort, and ſet to the numbers 10. 20. 30. &c. vnto 90 *gr.* the lines drawne through each of theſe degrees parallell to the equator, ſhall ſhew the declination of the Sunne, and repreſent the parallels of latitude.

If farther we diuide *A E*, *A Æ*, and his parallels in the like ſort, and then carefully draw a line through each 15 *gr.* ſo as it makes no angles; the lines ſo drawne ſhall be *ellipticall* and repreſent the houre-circles. The meridian *P Ɛ S* the

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houre of **12** at noone; that next vnto it drawne through *75 gr.* from the center the houres of **11** and **1**, that which is drawne through *60 gr.* from the center the houres of **10** and **2.** &c.

Then hauing reſpect vnto the latitude, we may number it from *E* Northward vnto *Z*, and there place the zenith: by which and the center the line drawne *Z A N* ſhall repreſent the verticall circle, paſſing through the zenith and nadir Eaſt and Weſt, and the line *M A H* croſſing it at right angles ſhall repreſent the horizon.

Theſe two being diuided in the ſame ſort as the ecliptique and the equator, the line drawne through each degree of the ſemidiameter *A Z*, parallell to the horizon, ſhall be the circles of altitude, and the diuiſions in the horizon and his parallels ſhall giue the azimuth.

Laſtly, if through 18 *gr.* in *A N*, be drawne a right line *I K* parallell to the horizon, it ſhall ſhew the time when the day breaketh, and the end of the twilight.

For example of this proiection, let the place of the Sunne be the laſt degree of ♉, the parallell paſſing through this place is *L D*, and therefore the meridian altitude *M L*, and the depreſſion below the horizon at midnight *H D*: the ſemidiurnall arke *L C*, the ſeminocturnall arke *C D*, the declination *A B*, the aſcentionall difference *B C*, the amplitude of aſcenſion *A C*. The difference betweene the end of twilight and the day breake is very ſmall; for it ſeemes the parallell of the Sunne doth hardly croſſe the line of twilight.

If the altitude of the Sunne be giuen, let a line be drawne for it parallell to the horizon; ſo it ſhall croſſe the parallell of the Sunne, and there ſhew both the azimuth and the houre of the day. As if the place of the Sunne being giuen as before, the altitude in the morning were found to be **20** *gr.* the line *F G* drawne parallell to the horizon through **20** *gr.* in A Z, would croſſe the parallell of the Sunne in ☉. Wherefore *F* ☉ ſheweth the azimuth, & *L* ☉ the quantitie of houres from the meridian. It ſeemes to be about halfe an houre paſt 6 in the morning, and yet more then halfe a point ſhort of the Eaſt.

The diſtance of two places may be alſo ſhewed by this proiection, their latitudes being knowne, and their difference of longitude.

For ſuppoſe a place in the Eaſt of Arabia, hauing **20** *gr.* of North latitude, whoſe difference of longitude from London is found by an eclipſe to be 5 *ho.* $\frac{1}{2}$. Let Z be the zenith of London, the parallell of latitude for that other place muſt be *L D*, in which the difference of longitude is *L* ⊙. Wherefore ⊙ repreſenting the ſite of that place, I draw through ⊙ a parallell to the horizon *M H*, croſſing the verticall A Z neare about 70 *gr.* from the zenith, which multiplied by 20, ſheweth the diſtance of London, and that place to be 1400 leagues. Or multiplied by 60, to be 4200 miles.

2 The Sphere may be proiected in *plano* by circular lines, as in the generall aſtrolabe of *Gemma Friſino*, by the help of the tangent on the ſide of the Sector.

For let the circle giuen repreſent the plane of the generall meridian as before; let it be diuided into foure parts, and croſſed at right angles with *E Æ* the equator, and *P S* the circle of the houre of 6, wherein *P* ſtands for the North pole, and *S* for the South pole. Let each quarter of the meridian be diuided into 90 *gr.* and ſo the whole into 360, beginning from *P*, and ſetting to the numbers of 10, 20, 30. &c. 90 at *Æ*, 180 at *S*, 270 at *E*, 360 at *P*. The ſemidiameters *A P*, *A Æ*, *A S*, A *E*, may be diuided according to the tangents of halfe their arkes, that is a tangent of 45 *gr.* which is alwayes equall to the Radius, ſhall giue the ſemidiameter of 90 *gr*; a tangent of 40 *gr.* ſhall giue 80 *gr.* in the ſemidiameter: a tangent of 35 *gr.* ſhall giue 70. &c. So that the ſemidiameters may be diuided in ſuch ſort as the tangent on the ſide of the Sector, the difference being onely in their numbers.

Hauing diuided the circumference and the ſemidiameters, we may eaſily draw the meridians and the parallels by the helpe of the Sector.

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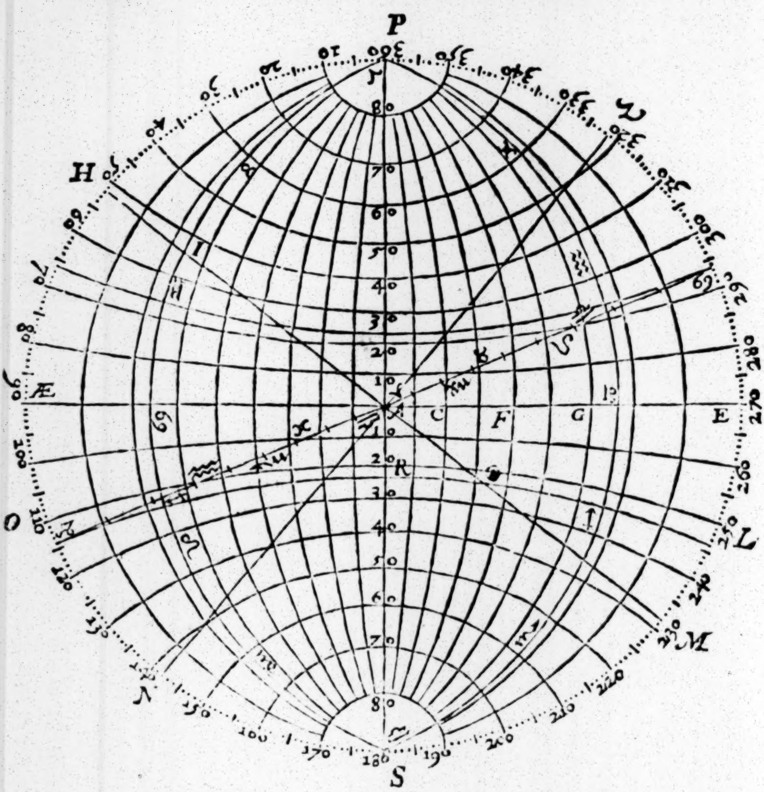

The meridians are to be drawne through both the poles *P* and *S*, and the degrees before graduated in the equator. The diſtance of the center of each meridian from *A* the center of the plane, is equall to the tangent of the ſame meridian, reckoned from the generall meridian *P Æ S E*, and the ſemidiameter equall to the ſecant of the ſame degree.

As for example, if I ſhould draw the meridian *P B S*, which is the tenth from *P Æ S*, the tangent of 10 *gr.* giueth me *A C*, and the ſecant of 10 *gr.* giueth me *S C*, whereof *C* is the center of the meridian *P B S*, and *C S* his ſemidiameter : ſo *A F* a tangent of *20 gr.* ſheweth *F* to be the center of *P D S*, the twentith meridian from *P Æ S*, and *A G* a tangent of 23 *gr.* 30 *M.* ſheweth *G* to be the center of *P ♋ S.* &c.

The parallels are to be drawne through the degrees, in *A P*, *A S*, and their correſpondent degrees in the generall meridian. The diſtance of the center of each parallell from *A* the center of the plane, is equall to the ſecant of the ſame parallell from the pole, and the ſemidiameter equall to the tangent of the ſame degree. As if I ſhould draw the parallell of 80 *gr.* which is the tenth from the pole *S*, firſt I open the compaſſes vnto *A C* the tangent of 10 *gr.* and this giueth me the ſemidiameter of this parallel, whoſe center is a little from *S*, in ſuch diſtance as the ſecant *S C* is longer then the radius *S A*.

The meridians and parallels being drawne, if we number 23 *gr.* 30 *m.* from *E* to ♋ Northward, from *Æ* to ♑ Southward, the line drawne from ♋ to ♑ ſhall be the ecliptique : which being diuided in ſuch ſort as the ſemidiameter *A P*, the firſt 30 *gr.* from *A* to ♋ ſhall ſtand for the ſine of ♈; the 30 *gr.* next following for ♉; the reſt for ♊. ♋. ♌. &c. in their order.

If farther we haue reſpect vnto the latitude, we may number it from *E* Northward vnto *Z*, and there place the zenith, by which and the center, the line drawne *Z A N* ſhall repreſent the verticall circle, and the line *M A H* croſſing it at right angles, ſhall repreſent the horizon ; and theſe diuided in the ſame ſort as *A P*, the circles drawne through each degree of the ſemidiameter *A Z*, parallell to the horizon, ſhall be the circles of altitude: and the circles drawne through the horizon and his poles, ſhall giue the azimuths.

For example of this proiection, let the place of the Sunne be in the beginning of ♒, the parallell paſſing through this place is ♒ ☉ *L*, and therefore the meridian altitude *M L*, and the depreſſion below the horizon at midnight *H* ♒, the ſemidiurnall arke *L* ☉, the ſemnocturnall arke *O* ♒, the declination *A R*, the aſcenſionall difference *R* ☉, the ampli-

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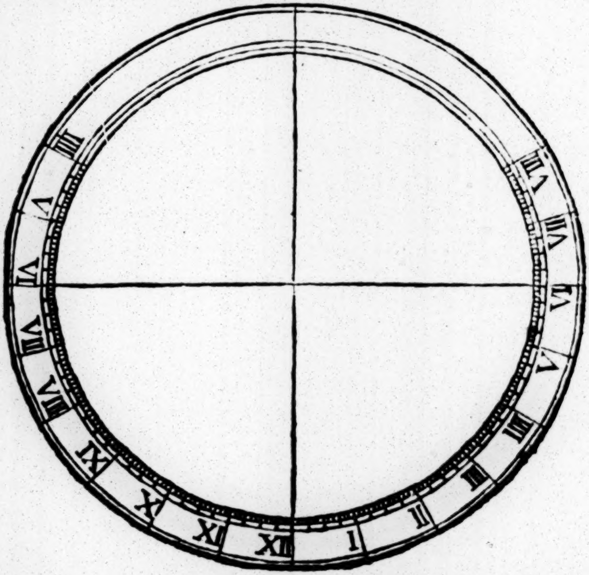
tude of aſcenſion *A*☉.

Or if *A* be put to repreſent the pole of the world, then ſhall *P Æ S E* ſtand for the equator, and *P* ♋ *S* ♑ for the ecliptique, and the reſt which before ſtood for meridians, may now ſerue for particular horizons, according to their ſeuerall eleuations. Then ſuppoſe the place of the Sunne giuen to be 24 *gr.* of ♉, his longitude ſhall be *PI*, his right aſcenſion *P H*, his declination *H I.* And if the place giuen be 19 *gr.* of ♌, his longitude ſhal be *PK*, his right aſcenſion *PN*, his declination N K. Againe, the declination brought to the horizon of the place, ſhall there ſhew the aſcentionall difference, amplitude of aſcenſion, and the like concluſions of the globe. But I intend not here to ſhew the vſe of the Aſtrolabe, but the vſe of the Sector in proiection.

And after this maner may a nocturnall be proiected to ſhew the houre of the night, whereof I will ſet downe a type for the vſe of Sea-men.



It conſiſts as you ſee of two parts, the one is a plane, diuided equally according to the 24 houres of the day, and each houre into quarters or minutes, as the plane will beare: the line from the center to XII, ſtands for the meridian, and XII ſtands for the houre of 12 at midnight. The other part is a rundle for ſuch ſtarres as are neare the North pole, together with the twelue moneths, and the dayes of each moneth fitted to the right aſcenſion of the ſtarres. Thoſe that haue occaſion to ſee the South pole, may do the like for the Southerne conſtellations, and put them in a rundle on the back of this plane, and ſo it may ſerue for all the world.

The vſe of this nocturnall is eaſie and ready. For looke vp to the pole, and ſee what ſtarres are neare the meridian, then place the rundle to the like ſituation, ſo the day of the moneth will ſhew the houre of the night.

3 The Sphere may be proiected in *plano* by circular lines, as in the particular Aſtrolabe of *Ioh. Stophlerin*, by help of the tangent, as before.

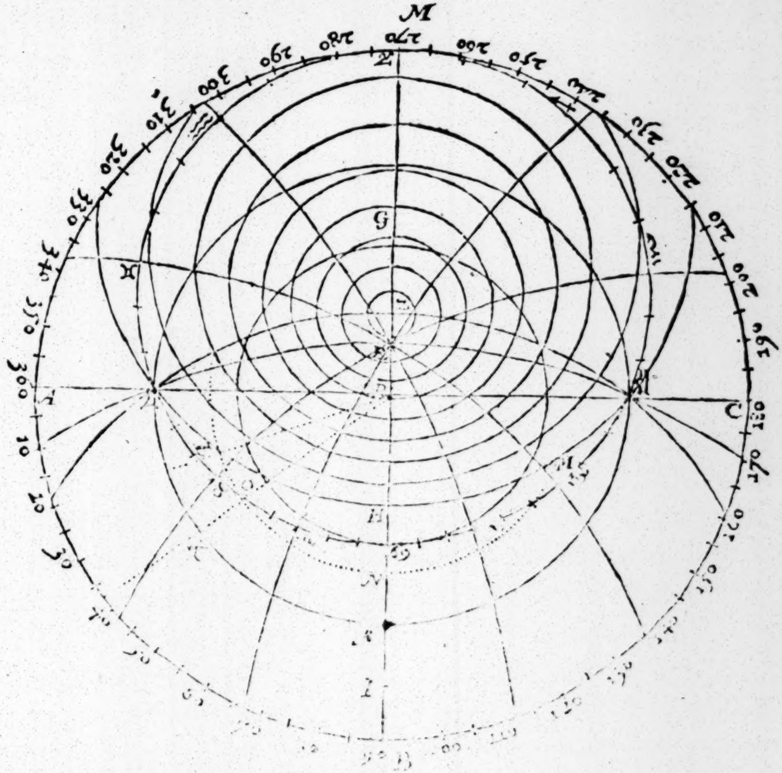
For let the circle giuen repreſent the tropique of ♑, let it be diuided into foure parts, and croſſed at right angles with AC the equinoctiall colure, and MB the ſolſtitiall colure, and generall meridian, the center *P* repreſenting the pole of the world. Let each quarter be diuided into 90 *gr.* and ſo the whole into 360, beginning from *A* towards *B*. The meridian *PM*, or *PB*, may be diuided according to the tangent of halfe his arke. So as the arke from the North pole to the tropique of ♑, being 90 *gr.* and 23 *gr.* 30 *m.* that is 113 *gr* 30 *m.* and the halfe arke 56 *gr.* 45 *m.* the meridian ſhall be diuided into 90 *gr.* and 23 *gr.* 30 *m.* in ſuch ſort as the tangent of 56 *gr.* 45 *m.* on the ſide of the Sector is diuided into degrees and halfe degrees; of which *PÆ* the arke of the equator 90 *gr.* from the pole, ſhall be giuē by the tangent of 45 *gr.* And *P 69* the arke of the Summer tropique 66 *gr.* 30 *m.* from the pole, ſhall be giuen by the tangent of 33 *gr.* 15 *m.* And the circles drawne vpon the center *P* through *Æ* and ♋, ſhall be the equator, and the Summer tropique.

Hauing the equator and both the tropiques, the ecliptique ♈ ♋ ♎ ♑ ſhall be drawne from the one tropique to the other, through the interſection of the equator and the equinoctiall colure. And it may be diuided firſt into the twelue Signes after this maner: *P E* the arke of the pole of the ecliptique 23 *gr.* 30 *m.* from the pole of the world, ſhall be giuen by the tangent of 11 *gr.* 45 *m.* The center of the circle of longitude paſſing through this pole *E* ♈ and ♎, ſhal be found at *D* (ſomewhat below *B*) by the tangent of 66 *gr.* 30 *m.* Then through *D* draw an occult line parallell to *A C*, and diuide it on each ſide from *D*, in ſuch ſort as the tangent is diuided on the ſide of the *Sector*, allowing 45 *gr.* to be equall to *D E*. So the thirtith degree from D toward the right hand, ſhall be the center of the circle of longitude paſſing through *E* ♉ and ♏. The ſixtith degree, the center of ♊ *E* ♐. The thirtith degree from D toward the left hand, the center of ♓ *E* ♍. The ſixtith, the center of ♒ *E* ♌. And the other intermediate degrees ſhal be the centers to diuide each Signe into 30 *gr.*

If farther we haue reſpect vnto the latitude, we may (the meridian being before diuided) number it from *P* Northward vnto *H*, and there place the North interſection of the meridian and horizon: then the complement of the latitude being numbred from *P* Southward vnto *Z*, ſhall there giue the zenith; and 90 *gr.* from *Z* Southward vnto *F*, ſhall there giue the South interſection of the meridian and horizon. The middle betweene *F* and *H* ſhall be *G* the center of the horizon ♈ *H* ♎ *F*, paſſing through the beginning of ♈ and ♎, vnleſſe there be ſome former error.

All parallels to the horizon may be found in like ſort by their interſections with the meridian, and the middle betweene thoſe interſections is alwayes the center.

The azimuths may be drawne as the circles of longitude were before. For the center of the firſt verticall ♈ *Z* ♎ will be found at *I* (ſomewhat neare vnto *B*) by the tangent of the latitude. And if through *I* we draw an occult line parallell to *A C*, and diuide it on each ſide from *I*, in ſuch ſort as the tangent is diuided on the ſide of the *Sector*, allowing 45 *gr.* to be equall



equall to 1*Z*; theſe diuiſions ſhall be the centers, and the diſtance from theſe diuiſions vnto *Z*, ſhall be the ſemidiameters whereon to deſcribe the reſt of the azimuths.

For example of this proiection, let ☉ the place of the Sun giuen be 10 *gr.* of ♉: a right line drawne from *P* through this place vnto the equator, ſhall there ſhew his right aſcenſion ♈*K*, and his declination *K* ☉. Then may we on the center *P* and ſemidiameter ☉ *P*, draw an occult parallell of declinatiõ, croſſing the horizon in *L* and *M*, the meridian in *G* and *N*.

So

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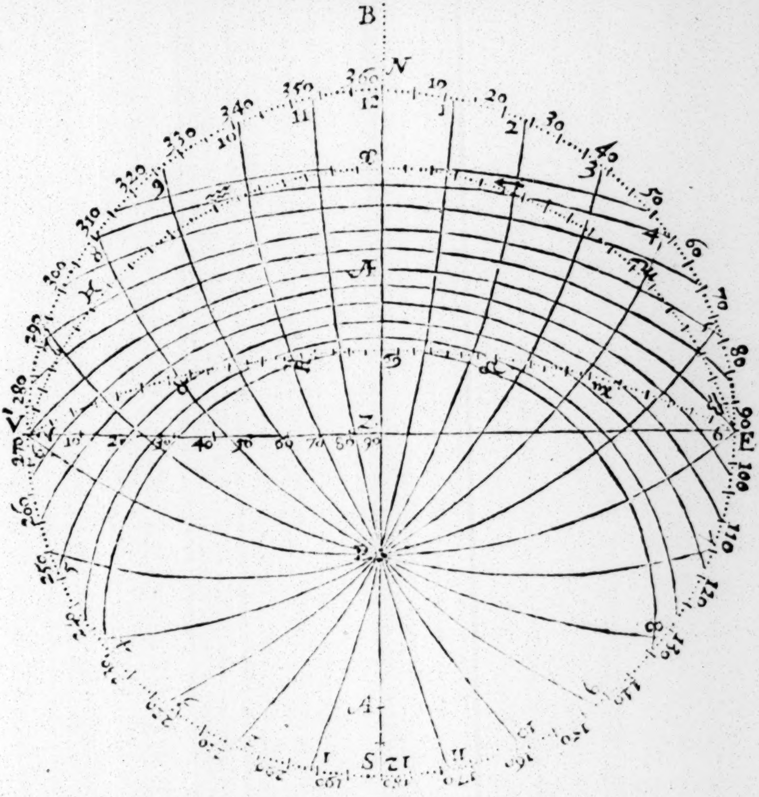
So the right lines *P L* and *P M* produced, ſhall ſhew the time of the Sunnes riſing and ſetting, ♈ *Q* the difference of aſcenſion, ≏ *R* the difference of deſcenſion, ♈ *L* the amplitude of his riſing, and ≏ *M* the amplitude of his ſetting. *L G M* ſheweth the length of the day, *L* N *M* the length of the night. *Z G* ſheweth his diſtance from the zenith at noone, H N his depreſſion below the horizon at midnight. And then hauing the altitude of the Sunne at any time of the day, the interſection of the parallell of altitude with the parallell of declination, ſheweth the azimuth, and a right line drawne from *P* through this interſectiō, giueth the houre of the day.

4 The Sphere may be proiected in *plano* by circular lines, after the maner of the old concaue hemiſphere, by the help of the tangent on the ſide of the Sector.

For let the circle giuen repreſent the plane of the horizon, let it be diuided into foure parts, and croſſed at right angles with *S* N the meridian, and *E V* the verticall; ſo as *S* may ſtand for the South, N for the North, *E* the Eaſt, *V* the Weſt part of the horizon, and the center *Z* repreſenteth the zenith. Let each quarter of the horizon be diuided into 90 *gr.* and ſo the whole into 360 *gr.* beginning from N, and ſetting to the numbers of 10.20.30.&c. 90 at *E*, 180 at *S*, 270 at *V*, 360 at N.

The ſemidiameters *Z* N, *Z S*, may be diuided according to the tangent of halfe their arkes: So as the arke from the zenith to the horizon being 90 *gr.* and the halfe arke 45 *gr.* the ſemidiameters are to be diuided in ſuch ſort as the tangent of 45 *gr.* as was ſhewed before in the ſecond proiection. And if from *Z* we draw circles through each of theſe diuiſions, they ſhall be parallels of altitude.

Then hauing reſpect vnto the latitude, we may (the meridian being before diuided) number it from *Z* to *Æ*, and there place the interſection of the meridian and equator. The complement of the latitude from *Z* vnto *P*, ſhall there giue the pole of the world, and 90 further from *P* ſhall there giue the other interſection of the meridian and equator.



The middle betweene theſe interſections ſhall be *A*, the center of the equator, paſſing through **E** and *V*, vnleſſe there be ſome former error. The interſections of the tropiques depend on the equator. From *Æ* **23** *gr.* 30 ***m.*** farther ſhal be ♑, the interſection of the meridian & the Southerne tropique. From *Æ* 23 *gr.* 30 ***m.*** nearer ſhall be ♋, the interſection of the meridian and the Northerne tropique. The interſections of the other intermediat parallels, ſhall be giuen in like ſort, by their degrees of diſtance ſrom the equator, and the middle betweene thoſe interſections is alwayes the center.

The houre circles may be here drawne as the azimuths in the third proiection. For the center of *E P V*, the houre of 6 will be found at *B* (ſomewhat neare vnto N) by the tangent of the latitude. And if through *B* we draw an occult line parallell vnto *E V*, and diuide it on each ſide from *B*, in ſuch ſort as the tangent is diuided on the ſide of the Sector, allowing 45 *gr.* to be equall to *B P*, and 15 *gr.* for euery houre: thoſe diuiſions ſhall be the centers, and the diſtance from theſe diuiſions vnto *P*, ſhall be the ſemidiameters, whereon to deſcribe the reſt of the houre circles.

The ecliptique may be drawne as the equator. For the center of that halfe which hath Southerne declination, ſhall be giuen by the tangent of the altitude, which the Sun hath in his entrance into ♑. And the center of the other halfe, by the tangent of his altitude, at his entrance into ♋. And it may be diuided, as in the former proiection, or elſe by tables calculated to that purpoſe.

To theſe circles thus drawne, if we ſhall adde the moneths of the yeare, and the dayes of each moneth, as we may well doe, at the horizon, on either ſide betweene the tropiques; this proiection ſhall be fitted for the moſt vſefull concluſions of the globe.

For the day of the moneth being giuen, the parallell that ſhooteth on it, doth ſhew what declination the Sunne hath at that time of the yeare. And where this parallell croſſeth the ecliptique, there is the place of the Sunne. Or the place of the Sunne being firſt giuen, the parallell which croſſeth it ſhall at the horizon ſhew the day of the moneth. Either of theſe then being giuen, or onely the parallell of declination, we may follow it firſt vnto the horizon, there the diſtance of the end of the parallell from *E* or *V*, ſheweth the amplitude; the ſame among the houre circles ſheweth the time when the Sunne riſeth or ſetteth. Then hauing the altitude of the Sunne at any time of the day, the interſection of the parallell of declination with the parallell of altitude, ſheweth the houre of the day; and a right line drawne from Z through this interſection to the horizon, giueth the azimuth.

Thus in either of theſe proiections, that which is otherwiſe moſt troubleſome, is eaſily done by the help of the *tangent* line: and what I haue ſaid of this line, the ſame may be wrought by ſcale and numbers out of the table of Tangents.

CHAP. IV.

Of the reſolution of right-line Triangles.

IN all Triangles there being ſix parts, viz. three angles and three ſides, any three of them being giuen, the reſt may be found by the Sector.

As in a Rectangle triangle,

1 *To finde the baſe, both ſides being giuen.*

Let the *Sector* be opened in the lines of *Lines* to a right angle, (as before was ſhewed *Cap.2. Prop.7.*) then take out the ſides of the triangle, and lay them, one on one line, the other on the other line, ſo as they meete in the center, and marke how farre they extend. For the line taken from the termes of their extenſion, ſhall be the baſe required, viz. the ſide oppoſite to the right angle.

Or adde the ſquares of the two ſides (as in *Prop.4.* Superf.) and the ſide of the compound ſquare ſhall be the baſe.

2 *To find the baſe by hauing the angles, and one of the ſides giuen.*

Take the ſide giuen, and turne it into the parallell line of his oppoſite angle; ſo the parallell Radius ſhall be the baſe.

3 *To find a ſide by hauing the baſe, and the other ſide giuen.*

Let the Sector be opened in the lines of *lines* to a right angle,

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angle , and the ſide giuen laid on one of thoſe lines from the center; then take the baſe with a paire of compaſſes, and ſetting one foote in the terme of the giuen ſide , turne the other to the other line of the Sector, and it ſhall there ſhew the ſide required.

Or take the ſquare of the ſide out of the ſquare of the baſe (as in *Prop.4. Superſ.*) and the ſide of the remaining ſquare ſhall be the ſide required.

4 *To find a ſide hauing the baſe and the angles giuen.*

Take the baſe giuen, and make it a parallell Radius, ſo the parallell *ſines* of the angles, ſhall be the oppoſite ſides required.

5 *To find a ſide by hauing the other ſide and the angles giuen.*

Take the ſide giuen, and turne it into his parallell *ſine* of his oppoſite angle ; ſo the parallell *ſine* of the complement ſhall be the ſide required.

6 *To find the angles by hauing the baſe and one of the ſides giuen.*

Firſt take out the baſe giuen, and laying it on both ſides of the Sector, ſo as they may meete in the center, and marke how farre it extendeth . Then take out the laterall Radius, and to it open the Sector in the termes of the baſe. This done, take out the ſide giuen , and place it alſo on the ſame lines of the Sector from the center. For the parallell taken in the termes of this ſide, ſhall be the ſine of his oppoſite angle.

Or take the baſe giuen , and make it a parallell Radius; then take the ſide giuen, and carrie it parallell to the baſe, till it ſtay in like *ſines* : ſo they ſhall giue the quantitie of the oppoſite angle.

7 *To finde the angles by hauing both the ſides giuen.*

Take out the greater ſide, and lay it on both ſides of the Sector, ſo as they meete in the center, and marke how farre it extendeth. Then take the other ſide, and to it open the Sector in the termes of the greater ſide; ſo the parallell Radius ſhall be the tangent of the leſſer angle. The third angle is alwayes knowne by the complement.

8 *The Radius being giuen, to find the tangent, and ſecant of any arke.*

9 *The tangent of any arke being giuen, to find the tangent thereof, and the Radius.*

10 *The ſecant of any arke being giuen, to find the tangent thereof, and the Radius.*

The tangent, and the ſecant, together with the Radius of euery arke, do make a right angle triangle; whoſe ſides are the *Radius* and *tangent*, and the baſe alwayes the *ſecant*; and the angles alwayes knowne by reaſon of the giuen arkes. Wherefore the ſolution is the ſame with thoſe before.

In any right-lined triangle whatſoeuer,

11 *To find a ſide by knowing the other two ſides, and the angle contained by them.*

Let the *Sector* be opened in the lines of *lines* to the angle giuen, then take out the ſides of the triangle, & laying them the one on the one line, the other on the other, ſo as they meete in the center, marke how far they extend. For the line taken betweene the termes of their extenſion, ſhall be the third ſide required.

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12 *To find a ſide by hauing the other two ſides, and one of the adiacent angles, ſo it be knowne which of the other angles is acute or oblique.*

Let the *Sector* be opened in the lines of *lines* to the angle giuen, and the adiacent ſide layd on one of thoſe lines from the center; then take the other ſide with a paire of compaſſes, and ſetting one foote in the terme of the former giuen ſide, turne the other to the other line of the *Sector* which here repreſenteth the ſide required, and it ſhall croſſe it in two places; but with which of them is the terme of the ſide required, muſt be iudged by the angle.

As if in the triangle following, the ſide *A C* being giuen, and the ſide *C D* and the angle *C A D* 18 *gr.* 40 *m.* it were required to find the ſide *A D*.

Firſt I open the Sector in the lines of *lines* to an angle 18 *gr.* 40 *m.* and laying the adiacent ſide from the center *A* it extendeth to 800 in *C*. Then I take the other ſide *CD* with the compaſſes, and ſetting one foote in *C*, and turning the other to the other line of the *Sector*, I find that it doth croſſe it both in *B* and D; ſo that it is vncertaine whither the ſide required be *AB* or *A*D, onely it may be iudged by the angle. For if the inward angle where they croſſe be obtuſe, the ſide required is the leſſer; if it be acute, it is the greater.

13 *To find a ſide by hauing the angles and one of the other ſides giuen.*

Take the ſide giuen, and turne it into the parallell ſine of his oppoſite angle; ſo the parallell ſines of the other angle ſhall be the oppoſite ſides required.

14 *To find the proportion of the ſides by hauing the three angles.*

Take the laterall ſines of the angles, and meaſure them in the line of *lines*. For the numbers belonging to thoſe lines do giue the proportion of the ſides.

15 *To finde an angle by knowing the three ſides.*

Let the two containing ſides be layd on the lines of the *Sector* from the center, one on one line, and the other on the other; and let the third ſide, which is oppoſite to the angle required, be fitted ouer in their termes: ſo ſhall the *Sector* be opened in thoſe lines to the quantitie of the angle required.

The quantitie of this angle is found as in *Cap.2. Prop.*8.

16 *To finde an angle by hauing two ſides and one adiacent angle.*

Firſt take out the ſide oppoſite to the angle giuen, and laying it on both ſides of the Sector, ſo as they meete in the center, marke how farre it extendeth; then take out the laterall ſine of the angle, and to it open the Sector in the termes of the firſt ſide: this done, take out the other ſide giuen, and place it alſo on the ſame lines of the Sector from the center, for the parallels taken in the termes of this ſide, ſhall be the ſine of the angle oppoſite to the ſecond ſide.

Or take out the ſide oppoſite to the angle giuen, and make it a parallell ſine of that angle; then take the other ſide giuen and carrie it parallell to the former, till it ſtay in like ſines: ſo they ſhall giue the quantitie of the angle oppoſite to the ſecond ſide.

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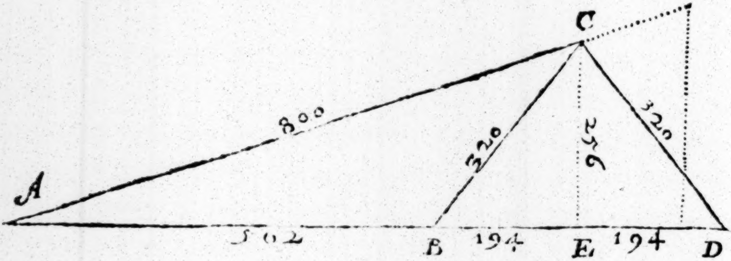
17 *To finde an angle by hauing two ſides, and the angle contained by them.*

Firſt find the third ſide by the 11.*Prop.* and then the angles may be found by the 15.or 16.*Prop.*

For practiſe in each of theſe caſes, we may vſe the examples following, wherein *C E A*, *C E B*, *C E D* are rectangle in *E*; the reſt conſiſt of oblique angles.

C A B	18 *gr.*	40 *m.*
A B C	126	52
A C B	34	28
A C D	108	12
A D C	53	8
B C D	73	44



For obſeruation of angles, the *Sector* may haue ſights ſet on the moueable foote; ſo that by looking through them, the edges of the Sector may be applied to the ſides of the angle.

For meaſuring of the ſides of leſſer triangles, any ſcale may ſuffice, either of feete, or inches, or leſſer parts. But for greater triangles, eſpecially for plotting of grounds, I hold it fit to vſe a chaine of foure perches in length, diuided into an hundred links. For ſo the length being multiplied into the bredth, the fiue laſt figures giue the content in roods and perches by this Table; the other figures toward the left hand, doe ſhew the number of acres directly.

As if in the former triãgle *ACD*, the length *A* D be 9 chaines and 50 links, the bredth *C E* be 2 chaines and 56 links; theſe multiplied giue the content for the long ſquare 2.43200, the halfe whereof for the triangle is 1.21600, that is 1 acre, 21600 parts of 100000, of which laſt fiue figures, 20000 giue 32 perches, and the remainder 1600 giue better then two perches more.

Links	*R*	*P*
100000	4	0
90000	3	24
80000	3	8
70000	2	32
60000	2	16
50000	2	0
40000	1	24
30000	1	8
20000		32
10000		16
9375		15
8750		14
8125		13
7500		12
6875		11
6250		10
5625		9
5000		8
4375		7
3750		6
3125		5
2500		4
1875		3
1250		2
625		1

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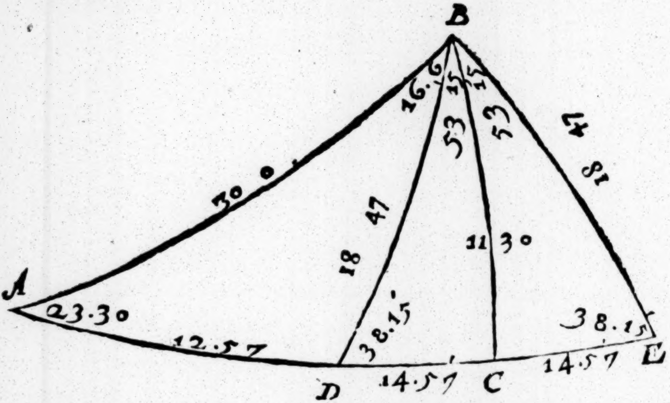
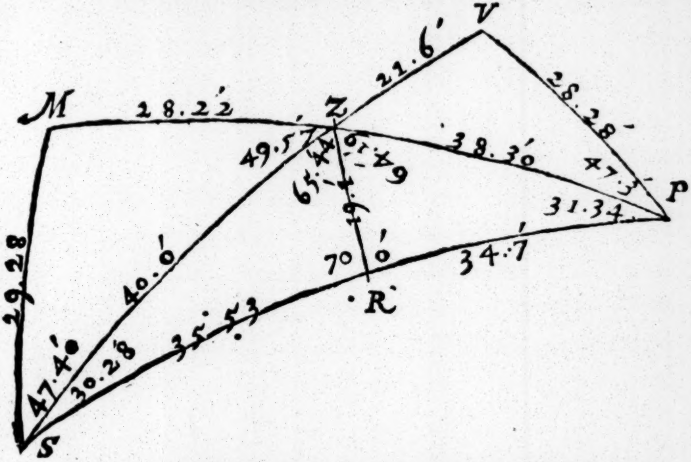
CHAP. V.

Of the reſolution of ſphericall triangles.

FOr our practiſe in ſphericall triangles, let *A* be the equinoctiall point, *A* B an arke of the ecliptique repreſenting the longitude of the Sunne in the beginning of ♉, *B C* an arke of the declination from the Sunne to the equator, and *A C* an arke of the equator repreſenting the right aſcenſion.



Let B D be an arke of the horizon repreſenting the amplitude of the Sunnes riſing from the Eaſt, and *B E* an arke of the horizon for his ſetting from the Weſt: ſo DC ſhall be the difference of aſcenſion, and *C E* the difference of deſcenſion; A D the oblique aſcenſion, and *A E* the oblique deſcenſion of the ſame place of the Sunne in our latitude at *Oxford* of 51 *gr.* 45 *m.* whoſe complement 38 *gr.* 15 *m.* is the angle at *E* and D. The triangles *A C B*, D C *B*, *E C B*, are rectangle in *C*: the other *A* D *B*, *A E B*, conſiſt euery way of oblique angles.



Or to fit an example nearer to the latitude of *London.* Let *Z P S* repreſent the zenith pole and Sun, *Z P* being 38 *gr.* 30 *m.* the complement of the latitude, *P S* 70 *gr.* the complement of the declination, and *Z S* 40 *gr.* the complement of the Suns altitude. The angle at *Z* ſhall ſhew the azimuth, and the angle at *P*, the houre of the day from the meridian. Then if from *Z* to *P S* we let downe a perpendicular *Z R*, we ſhall reduce the oblique triangle into two rectangle triangles *Z R P*, *Z R S.* Or if from *S* to *Z P* we ſet downe a perpendicular *S M*, we ſhall reduce the ſame *Z P S* into two other triangles, *S M Z*, *S M P*, rectangle at *M*: whatſoeuer is ſaid of any of theſe triangles, the ſame holdeth for all other triangles in the like caſes.

For the reſolution of each of theſe, there be ſeueral wayes. I onely chuſe thoſe which are fitteſt for the *Sector*, wherein if that be remembred which before is ſhewed in the generall vſe of the *Sector* concerning laterall and parallell entrance, it may ſuffice onely to ſet downe the proportion of the three parts giuen to the fourth required, and ſo I ſhew firſt by the *ſines* alone.

In a rectangle triangle

1 *To finde a ſide by knowing the baſe, and the angle oppoſite to the required ſide.*

As the Radius
is to the ſine of the baſe:
So the ſine of the oppoſite angle
to the ſine of the ſide required.

As in the rectangle *A C B*, hauing the baſe *A B*, the place of the Sunne 30 *gr.* from the equinoctiall point, and the angle *B A C* of 23 *gr.* 30 *m.* the greateſt declination, if it were required to find the ſide *B C* the declination of the Sunne.

Take either the laterall ſine of 23 *gr.* 30 *m.* and make it a parallell Radius; ſo the parallell ſine of 30 *gr.* taken and meaſured in the ſide of the Sector, ſhall giue the ſide required 11 *gr.* 30 *m.* Or take the ſine of 30 *gr.* and make it a parallell Radius; ſo the parallell ſine of 23 *gr.* 30 *m.* taken and meaſured in the laterall ſines, ſhall be 11 *gr.* 30 *m.* as before.

So in the triangle Z *P S* hauing Z *P* 38 *gr.* 30 *m.* and the angle *P* 31 *gr.* 34 *m.* giuen, we ſhall find the perpendicular Z *R* to be 19 *gr.* 1 *m*; or hauing *P S* 70 *gr.* and the ſaid angle *P* 31 *gr.* 34 *m.* giuen, we may finde the perpendicular *S M* to be 29 *gr.* 28 *m.*

2 *To finde a ſide by knowing the baſe and the other ſide.*

As the ſine of the complement of the ſide giuen
is to the Radius:
So the ſine of the complement of the baſe
to the ſine of the complement of the ſide required.

So in the rectangle *A C B*, hauing *A B* 30 *gr.* and *BC* 11 *gr.* 30 *m.* giuen, the ſide *A C* will be found 27 *gr.* 54 *m.*

Or in the rectangle *Z R P*, hauing Z *P* 38 *gr.* 30 *m.* and Z *R* 29 *gr.* 1 *m.* giuen, the ſide *R P* will be found 34 *gr.* 7 *m.*

3 *To find a ſide by knowing the two oblique angles.*

As the ſine of either angle
to the ſine of the complement of the other angle:
So is the Radius
to the ſine of the complement of the ſide oppoſite
to the ſecond angle.

So in the rectangle *A C B*, hauing *C A B* for the firſt angle *23 gr. 30 m.* and *A B C* for the ſecond *69 gr. 21 m.* the ſide *A C* will be found *27 gr. 54 m.* Or making *A B C* the firſt angle, and *C A* B the ſecond, the ſide BC will be found *11 gr. 30 m.*

4 *To finde the baſe by knowing both the ſides.*

As the Radius
to the ſine of the complement of the one ſide:
So the ſine of the complement of the other ſide,
to the ſine of the complement of the baſe required.

So in the rectangle *A C*B hauing *A C 27 gr. 54 m.* and BC *11 gr. 30 m.* the baſe *A* B will be found 30 *gr.*

5 *To finde the baſe by knowing the one ſide, and the angle oppoſite to that ſide.*

As the ſine of the angle giuen,
to the ſine of the ſide giuen:
So is the Radius
to the ſine of the baſe required.

So in the rectangle B C D, knowing the latitude and the declination, we may find the amplitude; as hauing B C the ſide of the declination *11 gr. 30 m.* and B DC the angle of the complement of the latitude 33 *gr. 15 m.* the baſe B D which is the amplitude, will be found to be 18 *gr. 47 m.*

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6 To find an angle by the other oblique angle, and the ſide oppoſite to the inquired angle.

As the Radius
to the ſine of the complement of the ſide:
So the ſine of the angle giuen,
to the ſine of the complement of the angle required.

So in the rectangle A C B, hauing the angle B A C 23 *gr.* 30 *m.* and the ſide A C 27 *gr.* 54 *m.* the angle A B C will be found 69 *gr.* 21 *m.*

7 To finde an angle by the other oblique angle, and the ſide oppoſite to the angle giuen.

As the ſine of the complement of the ſide
to the ſide of the complement of the angle giuen:
So is the Radius
to the ſine of the angle required.

So in the rectangle A C B, hauing B A C 23 *gr.* 30 *m.* and B C 11 *gr.* 30 *m.* the angle A B C will be found 69 *gr.* 21 *m.*

8 To finde an angle by the baſe, and the ſide oppoſite to the inquired angle.

As the ſine of the baſe
is to the Radius:
So the ſine of the ſide
to the ſine of the angle required.

So in the rectangle B C D, hauing B D 18 *gr.* 47 *m.* and B C 11 *gr.* 30 *m.* the angle B D C will be found 38 *gr.* 15 *m.*

Theſe eight Propoſitions haue been wrought by the *ſines* alone; thoſe which follow require ioynt help of the *tangent.*

And foraſmuch as the *tangent* could not well be extended beyond 63 *gr.* 30 *m.* I ſhall ſet downe two wayes for the reſolution of each Propoſition; if the one will not hold, the other may.

9 To

9 *To finde a ſide by hauing the other ſide, and the angle oppoſite to the inquired ſine.*

1 As the Radius
to the ſine of the ſide giuen:
So the tangent of the angle,
to the tangent of the ſide required.

2 As the ſine of the ſide giuen,
is to the Radius:
So the tangent of the complement of the angle,
to the tangent of the complement of the ſide required.

So in the rectangle ACB, hauing the right ſide AC 27 *gr.* 54 *m*, and the angle B A C 23 *gr*. 30 *m*. the ſide B C will be found to be 11 *gr*. 30 *m*.

10 *To find a ſide, by hauing the other ſide, and the angle adiacent next to the inquired ſide.*

1 As the tangent of the angle,
to the tangent of the ſide giuen:
So is the Radius
to the ſine of the ſide required.

2 As the tangent of the complement of the ſide,
to the tangent of the complement of the angle:
So is the Radius
to the ſine of the ſide required.

This and the like, where the tangent ſtandeth in the firſt place, are beſt wrought by parallell entrance. And ſo in the rectangle BC D, hauing B C the ſide of declination 11 *gr*. 30 *m*. and B D C the angle of the complement of the latitude 38 *gr*. 15 *m*. the ſide D C, which is the aſcenſionall difference, will be found 14 *gr*. 57 *m*.

By the aſcenſionall difference is giuen the time of the Sunnes riſing and ſetting, and length of the day; allowing an

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an houre for each 15 *gr.* and 4 minutes of time for each ſeuerall degree. As in the example the difference betweene the Sunnes aſcenſion in a right ſphere, which is alwayes at 6 of the clocke, and his aſcenſion in our latitude being 14 *gr.* 57 *m.* it ſheweth that the Sunne riſeth very neare an houre before 6, becauſe of the Northerne declination; or after 6, if the Sunne be declining to the Southward.

11 *To finde a ſide by knowing the baſe, and the angle adiacent next to the inquired ſide.*

1 As the Radius
to the ſine of the complement of the angle:
So is the tangent of the baſe,
to the tangent of the ſide required.

2 As the ſine of the complement of the angle
is to the Radius:
So the tangent of the complement of the baſe,
to the tangent of the complement of the ſide required.

So in the rectangle A C B, knowing the place of the Sun from the next equinoctiall point, and the angle of his greateſt declination, we may find his right aſcenſion: viz. the baſe A B 30 *gr.* and the angle BAC 23 *gr.* 30 *m.* being giuen, the right aſcenſion A C will be found 27 *gr.* 54 *m.*

12 *To finde the baſe by knowing the oblique angles.*

As the tangent of the one angle,
to the tangent of the complement of the other angle:
So is the Radius
to the ſine of the complement of the baſe.

So in the rectangle A C B, hauing B A C 23 *gr.* 30 *m.* and A B C 69 *gr.* 21 *m.* the baſe A B will be found 30 *gr.*

13 *To finde the baſe, by one of the ſides, and the angle adiacent next that ſide.*

1 As the Radius
is to the ſine of the complement of the angle:
So the tangent of the complement of the ſide,
to the tangent of the complement of the baſe.

2 As the ſine of the complement of the angle
is to the Radius:
So the tangent of the ſide giuen,
to the tangent of the baſe required.

So in the rectangle *A C B*, hauing *A C* 27 *gr.*54 *m.* and *B A C* 23 *gr.*30 *m.* the baſe A B will be found 30 *gr.*0 *m.*

14 *To find an angle, by knowing both the ſides.*

1 As the Radius
is to the ſine of the ſide next the inquired angle:
So the tangent of the complement of the oppoſite ſide,
to the tangent of the complement of the angle required.

2 As the ſine of the ſide next the inquired angle,
is to the Radius:
So the tangent of the oppoſite ſide,
to the tangent of the angle required.

So in the rectangle A C B, hauing A C 27 *gr.*54 *m.* and B C 11 *gr.*30 *m.* the angle at *A* will be found 23 *gr.*30 *m.* and the angle at B 69 *gr.*21 *m.*

15 *To finde an angle, by the baſe, and the ſide adiacent to the inquired angle.*

1 As the tangent of the complement of the ſide,
to the tangent of the complement of the baſe:

So is the Radius
to the ſine of the complement of the angle required.

2 As the tangent of the baſe,
to the tangent of the ſide:
So is the Radius,
to the ſine of the complement of the angle required.

So in the rectangle BCD, hauing the baſe BD 18 *gr.* 47 *m.* and the ſide BC 11 *gr.* 30 *m.* the angle D B C between them will be found 53 *gr.* 15 *m.*

16 *To find an angle, by knowing the other oblique angle, and the baſe.*

1 As the Radius,
to the ſine of the complement of the baſe:
So the tangent of the angle giuen,
to the tangent of the complement of the angle required.

2 As the ſine of the complement of the baſe,
is to the Radius:
So the tangent of the complement of the angle giuen,
to the tangent of the angle required.

So in the rectangle A C B, hauing the angle at *A* 23 *gr.* 30 *m.* and the baſe A B 30 *gr.* the angle A B C will be found 69 *gr.* 21 *m.*

Theſe ſixteen caſes are all that can fall out in a rectangle triangle: thoſe which follow do hold

In any ſphericall triangle whatſoeuer

17 *To find a ſide oppoſite to an angle giuen, by knowing one ſide, and two angles, whereof one is oppoſite to the giuen ſide, the other to the ſide required.*

As the ſine of the angle oppoſite to the ſide giuen,
is to the ſine of that ſide giuen:
So the ſine of the angle oppoſite to the ſide required,
to the ſine of the ſide required.

So in the triangle A B E, hauing the place of the *Sunne*, the latitude, and the greateſt declination, we may finde the amplitude. As hauing A B 30 *gr.* BA E 23 *gr.* 30 *m.* and AEB 38 *gr.* 15 *m.* the ſide B E which is the amplitude, will be found 18 *gr.* 47 *m.*

18 *To finde an angle oppoſite to a ſide giuen, by hauing one angle and two ſides, the one oppoſite to the giuen angle, the other to the angle required.*

As the ſine of the ſide oppoſite to the angle giuen,
is to the ſine of that angle giuen:
So the ſine of the ſide oppoſite to the angle required,
to the ſine of the angle required.

So in the triangle *Z P S*, hauing the azimuth, and latitude, and declination, we may find the houre of the day. As hauing *P Z S* 130 *gr.* 3 *m.* *PS* 70 *gr.* and *Z S* 40 *gr.* the angle *Z P S*, which ſheweth the houre from the meridian ſhall be found 31 *gr.* 34 *m.*

19 *To find an angle by knowing the three ſides.*

This propoſition is moſt vſefull, but moſt difficult of all others: as in Arithmetique, ſo by the *Sector*, yet may it be performed ſeuerall wayes.

1 According to *Regiomontanus* and others.

As the ſine of the leſſer ſide next the angle required,
to the difference of the verſed ſines of the baſe and difference of the ſides:
So is the Radius
to a fourth proportionall.

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Then as the ſine of the greater ſide next the angle required
is to that fourth proportionall :
So is the Radius
to the verſed ſine of the angle required.

So in the triangle *ZPS*, hauing the ſide *PS*, the cõplement of the declination 70 *gr.* 0 *m.* the ſide Z *P* the complement of the latitude 38 *gr.* 30 *m.* and the baſe Z *S* the complement of the altitude 40 *gr.* the angle of the houre of the day Z*PS* will be found 31 *gr.* 34 *m.* which is 2 *h.* 6 *m.* from the meridian.

For the baſe being 40 *gr.* 0 *m.* and the difference of the ſides 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* being 31 *gr.* 30 *m.* the difference of their verſed ſines wil be the ſame with the diſtance between the right ſine of 50 *gr.* and 58 *gr.* 30 *m.* This difference I take out, and make it a parallell ſine of the leſſer ſide 38 *gr.* 30 *m.* ſo the parallell Radius wil be the fourth proportionall. Then coming to the ſecond operation, I make this fourth proportionall, a parallel ſine of the greater ſide of 70 *gr.* 0 *m.* and take out his parallell Radius. For this meaſured from 90 *gr.* toward the center, will be the verſed ſine of 31 *gr.* 34 *m.*

In the like ſort in the ſame triangle Z *P S*, hauing the ſame complements giuen, the angle *P* Z *S* which is the azimuth from the North part of the meridian, will be found 130 *gr.* 3 *m.* For here the baſe oppoſite to the angle required being 70 *gr.* and the difference of the ſides 38 *gr.* 30 *m.* and 40 *gr.* being 1 *gr.* 30 *m.* the difference of their verſed ſines will be the ſame with the diſtance betweene the right ſines of 20 *gr.* and 88 *gr.* 30 *m.* This difference I take, and make it a parallel ſine of the leſſer ſide 38 *gr.* 30 *m.* ſo the parallell Radius will be the fourth proportionall. Then coming to the ſecond operation, I make this fourth proportionall a parallell ſine of the greater ſide 40 *gr.* and take out his parallell Radius. For this meaſured from 90 *gr.* beyond the center in the lines of *ſines* ſtretched forth at their full length, will be the verſed ſine of 130 *gr.* 3 *m.*

2. I may finde an angle by knowing three ſides, by that which I haue elſewhere demonſtrated vpon ***Barth.Pitiſcus***,

and that at one operation in this maner.

As the ſine of the greater ſide
is to the ſecant of the complement of the other ſide:
So the difference of ſines of the complement of the baſe, and the arke compounded of the leſſer ſide with the complement of the greater,
to the verſed ſine of the angle required.

So in the ſame triangle Z *P S*, hauing the ſame complements giuen, the angle at *P*, which ſheweth the houre from the meridian, will be found as before 31 *gr.* 34 *m.*

For the ſides being 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* I take the ſecant of the complement of 38 *gr.* 30 *m.* and make it a parallell ſine of 70 *gr*; then keeping the Sector at this angle, I conſider that the complement of 70 *gr.* being 20 *gr.* added vnto 38 *gr.* 30 *m.* the compounded ſide (which is here the meridian altitude) will be 58 *gr.* 30 *m*; and that the baſe being 40 *gr.* the difference of ſines of the compounded ſide and the complement of the baſe will be (as before) the diſtance betweene the ſines of 50 *gr.* and 58 *gr.* 30 *m.* Wherefore I take out this difference, and lay it on both the lines of *ſines* from the center: ſo the parallell taken in the termes of this difference, and meaſured from 90 *gr.* toward the center, doth giue the verſed ſine of 31 *gr.* 34 *m.*

The other angles *P Z S*, *P S Z*, may be found in the ſame ſort; but hauing the ſides and one angle, it will be ſooner done by that which we ſhewed before in the 18. *Prop.*

20 *To find a ſide by knowing the three angles.*

If for the greater angle we take his complement to 180 *gr.* the angles ſhall be turned into ſides, and the ſides into angles, & the operation ſhall be the ſame, as in the former *Prop.*

21 *To finde a ſide, by hauing the other two ſides, and the angle comprehended.*

This propoſition being the conuerſe of the nineteenth,

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may be wrought accordingly; but the beſt way both for it and thoſe which follow, is to reſolue them into two rectangles, by letting downe a perpendicular, as was ſhewed in the firſt *Prop.*

So in the triangle Z *P S*, hauing Z *P* the complement of the latitude, and *PS* the complement of the declination, with Z *P S* the angle of the houre from the meridian, we may find Z *S* the complement of the altitude of the Sunne.

For hauing let downe the perpendicular Z *R* by the firſt *Prop.* we haue two triangles, Z *R P*, Z *R S*, both rectangle at *R*. Then may we find the ſide *PR*, either by the ſecond, or tenth, or eleuenth *Prop*; which taken out of *P S*, leaueth the ſide *RS*: with this *R S* and Z *R* we may find the baſe Z *S* by the fourth *Prop.*

Or hauing let downe the perpendicular *S M*, we haue two rectangle triangles *S M Z*, *S M P*. Then may we find *M P*, from which if we take Z *P*, there remaineth *M* Z: but with *M* Z and *S M*, we may find the baſe Z *S*.

22 *To find a ſide, by hauing the other two ſides, and one of the angles next the inquired ſide.*

So in the triangle *Z P S*, hauing Z *P* the complement of the latitude, and *P S* the complement of the declination, with *P* Z *S* the angle of the azimuth, we may finde Z *S* the complement of the altitude of the Sunne.

For hauing Z *P*, and the angle at Z, we may to *S* Z produced, let downe a perpendicular *P V*. Then we haue two rectangle triangles, P V Z, P V S, wherein if we find the ſides V Z, V S, and take the one out of the other, there will remain the ſide inquired Z *S*.

23 *To find a ſide, by hauing one ſide, and the two angles next the inquired ſide.*

So in the triangle A B D, hauing A B the place of the ſun, and *B A D* the angle of the greateſt declination, and *A D B* the angle of the equator with the horizon, we may find *A D* the oblique aſcention.

For hauing let downe *B C* the perpendicular of declination, we haue two rectangle triangles, *A C B*, *D C* B. Then may we find *A C* the right aſcention, and D C the aſcentionall difference; and comparing the one with the other, there remaineth *A D*.

24 *To find a ſide, by hauing two angles, and the ſide incloſed by them.*

So in the triangle *Z P S*, hauing the angles at Z and P, with the ſide intercepted Z P, we may find the ſide P S. For hauing let downe the perpendicular P V, we haue two rectangles P V Z, P V S. Then may we find the angle V P Z, eiby the ſeuenth, or fifteenth, or ſixteenth *Prop.* which added to Z P S, maketh the angle V P S: with this V P S and P V, we may find the baſe PS, according to the 13 *Prop.*

25 *To find an angle by hauing the other two angles and the ſide incloſed by them.*

So in the triangle Z P *S*, hauing the angles at Z and P, with the ſide intercepted Z P, we may finde the other angle Z *S* P. For hauing let downe the perpendicular *Z R*, we haue two rectangles Z R P, Z R *S*. Then may we finde the angle P Z R by the ſixteenth *Prop.* and that compared with P Z *S*, leaueth the angle R Z *S*: with this *R Z S* and *Z R* we may find the angle required *Z S R*, according to the ſixth *Prop.*

26 *To finde an angle, by hauing the other two angles, and one of the ſides next the inquired angle.*

So in the triangle A B D, hauing the angles at *A* and *D*, with the ſide A B, we may find the angle A B D. For hauing let downe the perpendicular BC, we haue two rectangles, A C B,

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A C B, D C B. Then may we find the angles ABC, DBC, and take D B C out of A B C; for ſo there remaineth the angle required A B D.

27 *To finde an angle, by knowing two ſides, and the angle contained by them.*

So in the triangle Z P S, hauing the ſides Z P, P S, with the angle comprehended Z P S, we may find the angle PZS. For hauing let downe the perpendicular S M, we haue two rectangles *S MZ, S* M P. Then may we find the ſide M P, and taking Z P out of M P, there remaineth *M Z*: with this *M Z* and the perpendicular *M S*, we may finde the angle *MZ S*, by the fourteenth *Prop.* This angle M Z*S*, taken out of 180 *gr.* there remaineth P Z *S*.

28 *To find an angle by knowing the two ſides next it, and one of the other angles.*

So in the triangle *Z P S*, hauing the ſides *Z P* and *P S*, with the angle *P Z S*, we may find the angle Z *P S*. For hauing let downe the perpendicular *PV*, we haue two rectangles P V Z, P V S. Then may we find the angles V PZ, V P S; and taking V P *Z* out of V P S, there remaineth Z P S, which was required.

Theſe 28 caſes are all that can fall out in any ſphericall triangle: if any do not preſently vnderſtand them, let them once more reade ouer the vſe of the globes, and they ſhall ſoone become eaſie vnto them.

CHAP. VI.

Of the vſe of the Meridian line in Nauigation.

THe *Meridian* line is here ſet on the ſide of the Sector, ſtretched forth at full length, on the ſame plane with the line of *lines* and *Solids*, and is diuided vnequally toward 87 *gr.* (whereof 70 *gr.* are about one halfe) in ſuch ſort as the Meridian in the cart of *Mercators* proiection. The vſe of it may be

1 *To diuide a ſea-chart according to* Mercators *proiection.*

If a degree of the equator on the ſea-chart be equall to the hundred part of the line of *lines* in the *Sector*, the degrees of the *Meridian* vpon the *Sector*, ſhall giue the like degrees vpon the ſea-chart: if otherwiſe they be vnequall, then may the meridians of the ſea-chart be diuided in ſuch ſort as the line of *Meridians* is diuided on the *Sector*, by that which we ſhewed before in the 8 *Prop.* of the line of *lines*.

But to auoid error, I haue here ſet downe a Table, whereby the Meridian line may be diuided out of the degrees of the equator, ſuppoſing each degree to be ſubdiuided into a thouſand parts. By which Table, & the vſuall Table of *Sines*, *Tangents* and *Secants*, the proportions following may be alſo reſolued arithmetically. For the maner of diuiſion, let the equator (or one of the parallels if it be a particular chart) be drawne, and diuided, and croſſed with parallell meridians, as in the common ſea-chart: then looke into the Table, and let the diſtance of 40 *gr.* in the meridian, from the equator, be equall to 43 *gr.* 711 *parts* of the equator; let 50 *gr.* in the meridian from the equator, be equall to 57 *gr.* 909 *parts* of the equator; and ſo in the reſt.

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M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par
0	0	0	3	3	001	6	6	011	9	9	037	12	12	088
		100		3	101		6	111		9	138		12	190
		200		3	201		6	212		9	239		12	293
		300		3	301		6	312		9	341		12	395
		400		3	402		6	413		9	442		12	497
		500		3	502		6	514		9	543		12	600
		600		3	602		6	614		9	645		12	702
		700		3	702		6	715		9	746		12	805
		800		3	803		6	816		9	848		12	907
		900		3	903		6	916		9	949		13	010
1	1	000	4	4	003	7	7	017	10	10	051	13	13	112
	1	100		4	103		7	118		10	152		13	215
	1	200		4	204		7	219		10	254		13	318
	1	300		4	304		7	319		10	355		13	421
	1	400		4	404		7	420		10	457		13	523
	1	500		4	504		7	521		10	559		13	626
	1	600		4	605		7	622		10	661		13	729
	1	700		4	705		7	723		10	762		13	832
	1	800		4	805		7	824		10	864		13	935
	1	900		4	906		7	925		10	966		14	038
2	2	000	5	5	006	8	8	026	11	11	068	14	14	141
	2	100		5	106		8	127		11	170		14	244
	2	200		5	207		8	228		11	272		14	347
	2	300		5	307		8	329		11	374		14	450
	2	400		5	408		8	430		11	476		14	553
	2	500		5	508		8	531		11	578		14	656
	2	601		5	609		8	632		11	680		14	760
	2	701		5	709		8	733		11	782		14	863
	2	801		5	810		8	834		11	884		14	967
	2	901		5	910		8	936		11	986		15	070
3	3	001	6	6	011	9	9	037	12	12	088	15	15	174

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	part	M.	G	part
15	15	174	18	18	303	21	21	486	24	24	734	27	28	058
	15	277		18	408		21	593		24	844		28	171
	15	381		18	513		21	701		24	953		28	283
	15	485		18	619		21	808		25	063		28	396
	15	588		18	724		21	915		25	173		28	508
	15	692		18	830		21	023		25	282		28	621
	15	796		18	935		22	130		25	392		28	734
	15	900		19	041		22	238		25	502		28	847
	16	004		19	146		22	345		25	613		28	959
	16	107		19	251		22	453		25	723		29	072
16	16	211	19	19	356	22	22	561	25	25	833	28	29	186
	16	316		19	463		22	669		25	943		29	299
	16	420		19	569		22	777		26	054		29	413
	16	524		19	675		22	885		26	164		29	526
	16	628		19	781		22	993		26	275		29	640
	16	732		19	887		23	101		26	386		29	753
	16	836		19	993		23	210		26	497		29	867
	16	941		20	100		23	318		26	608		29	981
	17	045		20	206		23	427		26	719		30	095
	17	150		20	312		23	535		26	830		30	300
17	17	255	20	20	419	23	23	643	26	26	941	29	30	324
	17	359		20	525		23	752		27	052		30	438
	17	464		20	632		23	861		27	164		30	553
	17	568		20	738		23	970		27	275		30	667
	17	673		20	845		24	079		27	387		30	782
	17	778		20	952		24	188		27	499		30	897
	17	883		21	059		24	297		27	610		31	012
	17	988		21	165		24	406		27	722		31	127
	18	093		21	272		24	515		27	834		31	242
	18	198		21	379		24	624		27	946		31	357
18	18	303	21	21	486	24	24	734	27	28	058	30	31	473

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M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par
30	31	473	33	34	992	36	38	633	39	42	415	42	46	362
	31	588		35	111		38	757		42	544		46	496
	31	704		35	231		38	880		42	673		46	631
	31	820		35	350		39	004		42	802		46	766
	31	936		35	470		39	129		42	931		46	902
	32	052		35	590		39	253		43	061		47	037
	32	168		35	710		39	377		43	191		47	173
	32	284		35	830		39	502		43	320		47	309
	32	400		35	950		39	627		43	451		47	445
	32	517		36	071		39	752		43	581		47	581
31	32	633	34	36	191	37	39	877	40	43	711	43	47	718
	32	750		36	312		40	002		43	842		47	855
	32	867		36	433		40	128		43	973		47	992
	32	984		36	554		40	253		44	104		48	129
	33	101		36	675		40	379		44	235		48	267
	33	218		36	796		40	505		44	366		48	404
	33	336		36	917		40	631		44	498		48	542
	33	453		37	039		40	757		44	630		48	681
	33	571		37	161		40	884		44	762		48	819
	33	688		37	283		41	011		44	894		48	958
32	33	806	35	37	405	38	41	137	41	45	026	44	49	097
	33	924		37	527		41	264		45	159		49	236
	34	042		37	649		41	392		45	292		49	375
	34	161		37	771		41	519		45	425		49	515
	34	279		37	894		41	646		45	558		49	655
	34	397		38	017		41	774		45	691		49	795
	34	516		38	140		41	902		45	825		49	935
	34	635		38	263		42	030		45	959		50	076
	34	754		38	386		42	158		46	093		50	217
	34	873		38	509		42	287		46	227		50	358
33	34	992	36	38	633	39	42	415	42	46	362	45	50	499

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	part	M	Gr	part
45	50	499	48	54	860	51	59	481	54	64	412	57	69	711
	50	641		55	010		59	640		64	582		69	895
	50	783		55	160		59	800		64	753		70	080
	50	925		55	310		59	960		64	924		70	263
	51	068		55	460		60	120		65	096		70	449
	51	210		55	611		60	280		65	268		70	635
	51	353		55	762		60	441		65	440		70	821
	51	496		55	913		60	602		65	613		71	008
	51	639		56	065		60	763		65	786		71	195
	51	783		56	217		60	925		65	960		71	383
46	51	927	49	56	369	52	61	088	55	66	134	58	71	572
	52	071		56	522		61	250		66	308		71	761
	52	215		56	675		61	413		66	483		71	950
	52	360		56	828		61	577		66	659		72	140
	52	505		56	981		61	740		66	835		72	331
	52	650		57	135		61	904		67	011		72	522
	52	795		57	289		62	069		67	188		72	714
	52	941		57	444		62	234		67	365		72	906
	53	087		57	598		62	399		67	543		73	099
	53	233		57	754		62	564		67	721		73	292
47	53	380	50	57	909	53	62	730	56	67	900	59	73	486
	53	526		58	065		62	897		68	079		73	680
	53	673		58	221		63	063		68	258		73	875
	53	821		58	377		63	231		68	438		74	071
	53	968		58	534		63	398		68	618		74	267
	54	116		58	691		63	566		68	799		74	464
	54	264		58	848		63	734		68	981		74	661
	54	413		59	006		63	903		69	163		74	859
	54	562		59	164		64	072		69	345		75	057
	54	711		59	322		64	242		69	528		75	256
48	54	860	51	59	481	54	64	412	57	69	711	60	75	456

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr.	Par
60	75	456	63	81	749	66	88	725	69	96	575	72	105	579
	75	656		81	970		88	971		90	854		105	904
	75	857		82	191		89	219		97	135		106	230
	76	059		82	413		89	467		97	418		106	558
	76	261		82	635		89	716		97	701		106	888
	76	464		82	860		89	967		97	986		107	220
	76	667		83	084		90	218		98	272		107	553
	76	871		83	310		90	470		98	560		107	888
	77	076		83	536		90	723		98	849		108	226
	77	281		83	763		90	978		99	139		108	565
61	77	487	64	83	990	67	91	232	70	99	431	73	108	906
	77	694		84	219		91	489		99	724		109	249
	77	901		84	448		91	746		100	018		109	594
	78	109		84	678		92	005		100	314		109	941
	78	317		84	909		92	264		100	612		110	290
	78	526		85	141		92	525		100	910		110	641
	78	736		85	374		92	787		101	211		110	994
	78	947		85	607		93	050		101	513		111	349
	79	158		85	842		93	314		101	816		111	707
	79	370		86	077		93	579		102	121		112	066
62	79	583	65	86	313	68	93	846	71	102	427	74	112	428
	79	796		86	550		94	113		102	735		112	792
	80	010		86	788		94	382		103	044		113	158
	80	225		87	027		94	652		103	356		113	526
	80	441		87	267		94	923		103	668		113	897
	80	657		87	508		95	195		103	983		114	270
	80	874		87	749		95	468		104	299		114	645
	81	091		87	992		95	743		104	616		115	023
	81	310		88	235		96	019		104	936		115	403
	81	529		88	480		96	296		105	257		115	786
63	81	749	66	88	725	69	96	575	72	105	579	75	116	171

M	Gr.	Par	M	Gr.	Par	M	Gr.	Par	M	Gr.	Par	M	Gr.	part
75	116	171	78	129	075	81	145	650	84	168	947	87	208	705
	116	559		129	558		146	292		169	912		210	649
	116	949		130	045		146	942		170	893		212	668
	117	342		130	536		147	600		171	891		214	745
	117	737		131	031		148	265		172	907		216	909
	118	135		131	530		148	937		173	941		219	158
	118	536		132	034		149	618		174	994		221	498
	118	939		132	542		150	307		176	067		223	938
	119	345		133	055		151	003		177	160		226	486
	119	755		133	572		151	709		178	275		229	153
76	120	166	79	134	094	82	152	423	85	179	411	88	231	950
	120	581		134	620		153	147		180	569		234	891
	121	000		135	151		153	878		181	752		237	991
	121	420		135	687		154	620		182	960		241	268
	121	843		136	228		155	372		184	194		244	744
	122	270		136	775		156	132		185	454		248	445
	122	700		137	326		156	903		186	743		252	402
	123	133		137	883		157	685		188	062		256	652
	123	570		138	445		158	478		189	411		261	243
	124	009		139	012		159	281		190	793		266	235
77	124	452	80	139	585	83	160	096	86	192	210	89	271	705
	124	898		140	164		160	922		193	661		277	753
	125	348		140	748		161	761		195	151		284	517
	125	801		141	339		162	612		196	680		292	191
	126	258		141	936		163	475		198	251		301	058
	126	718		142	538		164	352		199	867		311	563
	127	182		143	147		165	242		201	529		324	455
	127	649		143	763		160	146		203	240		341	166
	128	121		144	385		267	065		205	005		365	039
	128	596		145	014		167	999		206	825		408	011
78	129	075	81	145	650	84	168	947	87	208	705	90	Infinite	

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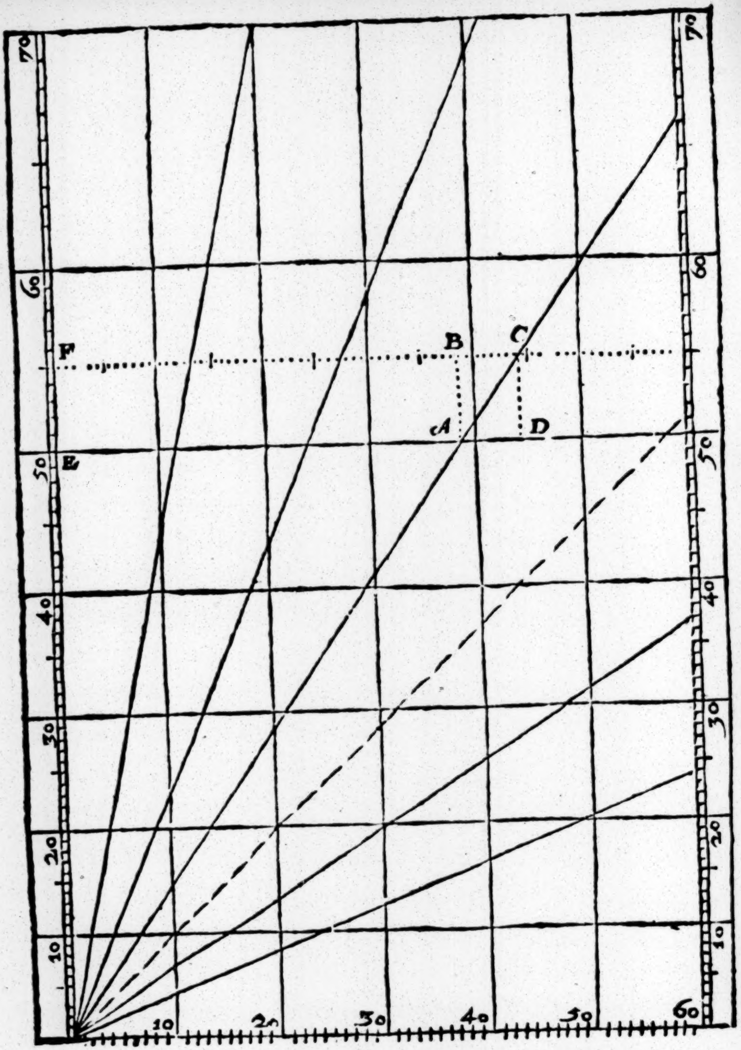
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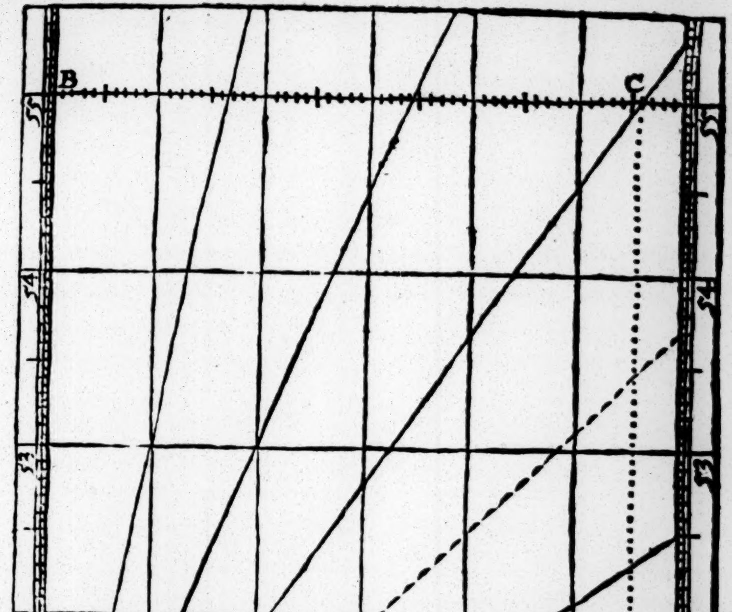

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If any deſire to haue his chart to agree with his *Sector*, he may make each degree of longitude equall to the tenth part of the line of *lines*, and diuide the meridian of his chart out of the Sector: ſo ſhall each degree of the chart, be ten times as large as the like degree on the *Sector*, and the worke be eaſie from the one to the other.

2 *To find how many leagues anſwer to one degree of longitude in euery ſeuerall latitude.*

In ſailing by the compaſſe, the courſe holds ſometime vpon a great circle, ſometime vpon a parallel to the equator; but moſt commonly vpon crooked lines winding towards one of the poles, which lines are well knowne by the name of *Rumbs*.

If the courſe hold vpon a great circle, it is either North or South, vnder ſome meridian, or Eaſt or Weſt, vnder the equator. And in theſe caſes, euery degree requires an allowance of twentie leagues, euery twentie leagues will make a degrees difference in the ſailing: ſo that here needs no further precept then the rule of proportion in the Chapter of *lines*.

But if the courſe hold Eaſt or Weſt, on any of the parallels to the equator;

As the Radius
  is to twentie leagues, the meaſure of one degree
  at the equator:
So the ſine of the complement of the latitude
  to the meaſure of leagues anſwering to one degree
  in that latitude.

Wherefore I take 20 leagues out of the line of *lines*, and make it a parallell Radius, by fitting it ouer in the ſines of 90 and 90: ſo his parallell ſine taken out of the complement of the latitude, and meaſured in the line of *lines*, ſhall ſhew the number of leagues required.

Thus in the latitude of 18 *gr.* 12 *m.* we ſhall find 19 leagues anſwering to one degree of longitude, and 18 leagues in the latitude of 25 *gr.* 15 *m.* and as in this Table.

Gr.	'	*Lg.*
0	0	20
18	12	19
25	15	18
31	48	17
36	52	16
41	25	19
45	34	14
49	28	13
53	8	12
55	38	11
60	0	10
63	15	9
66	25	8
69	30	7
72	32	6
75	31	5
78	28	4
81	23	3
84	15	2
87	8	1

This may be done more readily without opening the Sector, by doubling the ſine of the complement of the latitude, as may appeare in the ſame example.

It may alſo be done by the line of meridians, either vpon the Sector, or vpon the chart. For if we open a paire of compaſſes to the quantitie of one degree of longitude in the equator, and meaſure it in the meridian line, ſetting one foot as much aboue the latitude giuen, as the other falleth beneath it, ſo that the latitude may be in the middle betweene the feete of the compaſſes, the number of leagues intercepted ſhall be that which was required.

But if the courſe hold vpon any of the *rumbs*, betweene a parallell of the equator and the meridian, we are to conſider beſides the quarter of the world to which we tend, which muſt be alwayes knowne.

1 The difference of longitude at leaſt in generall.
2 The difference of latitude, and that in particular.
3 The *rumb* whereon the courſe holds.
4 The diſtance vpō the *rumb*, which is the diſtance, which we are here to conſider, and is alwayes ſomewhat greater then the like diſtance vpon a greater circle. And for theſe firſt I ſhew in generall this third *Prop.*

3 *To finde how many leagues do anſwer to one degree of latitude in euery ſeuerall Rumb.*

As the ſine of the complement of the *rumb* frō the meridian,
is to 20 leagues the meaſure of one degree at the meridian:
So the Radius
to the leagues anſwering to one degree vpon the Rumb

Wherfore I take 20 leagues out of the line of *lines*, and make it a parallell ſine of the complement of the Rumb from the meridian; ſo his parallell Radius taken and meaſured in the line of *lines*, ſhall ſhew the number of leagues required.

Thus in the firſt Rumb from the meridian, we ſhall finde 20 *lgs* 39 *parts* anſwering to one degree of latitude, and 21 *lgs* 65 *parts* in the ſecond Rumb, &c. as in this Table, where we ſubdiuide each league into a hundred parts, and ſhew beſides what inclination the rumb hath to the meridian.

This may be done more readily without opening the *Sector*, by doubling the ſecant of the latitude, as may appeare in the ſame example.

It may alſo be done vpon the chart, if we take the diſtance vpon the Rumb betweene two parallels, and meaſure it in the meridian line, as farre aboue the greater latitude as beneath the leſſer. For ſo the number of leagues intercepted, ſhall be that which was required.

This conſidered in generall, I ſhew more particularly in twelue *Prop.* following, how of theſe foure any two being giuen, the other two may be found, both by *Mercators* chart, and by this *Sector*.

Rumbs.	Inclinatiō to the meridian		Numb. of leags.	
	Gr.	Mi.	Lgs	Par
	2	49	20	02
	5	37	20	10
	8	26	20	22
1	11	15	20	39
	14	4	20	62
	16	52	20	90
	19	41	21	24
2	22	30	21	65
	25	19	22	12
	28	7	22	68
	30	56	23	32
3	33	45	24	05
	36	34	24	90
	39	22	25	87
	42	11	26	99
4	45	0	28	28
	47	49	29	78
	50	37	31	52
	53	26	33	57
5	56	15	36	00
	59	4	38	90
	61	52	42	43
	64	41	36	78
6	67	30	52	26
	70	19	59	37
	73	7	68	90
	75	56	82	31
7	78	45	102	52
	81	34	136	30
	84	22	205	24
	87	11	407	60
8	90	0	Infinita.	

1 By one latitude Rumb and diſtance to find the difference of latitudes.

As the Radius
to the ſine of the complement of the Rumb from the meridian:
So the diſtance vpon the Rumb,
to the difference of latitudes.

Let the place giuen be *A* in the latitude of 50 *gr.* *C* in a greater latitude, but vnknowne, the diſtance vpon the Rumb being 6 *gr.* betweene them, and the Rumb the third from the meridian.

Firſt I take 6 *gr.* for the diſtance vpon the Rumb, out of the line of *lines*, and make it a parallell Radius, by putting it ouer in the ſines of 90 and 90. Then keeping the *Sector* at this angle, I take out the parallell ſine of 56 *gr.* 15 *m.* which is the ſine of the complement of the third Rumb from the meridian, and meaſuring it in the line of *lines*, I find it to be 5 *gr.* and ſuch is the difference of latitude required.

Or I may take out the ſine of 56 *gr.* 15 *m.* for the complement of the third Rumb from the meridian, make it a parallell Radius; then keeping the *Sector* at this angle, I take 6 *gr.* for the diſtance, either out of the line of *lines*, or any other ſcale of equall parts, or elſe out of the meridian line, and lay it on both ſides of the *Sector* from the center, either on the line of *lines* or *ſines*: ſo the parallell taken from the termes of this diſtance, and meaſured in the ſame ſcale wherein the diſtance was meaſured, ſhall ſhew the difference of latitude to be 5 *gr.* as before.

But in ſhorter diſtances, ſuch as fall within the compaſſe of a dayes ſailing, this worke will hold much better. As may appeare by comparing the worke with the Table following: where the numbers in the front do ſignifie the leagues; thoſe in the ſide, the Rumb; and the reſt in the middle, the difference of latitude.

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Lgs	100		80		60		40		20	19	18	17	16	15
Rum.	G.	M	G.	M	G.	M	G.	M	M	M	M	M	M	M
	5	0	4	0	3	0	2	0	60	57	54	51	48	45
	4	59	3	59	2	59	1	59	60	57	54	51	48	45
	4	58	3	58	2	59	1	59	60	57	54	51	48	45
	4	56	3	57	2	58	1	58	59	56	53	50	47	44
1	4	54	3	55	2	56	1	57	59	56	53	50	47	44
	4	51	3	53	2	55	1	56	58	56	52	50	47	43
	4	47	3	50	2	52	1	55	57	55	52	49	46	43
	4	42	3	46	2	49	1	53	56	54	51	48	45	42
2	4	37	3	42	2	46	1	51	55	53	50	47	44	41
	4	31	3	37	2	43	1	48	54	52	49	46	43	40
	4	25	3	32	2	39	1	46	53	50	48	45	42	39
	4	17	3	26	2	34	1	43	51	49	46	44	41	38
3	4	10	3	20	2	30	1	40	50	47	45	42	40	37
	4	1	3	13	2	25	1	36	48	46	43	41	39	36
	3	52	3	5	2	19	1	32	46	44	42	39	37	35
	3	42	2	58	2	13	1	28	44	42	40	38	36	33
4	3	32	2	50	2	7	1	25	42	40	38	36	34	32
	3	22	2	41	2	1	1	21	40	38	36	34	32	30
	3	10	2	32	1	54	1	16	38	36	34	32	30	28
	2	59	2	23	1	47	1	12	36	34	32	30	29	27
5	2	47	2	14	1	40	1	7	33	32	30	28	27	25
	2	34	2	3	1	32	1	2	31	29	28	26	25	23
	2	22	1	53	1	25	0	57	28	27	25	24	23	22
	2	8	1	43	1	17	0	52	26	24	23	22	21	19
6	1	55	1	32	1	8	0	46	23	22	21	20	18	17
	1	41	1	20	1	0	0	40	20	19	18	17	16	15
	1	27	1	9	0	52	0	35	17	16	16	15	14	13
	1	13	0	58	0	44	0	30	15	14	13	12	12	11
7	0	59	0	47	0	35	0	24	12	11	11	10	9	9
	0	44	0	36	0	26	0	18	9	8	8	7	7	7
	0	30	0	24	0	18	0	12	6	6	5	5	5	4
	0	15	0	12	0	9	0	9	3	3	3	3	2	2
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0

14	13	12	11	10	9	8	7	6	5	4	3	2	1	Lg
M	M	M	M	M	M	M	M	M	M	M	M	M	M	Rum.
40	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
41	38	35	32	29	26	24	21	18	15	12	9	6	3	1
41	38	35	32	29	26	23	20	17	15	12	9	6	3	
40	37	34	32	29	26	23	20	17	14	11	9	6	3	
40	37	34	31	28	25	23	20	17	14	11	8	6	3	
39	36	33	31	28	25	22	19	17	14	11	8	6	3	2
38	35	33	30	27	24	22	19	16	14	11	8	5	3	
37	34	32	29	26	24	21	19	16	13	11	8	5	3	
36	33	31	28	26	23	21	18	15	13	10	8	5	3	
35	32	30	27	25	22	20	17	15	12	10	7	5	2	3
34	30	29	26	24	22	19	17	14	12	10	7	5	2	
33	30	28	26	23	21	19	16	14	12	9	7	5	2	
31	29	27	24	22	20	18	16	13	11	9	7	4	2	
30	28	25	23	21	19	17	15	13	11	8	6	4	2	4
28	26	24	22	20	18	16	14	12	10	8	6	4	2	
27	25	23	21	19	17	15	13	11	10	8	6	4	2	
25	23	21	20	18	16	14	13	11	9	7	5	4	2	
23	22	20	18	17	15	13	12	10	8	7	5	3	2	5
22	20	18	17	15	14	12	11	9	8	6	5	3	2	
20	18	17	16	14	13	11	10	8	7	6	4	3	1	
18	17	15	14	13	12	10	9	8	6	5	4	3	1	
16	15	14	13	11	10	9	8	7	6	5	3	2	1	6
14	13	12	11	10	9	8	7	6	5	4	3	2	1	
12	11	10	10	9	8	7	6	5	4	3	3	2	1	
10	9	9	8	7	7	6	5	4	4	3	2	1	1	
8	8	7	6	6	6	5	4	3	3	2	2	1	1	7
6	6	5	5	4	5	4	3	3	2	2	1	1	1	
4	4	4	3	3	3	2	2	2	1	1	1	1	0	
2	2	2	2	1	1	1	1	1	1	1	1	1	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	8

In the Chart let a meridian *A B* be drawne through *A*, and in *A* with *A* B make an angle of the Rumb *B A C*. Then open the compaſſes, accrding to the latitude of the places, to *E F* the quantitie of 6 *gr.* in the meridian, transferring them into the Rumb from *A* to *C*, and through *C* draw the parallell *B C*, croſſing the meridian *A B* in *B*: ſo the degrees in the meridian from *A* to *B*, ſhall ſhew the difference of latitude to be 5 *gr.*

2 *By the Rumb and both latitudes to find the diſtance vpon the Rumb.*

As the ſine of the complement of the Rumb from the meridian,
is to the Radius:
So the difference of latitudes,
to the diſtance vpon the Rumb.

As if the places giuen were *A* in the latitude of 50 *gr.* C in the latitude of 55 *gr.* and the Rumb the third from the meridian.

Here I may take 5 *gr.* for the difference of latitude out of the line of *lines*, and put it ouer in the ſine of 56 *gr.* 15 *m.* for the complement of the third Rumb from the meridian. Then keeping the *Sector* at this angle, I take out the parallell Radius, and meaſuring it in the line of *lines*, I find it to be 6 *gr.* and ſuch is the diſtance vpon the Rumb, which was required.

Or I may take the laterall Radius, and make it a parallell ſine of 56 *gr.* 15 *m.* the complement of the Rumb from the meridian: then keeping the Sector at this angle, I take 5 *gr.* for the difference of latitude, either out of the line of *lines*, or out of ſome other ſcale of equall parts, and lay it on both ſides of the *Sector* from the center, either on the line of *lines* or of *ſines*: ſo the parallell taken from the termes of this difference, and meaſured in the ſame ſcale with the difference, ſhall ſhew the diſtance vpon the Rumb to be 6 *gr.* or 120 leagues.

Or keeping the *Sector* at this angle, I may take the difference betweene 50 *gr.* and 55 *gr.* out of the *Meridian* line, and meaſuring it in the equator, I ſhal find it to be equall to 8 *gr.* 22 *p.* of the equator. Wherefore I take the parallell betweene 822 and 822 out of the line of *lines*, and meaſuring it in the line of *lines* I ſhall find it to be 989; which ſhewes that according to this proiection, the diſtance vpon this third Rumb, anſwerable to the former diſtance of latitudes, will be equall to 9 *gr.* 89 *p.* of the equator.

Or the Sector remaining at this angle, I may take the difference betweene 50 *gr.* and 55 *gr.* out of the *Meridian* line, and lay it from the center on both ſides of the *Sector*, either on the line of *lines* or of *ſines*: ſo the parallell taken from the termes of this difference, ſhall be the very line of diſtance required, the ſame with *A C* or *E F* vpon the chart; which may ſerue for the better pricking downe of the diſtance vpon the Rumb, without taking it forth of the *Meridian* line, as in the former *Prop.*

Or if the Rumb fall nearer to the equator, that the laterall Radius cannot be fitted ouer in it, this propoſition may be wrought by parallell entrance.

For if I firſt take out the ſine of 56 *gr.* 15 *m.* and make it a parallell Radius, by fitting it ouer in the ſines of 90 and 90, or in the ends of the lines of *lines*, and then take 5 *gr.* for the difference of latitudes out of the line of *lines*, and carrie it parallell to the former, I ſhall find it to croſſe both lines of *lines* in the points of 6: and ſo it giues the ſame diſtance as before.

Or if the diſtance be ſmall, it may be found by the former Table. For the Rumb being found in the ſide of the Table, and the difference of latitude in the ſame line; the top of the columne wherein the difference of latitude was found, ſhall giue the number of leagues in the diſtance required.

Or we may finde this diſtance in the Table of Rumbs in the fift *Prop.* following. For according to the example looke into the Table of the third Rumb for 5 *gr.* of latitude, and there we ſhall finde 6 *gr.* 01 *parts* vnder the title of diſtance.

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So if the difference of latitude vpon the ſame Rumb were 50 *gr.* the diſtance would be 60 *gr.* 13 *parts.* If the difference of latitude vpon the ſame Rumb were onely ½ of a degree, the diſtance would be onely 60 *parts*, ſuch as 100 doe make a degree.

In the chart let a Meridian *A B* be drawne through *A*, and parallels of latitude through *A* and *C*; & then in *A* with *A B* make an angle of the Rumb *B A C*: ſo the diſtance taken from *A* to *C*, and meaſured in the Meridian line, according to the latitude of the places, ſhall be found to be 6 *gr.* or 120 leagues. And ſuch is the diſtance required.

3 *By the diſtance and both latitudes to find the Rumb.*

As the diſtance vpon the Rumb,
  to the difference of latitudes:
So is the Radius (ridian.
  to the ſine of the complement of the Rumb from the Me-

As if the places giuen were *A* in the latitude of 50 *gr.* *C* in the latitude of 55 *gr.* the diſtance betweene them being 6 *gr.* vpon the Rumb. Firſt I take 6 *gr.* for the diſtance vpon the Rumb, and lay it on both ſides of the *Sector* from the center; then out of the ſame ſcale I take 5 *gr.* for the difference of latitude, and to it open the *Sector* in the termes of the former diſtance: ſo the parallell Radius taken and meaſured in the *ſines*, doth giue 56 *gr.* 15 *m.* the complement whereof 33 *gr.* 45 *m.* is the angle of the Rumbs inclination to the Meridian, which was required.

In the chart let a meridian *A B* be drawne through *A*, and parallels of latitude both through *A* and *C*; then open the compaſſes according to the latitude of the places to *E F* the quantitie of 6 *gr.* in the meridian, and ſetting one foote in *A*, turne the other till it croſſe the parallell *B C* in *C*, and draw the right line *A C*: ſo the angle *B A C* ſhall ſhew the inclination of the Rumb to the Meridian to be 33 *gr.* 45 *m.* as before.

Theſe three laſt *Prop.* depend one on the other, and may be wrought as truly by the common ſea-chart as by this of *Mercators* proiection: and therefore in working them by the *Sector*, the diſtance and the difference of latitudes may as well or better be taken out of the line of *lines* (which here repreſenteth the equator) or any other line of equall parts, as out of the inlarged degrees in the *meridian* line. But in the propoſitions following, the difference of longitude muſt be taken out of the equator; the difference of latitudes and diſtance vpon the Rumb, muſt alwayes be taken out of the *meridian* line; which I therefore call the proper difference, and proper diſtance.

4 *By the longitude and latitude of two places to find the Rumb.*

As if the places giuen were *A* in the latitude of 50 *gr.* *C* in the latitude of 55 *gr.* and the difference of longitude betweene them were 5 *gr.* 30 *m.*

In the chart let meridians and parallels be drawn through *A* and *C*, and a ſtraight line for the Rumb from *A* to *C*; then by that we ſhewed *Cap.2.Prop.9.* inquire the quantitie of the angle *B A C*, and it ſhall be found to be 33 *gr.* 45 *m.* which is the third Rumb from the Meridian. Wherefore the proportion holds for the *Sector*,

As *A B* the proper difference of latitude,
is to *B C* the difference of longitude:
So *A B* as Radius,
to *B C* the tangent of the Rumb from the Meridian.

According to this I take the proper difference of latitude from 50 *gr.* to 55 *gr.* out of the line of *meridians*, and lay it on both ſides of the *Sector* from the center; then I take the difference of longitude 5 *gr.* $\frac{1}{2}$ out of the line of *lines*, and to it open the *Sector* in the termes of the former difference of latitudes: ſo the parallell Radius taken from betweene 90 and 90, and meaſured in the greater *tangent* on the ſide of the *Se-*

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ctor,doth giue 33 *gr*.45 *m*. for the Rumb required.

But if the Rumb fall nearer to the equator;

As *A D* the difference of longitudes,
is to *D C* the proper difference of latitudes:
So *A D* as Radius,
to *D C* the tangent of the rumb from the equator.

According to this I take the former difference of latitudes from 50 *gr*.to 55 *gr*. out of the line of *Meridians*, and to it open the *Sector* in the termes of the difference of longitude reckoned in the line of *lines* from the center: ſo the parallell Radius taken and meaſured in the *tangent*,doth giue 56 *gr*.15 *m*.for the rumb from the equator; which is the complement to the former 33 *gr*.45 *m*: and ſo both wayes it is found to be the third rumb from the Meridian.

But if this rumb were to be found in the common ſea-chart,it ſhould ſeeme to be aboue 47 *gr*. which is more then the fourth rumb from the meridian.

5 *By the Rumb and both latitudes to find the difference of longitude.*

As if the places giuen were *A* in the latitude of 50 *gr*. and *C* in the latitude of 55 *gr*. and the rumb the third from the meridian.

In the chart, let a meridian be drawne through *A*, and a parallell of latitude through *C*; then in *A* with *A B* make the angle of the rumb from the meridian *B A C*, (as was ſhewed *Cap*.2.*Prop*.10.) So the degrees in the parallell betweene B and *C*,ſhall be found to be 5 *gr*.$\frac{1}{2}$, the difference of longitude which was required Wherefore the proportion holds for the *Sector*.

As *A B* the Radius,
to B C the tangent of the rumb from the meridian:
So *A B* as proper difference of the latitudes,
to *B C* the difference of longitude.

According to this we may take the tangent of the Rumb which is here 33 *gr.*45 *m.*from the meridian, out of the greater *tangent* on the ſide of the *Sector*; and putting it ouer betweene 90 and 90, make it a Radius: then keeping the *Sector* at this angle, take the proper difference of latitudes from 50 *gr.*to 55 *gr.*out of the line of *Meridians*, and lay it on both ſides of the *Sector* from the center: ſo the parallel taken from the termes of this difference, and meaſured in the line of *lines*, ſhall ſhew the difference of longitude to be 5 *gr.*$\frac{1}{2}$.

Or if the Rumb fall nearer the equator.

As *D C* the tangent of the *R*umb from the equator,
  to *A D* the *R*adius:
So *D C* as proper difference of the latitudes,
  to *A D* the difference of longitude.

According to this we may beſt work by parallell entrance, firſt taking 56 *gr.*15 *m.*for the angle of the *R*umb from the equator, out of the greater *tangent*, and make it a parallell *R*adius: then take the proper difference of latitudes out of the line of *meridians*, and carrie it parallell to the former: ſo we ſhall find it to croſſe the line of *lines* in 5 *gr.*$\frac{1}{2}$. And this is the difference of longitude required, the ſame as before.

But if this difference were to be found by the common ſea-chart, it ſhould ſeeme to be only 3 *gr.*20 *m.*which is more then 2 *gr.*leſſe then the truth. And yet this error would be greater, if either the latitude be greater, or the *R*umb fall nearer the equator: as may appeare by comparing the common ſea-chart with the Tables following.

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The first Rumbe from the Meridian.					North and by East, South and by East,					North and by West, South and by West.				
La	Long.		Dist.		La	Long.		Dist.		La	Long.		Dist.	
Gr	Gr.	P.	Gr.	P	Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	6	26	30	59	60	15	01	61	18
1		20	1	02	31	6	49	31	61	61	15	41	62	20
2		40	2	04	32	6	72	32	63	62	15	83	63	21
3		60	3	06	33	6	96	33	65	63	16	26	64	23
4		80	4	08	34	7	20	34	67	64	16	71	65	25
5	1	00	5	10	35	7	44	35	69	65	17	17	66	27
6	1	20	6	12	36	7	68	36	71	66	17	65	67	29
7	1	40	7	14	37	7	92	37	73	67	18	15	68	31
8	1	60	8	16	38	8	17	38	75	68	18	67	69	33
9	1	80	9	18	39	8	43	39	77	69	19	21	70	35
10	2	00	10	20	40	8	70	40	78	70	19	78	71	37
11	2	20	11	22	41	8	96	41	80	71	20	37	72	39
12	2	40	12	24	42	9	22	42	82	72	21	00	73	41
13	2	61	13	25	43	9	50	43	84	73	21	66	74	43
14	2	81	14	27	44	9	76	44	86	74	22	36	75	45
15	3	02	15	29	45	10	04	45	88	75	23	10	76	47
16	3	22	16	31	46	10	33	46	90	76	23	90	77	49
17	3	43	17	33	47	10	62	47	92	77	24	7[illegible]	78	51
18	3	64	18	35	48	10	91	48	94	78	25	67	79	53
19	3	85	19	37	49	11	21	49	96	79	26	67	80	55
20	4	06	20	39	50	11	52	50	98	80	27	76	81	57
21	4	27	21	41	51	11	83	52	0	81	28	97	82	59
22	4	49	22	43	52	12	15	53	2	82	30	32	83	61
23	4	70	23	45	53	12	47	54	4	83	31	84	84	63
24	4	92	24	47	54	12	81	55	6	84	33	61	85	62
25	5	14	25	49	55	13	16	56	8	85	35	69	86	67
26	5	36	26	51	56	13	50	57	10	86	38	24	87	69
27	5	58	27	53	57	13	86	58	12	87	41	52	88	71
28	5	80	28	55	58	14	23	59	14	88	46	15	89	73
29	6	03	29	57	59	14	62	60	16	89	54	06	90	75
30	6	26	30	59	60	15	01	61	18	90				

The ſecond Rumbe from the Meridian.					North North-eaſt. South South-eaſt.					North North-weſt South South-weſt				
La	Long.		Diſt.		La	Long		Diſt.		La	Long.		Diſt.	
Gr	Gr.	P	Gr.	P	Gr	Gr.	P	Gr.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	13	03	32	47	60	31	25	64	94
1	0	42	1	08	31	13	51	33	55	61	32	09	66	03
2	0	83	2	16	32	14	00	34	64	62	32	96	67	11
3	1	24	3	25	33	14	49	35	72	63	33	86	68	19
4	1	65	4	33	34	15	00	36	80	64	34	79	69	27
5	2	07	5	41	35	15	50	37	88	65	35	75	70	35
6	2	49	6	49	36	16	00	38	97	66	36	75	71	44
7	2	91	7	57	37	16	51	40	05	67	37	80	72	52
8	3	32	8	66	38	17	03	41	13	68	38	88	73	60
9	3	74	9	74	39	17	56	42	21	69	40	00	74	68
10	4	16	10	82	40	18	10	43	30	70	41	19	75	77
11	4	59	11	90	41	18	65	44	38	71	42	43	76	85
12	5	01	12	99	42	19	20	45	46	72	43	74	77	93
13	5	43	14	07	43	19	76	46	54	73	45	11	79	01
14	5	85	15	15	44	20	33	47	62	74	46	57	80	10
15	6	28	16	23	45	20	92	48	71	75	48	12	81	18
16	6	71	17	32	46	21	50	49	79	76	49	78	82	26
17	7	14	18	40	47	22	11	50	87	77	51	55	83	34
18	7	58	19	48	48	22	72	51	95	78	53	46	84	42
19	8	01	20	56	49	23	35	53	03	79	55	54	85	51
20	8	45	21	65	50	23	98	54	12	80	57	82	86	59
21	8	90	22	73	51	24	63	55	20	81	60	33	87	67
22	9	34	23	81	52	25	30	56	28	82	63	13	88	76
23	9	79	24	89	53	25	98	57	37	83	66	32	89	84
24	10	24	25	98	54	26	68	58	45	84	69	99	90	92
25	10	70	27	06	55	27	39	59	53	85	74	32	92	00
26	11	16	28	14	56	28	12	60	61	86	79	63	93	09
27	11	62	29	22	57	28	87	61	70	87	86	46	94	17
28	12	08	30	31	58	29	64	62	78	88	96	10	95	25
29	12	55	31	39	59	30	44	63	86	89	112	57	96	33
30	13	03	32°	47	60	31	25	64	94	90				

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The third Rumbe from the Meridian. }					*North-eaſt by North, South-eaſt by South,*					*North-weſt by North, South-weſt by South.*				
La	*Long.*		*Diſt.*		*La*	*Long.*		Diſt.		*La*	*Long.*		Diſt.	
Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P
0		0		0	30	21	03	36	08	60	50	42	72	16
1	0	66	1	20	31	21	80	37	28	61	51	78	73	36
2	1	33	2	40	32	22	58	38	49	62	53	18	74	56
3	2	00	3	61	33	23	38	39	69	63	54	63	75	77
4	2	67	4	81	34	24	18	40	89	64	56	12	76	97
5	3	34	6	01	35	25	00	42	09	65	57	68	78	17
6	4	01	7	22	36	25	82	43	30	66	59	29	79	37
7	4	68	8	42	37	26	64	44	50	67	60	96	80	58
8	5	36	9	62	38	27	48	45	70	68	62	71	81	78
9	6	03	10	82	39	28	34	46	90	69	64	53	82	98
10	6	71	12	03	40	29	21	48	11	70	65	44	84	19
11	7	39	13	23	41	30	09	49	31	71	68	45	85	39
12	8	07	14	43	42	30	98	50	51	72	70	55	86	59
13	8	76	15	64	43	31	88	51	71	73	72	77	87	79
14	9	44	16	84	44	32	80	52	92	74	75	12	89	00
15	10	13	18	04	45	33	74	54	12	75	77	62	90	20
16	10	83	19	24	46	34	69	55	32	76	80	30	91	40
17	11	53	20	45	47	35	67	56	52	77	83	15	92	61
18	12	23	21	65	48	36	66	57	73	78	86	25	93	81
19	12	93	22	85	49	37	67	58	93	79	89	60	95	01
20	13	64	24	05	50	38	69	60	13	80	93	27	96	22
21	14	35	25	26	51	39	74	61	33	81	97	32	97	42
22	15	07	26	46	52	40	82	62	54	82	101	85	98	62
23	15	80	27	66	53	41	91	63	74	83	106	97	99	82
24	16	53	28	86	54	43	03	64	94	84	112	90	101	03
25	17	26	30	07	55	44	19	66	15	85	119	90	102	23
26	18	00	31	27	56	45	37	67	35	86	128	45	103	43
27	18	75	32	47	57	46	58	68	55	87	139	47	104	64
28	19	50	33	67	58	47	82	69	75	88	155	00	105	84
29	20	26	34	88	59	49	11	70	96	89	181	58	107	04
30	21	03	36	08	60	50	42	72	16	90				

The fourth Rumbe from the Meridian.					North-eaſt, South-eaſt,					North-weſt, South-weſt.				
La	Long.		Diſt.		La	Long.		Diſt.		La	Long.		Diſt.	
Gr	Gr.	P	Gr.	P	Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	31	4	42	43	60	75	46	84	85
1	1	00	1	41	31	32	63	43	84	61	77	49	86	27
2	2	00	2	83	32	33	81	45	25	62	79	58	87	68
3	3	00	4	24	33	34	99	46	67	63	81	75	89	09
4	4	00	5	66	34	36	19	48	07	64	83	99	90	51
5	5	01	7	07	35	37	41	49	50	65	86	31	91	92
6	6	01	8	49	36	38	63	50	91	66	88	73	93	34
7	7	02	9	90	37	39	88	52	33	67	91	23	94	75
8	8	03	11	31	38	41	14	53	74	68	93	85	96	17
9	9	04	12	73	39	42	42	55	15	69	96	58	97	58
10	10	05	14	14	40	43	71	56	57	70	99	43	98	99
11	11	07	15	56	41	45	03	57	98	71	102	43	100	41
12	12	09	16	97	42	46	36	59	40	72	105	58	101	82
13	13	11	18	38	43	47	72	60	81	73	108	91	103	24
14	14	14	19	80	44	49	10	62	22	74	112	43	104	65
15	15	17	21	21	45	50	50	63	64	75	116	17	106	06
16	16	21	22	63	46	51	93	65	05	76	120	17	107	48
17	17	25	24	04	47	53	38	66	46	77	124	45	108	89
18	18	30	25	45	48	54	86	67	88	78	129	08	110	31
19	19	36	26	87	49	56	37	69	29	79	134	10	111	72
20	20	42	28	28	50	57	91	70	71	80	139	59	113	14
21	21	49	29	70	51	59	48	72	12	81	145	65	114	55
22	22	56	31	11	52	61	09	73	54	82	152	42	115	96
23	23	64	32	53	53	62	73	74	95	83	160	10	117	38
24	24	73	33	94	54	64	41	76	37	84	168	95	118	79
25	25	28	35	35	55	66	13	77	78	85	179	41	120	21
26	26	94	36	77	56	67	90	79	20	86	192	21	121	62
27	28	06	38	18	57	69	71	80	61	87	208	71	123	04
28	29	18	39	60	58	71	57	82	02	88	231	95	124	45
29	30	32	41	01	59	73	49	83	44	89	271	71	125	86
30	31	47	42	43	60	75	46	84	85	90				

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The fift Rumbe from the Meridian. } *North-eaſt and by Eaſt, South-eaſt and by Eaſt,* *North-weſt and by Weſt, South-weſt and by Weſt,*

La	Long.		Diſt.		La	Long.		Diſt.		La	Long.		Diſt.	
Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	G.	P.	Gr	Gr.	P.	G.	P.
0		0		0	30	47	10	54	00	60	112	93	108	00
1	1	49	1	80	31	48	84	55	80	61	115	97	109	80
2	2	99	3	60	32	50	60	57	60	62	119	10	111	60
3	4	49	5	40	33	52	37	59	40	63	122	34	113	40
4	6	00	7	20	34	54	16	61	20	64	125	70	115	20
5	7	50	9	00	35	55	98	63	00	65	129	18	117	00
6	9	00	10	80	36	57	82	64	80	66	132	78	118	80
7	10	50	12	60	37	59	68	66	60	67	139	54	120	60
8	12	01	14	40	38	61	57	68	40	68	140	45	122	40
9	13	52	16	20	39	63	48	70	20	69	144	53	124	20
10	15	04	18	00	40	65	42	72	00	70	148	81	126	00
11	16	56	19	80	41	67	39	73	80	71	153	30	127	80
12	18	09	21	60	42	69	39	75	60	72	158	00	129	60
13	19	62	23	40	43	71	42	77	40	[illegible]	163	00	131	40
14	21	16	25	20	44	73	48	79	20	74	168	26	133	20
15	22	70	27	00	45	75	58	81	00	75	173	86	135	00
16	24	62	28	80	46	77	72	82	80	76	179	84	136	80
17	25	82	30	60	47	79	89	84	60	77	186	26	138	60
18	27	39	32	40	48	82	10	86	40	78	193	17	140	40
19	28	97	34	20	49	84	36	88	20	79	200	69	142	20
20	30	55	36	00	50	86	67	90	00	80	208	91	144	00
21	32	15	37	80	51	89	03	91	80	81	217	98	145	80
22	33	76	39	60	52	91	43	93	60	82	228	13	147	60
23	35	38	41	40	53	93	88	95	40	83	239	61	149	40
24	37	01	43	20	54	96	40	97	20	84	252	85	151	20
25	38	66	45	00	55	98	98	99	00	85	268	51	153	00
26	40	32	46	80	56	101	62	100	80	86	287	67	154	80
27	42	00	48	60	57	104	33	102	60	87	312	36	156	60
28	43	67	50	40	58	107	12	104	40	88	345	15	158	40
29	45	38	52	20	59	109	98	106	20	89	406	72	160	20
30	47	10	54	00	60	112	93	108	00	90				

The ſixt Rumbe from the Meridian.					Eaſt North-eaſt, Weſt North-weſt.					Eaſt South-eaſt, Weſt South-weſt.				
La	Long.		Diſt		La	Long.		Diſt.		La	Long.		Diſt	
Gr	G.	P.	Gr.	P.	Gr	Gr.	P.	G.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	75	98	78	39	60	182	18	156	78
1	2	41	2	61	31	78	78	81	00	61	187	07	155	40
2	4	83	5	23	32	81	61	83	62	62	192	13	162	01
3	7	25	7	84	33	84	48	86	23	63	197	36	164	62
4	9	66	10	45	34	87	37	88	84	64	202	77	167	24
5	12	08	13	06	35	90	30	91	46	65	208	38	169	85
6	14	51	15	68	36	93	27	94	07	66	214	20	172	46
8	16	94	18	29	37	96	27	96	68	67	220	25	175	08
7	19	37	20	90	38	99	31	99	30	68	226	57	177	69
9	21	81	23	52	39	102	40	101	91	69	233	15	180	30
10	24	26	26	13	40	105	53	104	52	70	240	06	182	92
11	26	71	28	74	41	108	71	107	14	71	247	27	185	53
12	29	17	31	36	42	111	93	109	75	72	254	90	188	14
13	31	65	33	97	43	115	20	112	36	73	262	92	190	75
14	34	14	36	58	44	118	53	114	97	74	271	43	193	37
15	36	63	39	20	45	121	92	117	59	75	280	46	195	98
16	39	1	41	81	46	125	36	120	20	76	290	11	198	59
17	41	65	44	42	47	128	87	122	81	77	300	46	201	21
18	44	18	47	03	48	132	44	125	43	78	311	62	203	82
19	46	73	49	65	49	136	09	128	04	79	323	73	206	43
20	49	29	52	26	50	139	81	130	65	80	337	00	209	05
21	51	87	54	87	51	143	60	133	27	81	351	64	211	66
22	54	47	57	49	52	147	47	135	88	82	368	00	214	27
23	57	08	60	10	53	151	44	138	49	83	386	51	216	89
24	59	71	62	71	54	155	50	141	10	84	407	89	219	50
25	62	36	65	33	55	159	66	143	72	85	433	13	222	11
26	65	04	67	94	56	163	93	146	33	86	464	05	224	73
27	67	74	70	55	57	168	31	148	95	87	503	88	227	34
28	70	46	73	17	58	172	80	151	56	88	560	00	229	95
29	73	20	75	78	59	177	42	154	17	89	656	08	232	56
30	75	98	78	39.	60	182	18	156	78	90				

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The ſeuenth Rumbe from the Meridian. } Eaſt and by North, Weſt and by North, Eaſt and by South, Weſt and by South.

La.	Long.		Diſt.		La.	Long.		Diſt.		La.	Long.		Diſt.	
Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P	Gr.	P	Gr	Gr.	P.	Gr.	P.
0		0		0	30	158	23	153	77	60	379	35	307	55
1	5	02	5	12	31	164	06	158	90	61	389	56	312	67
2	10	05	10	25	32	169	96	164	02	62	400	10	317	80
3	15	08	15	38	33	175	92	169	15	63	410	98	322	93
4	20	12	20	50	34	181	95	174	28	64	422	26	328	05
5	25	16	25	63	35	188	04	179	40	65	433	94	333	18
6	30	21	30	75	36	194	22	184	53	66	446	03	338	30
7	35	27	35	88	37	200	48	189	65	67	458	66	343	43
8	40	34	41	00	38	206	82	194	78	68	471	80	348	55
9	45	42	46	13	39	213	24	199	90	69	485	52	353	68
10	50	52	51	26	40	219	76	205	03	70	499	89	358	81
11	55	63	56	38	41	226	37	210	16	71	514	94	363	93
12	60	77	61	51	42	233	08	215	28	72	530	79	369	86
13	65	92	66	63	43	239	90	220	41	73	547	52	374	18
14	71	09	71	76	44	246	84	225	53	74	505	22	379	31
15	76	28	76	88	45	253	89	230	66	75	584	03	384	43
16	81	50	82	01	46	261	05	235	79	76	604	13	389	56
17	86	75	87	14	47	268	36	240	91	77	625	67	394	69
18	92	02	92	26	48	275	80	246	04	78	648	91	399	81
19	97	31	97	39	49	283	40	251	16	79	674	15	404	94
20	102	64	102	51	50	291	13	256	29	80	701	75	410	06
21	108	01	107	64	51	299	03	261	41	81	732	25	415	19
22	113	42	112	77	52	307	11	266	54	82	766	30	420	32
23	118	87	117	89	53	315	37	271	69	83	804	86	425	44
24	124	35	123	02	54	323	82	276	79	84	849	38	430	57
25	129	87	128	14	55	332	48	281	92	85	901	98	435	69
26	135	44	133	27	56	341	36	287	04	86	966	31	440	82
27	141	05	138	40	57	350	47	292	17	87	1049	26	445	94
28	146	71	143	52	58	359	81	297	30	88	1166	11	451	07
29	152	44	148	65	59	369	45	302	42	89	1366	23	456	20
30	158	23	153	77	60	379	35	307	55	90				

The eight Rumbe of Eaſt and Weſt, with the Longitude anſwering to one degree of diſtance, and the diſtance belonging to one degree of Longitude.

La	Long.		Diſt.		La	Long.		Diſt.		La	Long.		Diſt.	
Gr	Gr.	P.	Parts.		Gr	Gr.	P.	Parts.		Gr	Gr.	P.	Parts.	
0		0	100	00	30	1	25	86	60	60	2	00	50	00
1	1	00	99	98	31	1	17	85	71	61	2	06	48	48
2	1	00	99	94	32	1	18	84	80	62	2	13	46	94
3	1	00	99	86	33	1	19	83	86	63	2	20	45	40
4	1	00	99	75	34	1	21	82	90	64	2	28	43	83
5	1	00	99	62	35	1	22	81	91	65	2	37	42	26
6	1	01	99	45	36	1	24	80	90	66	2	46	40	67
7	1	01	99	25	37	1	25	79	86	67	2	56	39	07
8	1	01	99	02	38	1	27	78	80	68	2	67	37	46
9	1	01	98	76	39	1	29	77	71	69	2	79	35	83
10	1	02	98	48	40	1	31	76	60	70	2	92	34	20
11	1	02	98	16	41	1	33	75	47	71	3	07	32	55
12	1	02	97	81	42	1	35	74	31	72	3	24	30	90
13	1	03	97	43	43	1	37	73	13	73	3	42	29	23
14	1	03	97	03	44	1	39	71	93	74	3	63	27	56
15	1	03	96	59	45	1	41	70	71	75	3	86	25	88
16	1	04	96	12	46	1	44	69	46	76	4	13	24	19
17	1	04	95	63	47	1	47	68	20	77	4	44	22	49
18	1	05	95	10	48	1	49	66	91	78	4	81	20	79
19	1	06	94	55	49	1	52	65	6	79	5	24	19	08
20	1	06	93	97	50	1	55	64	28	80	5	76	17	36
21	1	07	93	35	51	1	59	62	93	81	6	39	15	64
22	1	08	92	72	52	1	62	61	56	82	7	18	13	91
23	1	09	92	05	53	1	66	60	18	83	8	20	12	18
24	1	09	91	35	54	1	70	58	77	84	9	57	10	45
25	1	10	90	63	55	1	74	57	35	85	11	47	8	71
26	1	11	89	88	56	1	79	55	92	86	14	33	6	97
27	1	12	89	10	57	1	84	54	46	87	19	11	5	2[illegible]
28	1	13	88	29	58	1	89	[illegible]2	99	88	28	65	3	49
29	1	14	87	46	59	1	94	51	50	89	57	30	1	74
30	1	15	86	60	60	2	00	50	00	90				0

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Theſe tables are calculated for each of the Rumbs. The firſt ſeuen haue three columnes, and of them the firſt containeth the degrees of Latitude, from the Equinoctiall to the Pole: the ſecond doth giue the difference of Longitude; and the third the diſtance, both of them belonging to that Rumb and latitude.

As in the Table of the third Rumb; at the *latitude* of 50 *Gr.* I find vnder the title of *Longitude* 38 *Gr.* 69 *parts*, and vnder the title of D*iſtance* 60 *Gr.* 13 *parts*. This ſhewes that if the courſe held conſtantly on the third Rumb from the Equinoctiall to the Latitude of 50 *Gr.* the difference of Longitude would be 38 *Gr.* 69 *parts* of a 100, and the diſtance vpon the Rumbe 60 *Gr.* 13 *parts*. For here I reckon the diſtance by degrees, rather then by leagues or miles, and ſubdiuide each degree into 100 parts, rather then into 60 minutes, for the more eaſe in calculation, and withall to make the calculation to agree the better, both with this, and my *Croſſe ſtaffe*, and other inſtruments.

The vſe of theſe Tables, for the finding of the difference of Longitude, is this. Turne to the table of the *R*umb, and there ſee what longitude belongeth to either latitude, then take the one longitude out of the other, the remainder will be the difference of longitude required.

As in the former example, where the places giuen were *A*, in the latitude of 50 *Gr.* *C* in the latitude of 55 *Gr.* and the *R*umb the third from the meridian: I looke into the table of the third *R*umb and there find,

Latitude 50 *gr.*	Longitude	38 *gr.*	69 *parts.*
Latitude 55.	Longitude	44.	19.
Therefore the diff.	of Longitude	5	50

There is another vſe of theſe tables, for the deſcribing of the *R*umbs both on the *Globe*, aud all ſorts of *Charts*. For hauing drawne the circles of longitude and latitude, and finding by the tables, the difference of longitude belonging to each *R*umb and latitude: If we make a prick in the chart, at euery degree of latitude, according to that difference of longitude, and draw lines through thoſe prickes, ſo as they make no angles, the lines ſo drawne ſhall be the *Rumbs* required.

The vſe of the eight *Rumb* is ſomething different from the reſt. For there being here no change of latitude, I haue ſet to each latitude, the difference of longitude, belonging to one degree of diſtance, and the diſtance belonging to one degree of longitude.

As if two places ſhall be 20 leagues, or one degree diſtant one from the other, in the latitude of 50 *gr.* the difference of longitude betweene them will be 1 *gr.* 55 *parts.* But if they differ one degree in longitude, the diſtance betweene them will be onely 64 parts, which fall ſhort of 13 leagues, or at the moſt 6428 parts, ſuch as 10000 do make a degree.

6 *By the difference of longitude, Rumb, and one latitude, to find the other latitude.*

As if the places giuen were *A*, in the latitude of 50 *gr.* *C* in a greater latitude but vnknowne, the difference of longitude 5 *gr.* ½, and the Rumb the third from the Meridian.

In the chart let *A B*, D*C*, meridians, be drawne through *A* and *C*, according to the difference of longitude, one 5 *gr.* ½ from the other; and a parallell of latitude through *A*, croſſing the meridian C D in D : then in *A*, with *A B*, make an angle of the Rumb *B A C*: ſo the degrees in the meridian betweene D and *C*, ſhall be found to be 5 *gr.* the proper difference of latitude which was required. Wherefore the proportion holds for the *Sector*,

As A D the Radius,
to DC the tangent of the Rumb from the equator:
So A D as difference of longitude,
to DC the proper difference of latitude.

According to this, I take 56 *gr.* 15 *m.* for the angle of the Rumb from the equator, out of the greater *Tangent*, and make

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make it a parallell Radius. Then I reckon 5 *gr.* ½ in the line of *lines* from the center, for the difference of longitude. So the parallell taken from the termes of this difference, and meaſured in the line of *meridians*, ſhal reach from 50 *gr.* the latitude giuen, to 55 *gr.* which is the latitude required.

Or if the Rumb fall nearer to the meridian.

As *BC* the tangent of the Rumb from the meridian,
is to *AB* the Radius:
So *BC* as difference of longitude,
to *A*D the proper difference of latitude.

According to this we may beſt work by parallel entrance; firſt take 33 *gr.* 45 *m.* for the angle of the Rumb from the meridian, out of the greater *Tangent*, and make it a parallell Radius; then take 5 *gr.* ½ for the difference of longitude out of the line of *lines*, and carrie it parallell to the former, till the feete of the compaſſes ſtay in like points: ſo the line between the center and the place of this ſtay, being taken and meaſured in the line of *meridians* from 50 *gr.* forward, ſhall ſhew the latitude required to be 55 *gr.* as in the former way.

The like may be found by the tables of Rumbs. For in the table of the third Rumb, at the latitude of 50 *gr.* I finde the longitude of 38 *gr.* 69 *p*; to this if I adde 5 *gr.* 50 *p.* for the difference of longitude giuen, the compound longitude will be 44 *gr.* 19 *p.* and this anſwers to the latitude of 55 *gr.*

But if this difference of latitude were to be found by the common ſea-chart, it ſhould ſeeme to be 8 *gr.* 13 *m*; and ſo the ſecond latitude ſhould be 58 *gr.* 13 *m.* which is aboue 3 *gr.* more then the truth.

7 *By one latitude, rumb, and diſtance, to find the difference of longitude.*

As if the places giuen were *A* in the latitude of 50 *gr.* *C* in a greater latitude but vnknowne, the diſtance vpon the Rumb being 6 *gr.* betweene them, and the Rumb the third from the meridian.

In the chart, let a meridian *A B*, and a parallell A D be drawne through *A*;and in *A*,with A B,make an angle BAC for the Rumb from the meridian; then open the compaſſes according to the latitude of the places to *E F*, the quantitie of *6 gr.*in the meridian, transferring them into the Rumb from *A* to *C*, and through *C* draw another meridian D C, croſſing the parallell drawne through *A* in D : ſo the degrees intercepted in the parallell from *A* to D, ſhall ſhew the difference of longitude required to be about *5 gr.*$\frac{1}{2}$. Wherefore the proportion holds for the *Sector*.

As *A C* the Radius,
is to AD,equall to B C,the ſine of the Rumb from the (meridian:
So *A C* as proper diſtance vpon the *Rumb*,
to A D the difference of longitude.

According to this I take the ſine of *33 gr.45 m.* for the angle of the *Rumb* from the meridian, and make it a parallell *Radius*; then keeping the *Sector* at this angle, I take *6 gr.*for the diſtance out of the *meridian* line, according to the eſtimated latitudes of both places, and lay it on both ſides of the *Sector* from the center: ſo the parallell taken from the termes of this diſtance,and meaſured in the lines of *lines*, ſhall ſhew the difference of longitude to be about *5.gr.*$\frac{1}{2}$.

In this, and ſome of the *Prop.*following, where there is but one latitude knowne, there may be ſometimes an error of a minute or two,in the eſtimation of the proper diſtance, yet it may be rectified at a ſecond operation.

This propoſition may alſo be wrought by the Tables of *Rumbs*. For according to the example, in the Table of the third *Rumb*,at the latitude of *50 gr.* I find the longitude of 38 *gr.69 p.* and the diſtance of *60 gr.13 p.* to this I adde *6 gr.* for the diſtance giuen; ſo the compound diſtance will be *66 gr.13 p.* and this anſwers to the longitude of *44 gr.19 p*; then if I take the one longitude out of the other, the difference will be *5 gr.50 p.*as before.

But if this difference were to be found by the common ſea-chart, it ſhould ſeeme to be onely 3 *gr.* 20 *m.* which is

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more then **2** *gr.* leſſe then the truth.

8 *By one latitude, Rumb, and difference of longitudes, to find the diſtance.*

As if the places giuen were *A*, in the latitude of *50 gr.* *C* in a greater latitude but vnknowne, the difference of longitude betweene them being 5 *gr.* ½, and the Rumb the third from the meridian.

In the chart let A B, D C, meridians be drawne through *A* and *C*, according to the difference of longitude, and a parallell of latitude through *A*, croſſing the meridian DC in D; then in *A*, with A B, make an angle of the Rumb BAC: ſo the diſtance on the Rumb from *A* to *C* taken and meaſured in the *meridian*, according to the eſtimated latitude of the places, ſhall be found to be *6 gr.* Wherefore the proportion holds for the *Sector.*

As A D, equall to BC, the ſine of the Rumb from the meridian,
is to *A C* the Radius:
So **A** D as difference of longitudes,
to *A C* the proper diſtance vpon the Rumb.

According to this, I take the laterall Radius, and make it a parallell ſine of 33 *gr.* 45 *m.* which is here the angle of the Rumb from the meridian; then I reckon *5 gr.* ½ in the lines of *lines* from the center, for the difference of longitude: ſo the parallell taken from the termes of this difference, and meaſured in the line of *meridians*, according to the latitudes of the places, ſhall there ſhew the diſtance required to be about *6 gr.* which are **120** leagues.

Or if the Rumb fall nearer to the meridian, that the lateral *R*adius cannot be fitted ouer in his ſine, this *Prop.* muſt be wrought by parallell entrance, and ſo alſo it giues the ſame diſtance as before.

Or we may find this diſtance by the Table of *R*umbs. For in the table of the third *R*umb, at the latitude of *50 gr.* I find the longitude of 38 *gr.* 69 *p.* and the diſtance of *60 gr.* 13 *p.*

To this longitude here found, I adde 5 *gr.* 50 *p.* for the difference of longitude giuen: ſo the compound longitude will be 44 *gr.* 19 *p.* and this anſwers to the diſtance of 66 *gr.* 15 *p.* Then if I take the one diſtance out of the other, the remainder will be 6 *gr.* 02 *p.* for the diſtance required.

But if this diſtance were to be meaſured on the common ſea-chart, it ſhould ſeeme to be almoſt 10 *gr.* or at the leaſt 197 leagues, aboue 77 leagues more then the truth.

9 *By one latitude, diſtance, and difference of longitudes, to find the Rumb.*

As if the places giuen were *A*, in the latitude of 50 *gr.* *C* in a greater latitude but vnknowne, the difference of longitude betweene them being 5 *gr.* $\frac{1}{2}$, and the diſtance 6 *gr.* vpon the *R*umb.

In the chart let AB, DC, meridians, be drawne through *A* and *C*, and a parallell of latitude through *A*; then open the compaſſes according to the latitudes of the places, to *E F* the quantitie of 6 *gr.* in the *meridian*, and ſetting the one foote in *A*, the other foote ſhall croſſe the other meridian in C; and if we draw the right line *A C*, the angle BAC ſhall ſhew the inclination of the *R*umb to the meridian to be about 33 *gr.* 45 *m.* Wherefore the proportion holds for the *Sector.*

As *A C* the proper diſtance vpon the *R*umb,
  is to A D the difference of longitude:
So *A C* as *R*adius,
  to AD, equall to B C, the ſine of the *R*umb from the meridian.

According to this, I take the proper diſtance 6 *gr.* out of the line of *meridians*, and lay it on both ſides of the *Sector* from the center; then I take the difference of longitude 5 *gr.* $\frac{1}{2}$ out of the line of *lines*, and to it open the *Sector* in the terms of the former diſtance: ſo the parallell *R*adius taken from betweene 90 and 90, and meaſured in the *ſines*, doth giue about 33 *gr.* 45 *m.* for the *R*umb required.

But if this *R*umb were to be found by the common ſea-

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chart, it ſhould ſeeme to be aboue 66 *gr*. and ſo almoſt the ſixt *R*umb from the meridian.

10 *By the longitude and latitude of two places, to find their diſtance vpon the Rumb.*

Let the *Sector* be opened in the lines of *lines*, vnto a right angle (as was ſhewed before *Cap.2.Prop.7*;) then take out the proper difference of latitude, and lay it on the one line, and the difference of longitude, and lay it on the other line, ſo as they may both meete in the center, marking how far they extend. For the line taken from the termes of their extenſion, and meaſured in the *meridian*, according to their latitudes, ſhall ſhew the diſtance required.

So if the places giuen were *A* and *C*, *A* in the latitude of 50 *gr*. *C* in the latitude of 55 *gr*. the proper difference of latitude ſhall be the line *A B*, and let *B C* the difference of longitude be 5 *gr*.$\frac{1}{2}$, we ſhall find that *A C* the diſtance vpon the Rumb is about 6 *gr*. which make 120 leagues.

For in the chart, let an occult meridian be drawne through *A*, and a parallell of latitude through *C*, croſſing the former meridian in *B*, and a right line for the Rumb from *A* to *C*, ſo haue we a rectangle triangle *A B C*, whoſe baſe *A C*, taken and meaſured in the meridian from *E* below 50 *gr*. to *F*, as much aboue 55 *gr*. doth containe the quantitie of 6 *gr*.

In the ſame maner the *Sector* being opened to a right angle, in the lines of *lines*: if we take the difference of latitude out of the line of *meridians*, in his proper place from 50 *gr*. to 55 *gr*. and place it on one of the ſides from the center, to reſemble *A B*, then reckon the difference of longitude on the other perpendicular line from the center to 5 *gr*.$\frac{1}{2}$, in ſtead of *B C*, we ſhall haue the like rectangle triangle on the *Sector*, to that which we had before on the chart; and if we take out the baſe of it, and meaſure it in the line of *meridians* from below 50 *gr*. to as much aboue 55 *gr*. we ſhall finde as before, that it containeth about 6 *gr*. or 120 leagues.

But if this diſtance were to be meaſured on the common ſea-chart, it ſhould ſeeme to be almoſt 7 *gr.* $\frac{1}{4}$, or 145 leagues; which is 25 leagues more then the truth.

11 *By the latitude of two places, and the diſtance vpon the Rumb, to find the difference of longitude.*

Let the *Sector* be opened in the lines of *lines* to a right angle, then take out the proper difference of latitudes, and lay it on one of the lines from the center, then take the proper diſtance with a paire of compaſſes, and ſetting one foote in the termes of the difference, turne the other foote to the other line of the *Sector*, and it ſhall there ſhew the difference of longitude required.

So if the places giuen were *A*, in the latitude of 50 *gr.* *C* in the latitude of 55 *gr.* with 6 *gr.* of diſtance one from another, we ſhall find their difference of longitude to be about 5 *gr.* $\frac{1}{2}$.

For in the chart let a meridian *A B* be drawne for the one, and *B C*, *A D*, parallels of latitude for them both. Then open the compaſſes according to the latitude of the places, to *E F* the quantitie of 6 *gr.* in the *meridian*, and ſetting one foote in *A*, hauing latitude of 50 *gr.* turne the other to the parallell of 55 *gr.* and it ſhall there cut off the required difference of longitude *B C* 5 *gr.* $\frac{1}{2}$.

In the ſame maner, the *Sector* being opened to a right angle, in the lines of *lines*: if we take the difference of latitude out of the line of *meridians* in his proper place from 50 *gr.* vnto 55 *gr.* and place it on one of the lines from the center; then take 6 *gr.* the diſtance vpon the Rumb out of the ſame line of *meridians*, according to the latitudes of the places, and ſet the one foote in the terme of the former difference, turning the other foote to the other perpendicular line, we ſhall finde that it will croſſe it about 5 *gr.* $\frac{1}{2}$ from the center: which is the difference of longitude required.

But if this difference of longitude were to be found by the common ſea-chart, it would ſeeme to be only 3 *gr.* 20 *m.* which is more then 2 *gr.* 10 *m.* leſſe then the truth.

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12 *By one latitude, diſtance and difference of longitudes, to finde the difference of latitudes.*

Let the *Sector* be opened in the line of *lines* to a right angle, and let the difference of longitude be reckoned in one of thoſe lines from the center; then take the proper diſtance with a paire of compaſſes, and ſetting the one foote in the terme of the former difference, turne the other foote to the other line of the *Sector*, and it ſhal thence cut off a line, equal to the proper difference of latitude required.

So if the places giuen were *A* and *C*, *A* in the latitude of 50 *gr.* *C* in a greater latitude but vnknowne, the difference of longitude betweene them 5 *gr.* $\frac{1}{2}$, and the diſtance vpon the Rumb 6 *gr.* or 120 leagues, we ſhall find the difference of latitude to be 5 *gr.*

For in the chart, let occult meridians be drawne through *A* and *C*, and a parallell of latitude through *A*; then open the compaſſes according to the eſtimated latitudes of the places to *E F* the quantitie of 6 *gr.* in the meridian, and ſetting the one foote in *A*, turne the other to the meridian drawne through *C*, and it ſhall there cut off the line *D C*, which is the difference of latitude required.

In the ſame maner, the *Sector* being opened to a right angle, in the lines of *lines*, if in the one line we reckon the difference of longitude from the center to 5 *gr.* $\frac{1}{2}$, then taking 6 *gr.* for the diſtance out of the line of *Meridians*, according to the latitude of the places, we ſet the one foote in the terme of the giuen difference, and turne the other foote to the other perpendicular line, we ſhall finde that it cuts a line from it, which taken and meaſured in the line of *meridians*, from 50 *gr.* on forward, doth ſhew the difference of latitude to be as before 5 *gr.*

But if this difference of latitude were to be found by the common ſea-chart, it would ſeeme to be only 2 *gr.* 25 *m.* which is 2 *gr.* 35 *m.* leſſe then the truth. Such is the difference betweene both theſe charts.

THE

THE THIRD BOOKE

Containing the vſe of the particular *Lines.*

THE lines of ***lines***, of ***ſuperficies***, of ***ſolids***, of ***ſines***, with the laterall lines of ***tangents*** and ***meridians***, whereof I haue hitherunto ſpoken, are thoſe which I principally intended: that little roome on the ***Sector*** which remaineth, may be filled vp with ſuch particular lines as each one ſhall think conuenient for his purpoſe. I haue made choiſe of ſuch as I thought might be beſt prickt on without hindring the ſight of the former, viz. lines of ***Quadrature***, of ***Segments***, of ***Inſcribed bodies***, of ***Equated bodies***, and of ***Mettals***.

CHAP. I.

Of the lines of Quadrature.

THe lines of ***quadrature*** may be knowne by the letter ***Q***, and by their place betweene the lines of ***ſines***. ***Q*** ſignifieth the ſide of a ***ſquare***; 5 the ſide of a ***pentagon*** with fiue equall ſides, 6 of an ***hexagon*** with ſix equall ſides, and ſo 7, 8, 9, and 10. *S* ſtands for the Semidiameter of a circle, and 90 for a line equall to 90 *gr.* in the circumference. The vſe of them may be

1 ***To make a ſquare equall to a circle giuen.***
2 ***To make a circle equall to a ſquare giuen.***

If the circle be firſt giuen, take his ſemidiameter, and to it open the ***Sector*** in the points at *S*: ſo the parallell taken from betweene the points at *Q*, ſhall be the ſide of the ſquare required.

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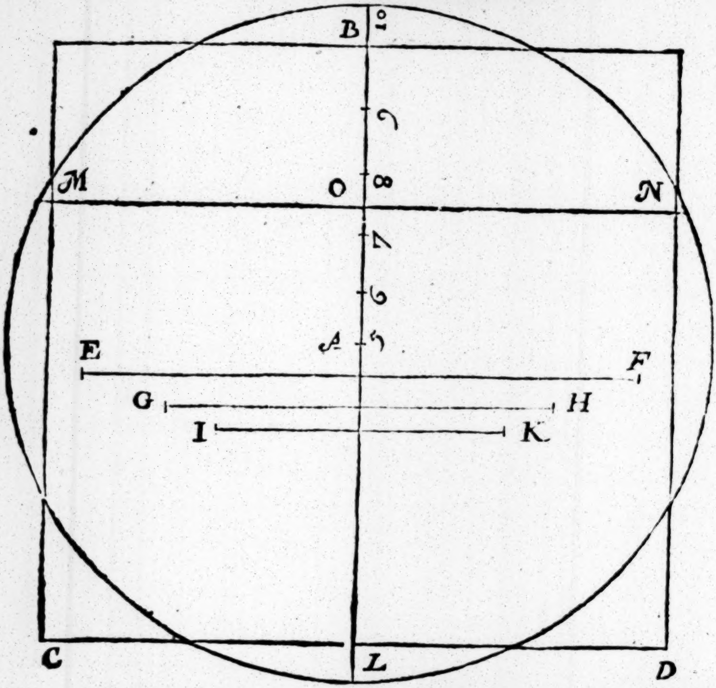

If the ſquare be giuen take his ſide, and to it open the *Sector*, in the points at *Q*: ſo the parallell taken from betweene the points at *S*, ſhall be the Semidiameter of the circle required.

Let the Semidiameter of the circle giuen be *A B*, the ſide of the ſquare equall vnto it ſhall be found to be *C D*.

3 *To reduce a circle giuen, or a ſquare into an equall pentagon, or other like ſided and like angled figure.*

Take the ſide of the figure giuen, and fit it ouer in his due points: ſo the parallels taken from betweene the points of the other figures, ſhall be the ſides of thoſe figures: which being made vp with equall angles, ſhall be all equall one to the other.

Let the Semidiameter of the circle giuen be *AB*, the ſide of an *hexagon* equal to this circle, ſhall by theſe meanes be found to be *GH*; and the ſides of an *octagon* to be *IK*. Other planes not here ſet downe, may firſt be reduced into a ſquare, by the ſixt *Prop. Superf.* and then into a circle, or other of theſe equall figures, as before.

4 *To find a right line, equall to the circumference of a circle, or other part thereof.*

Take the Semidiameter of the circle giuen, and to it open the *Sector* in the points at *S*; ſo the parallell taken from betweene the points at 90 in this line, ſhall be the fourth part of the circumference: which being knowne, the other parts may be found out by the ſecond and third *Prop.* of *lines*.

Thus if the Semidiameter of the circle giuen be *A B*, the right line *E F* ſhall be found to be the fourth part of the circumference. Therefore the double of *E F* ſhall be equall to the circumference of 180 *gr*; and the halfe of *E F* ſhal be the circumference of 45 *gr.* and ſo in the reſt.

CHAP. II.

Of the lines of Segments.

THe lines of *ſegments* which are here placed betweene the lines of *ſines* and *ſuperficies*, and are numbred by 5, 6, 7, 8, 9 10, do repreſent the diameter of a circle, ſo diuided into a hundred parts, as that a right line drawne through theſe parts, perpendicular to the diameter, ſhall cut the circle into two ſegments, of which the greater ſegment ſhall haue that proportion to the whole circle, as the parts cut haue to 100. The vſe of them may be

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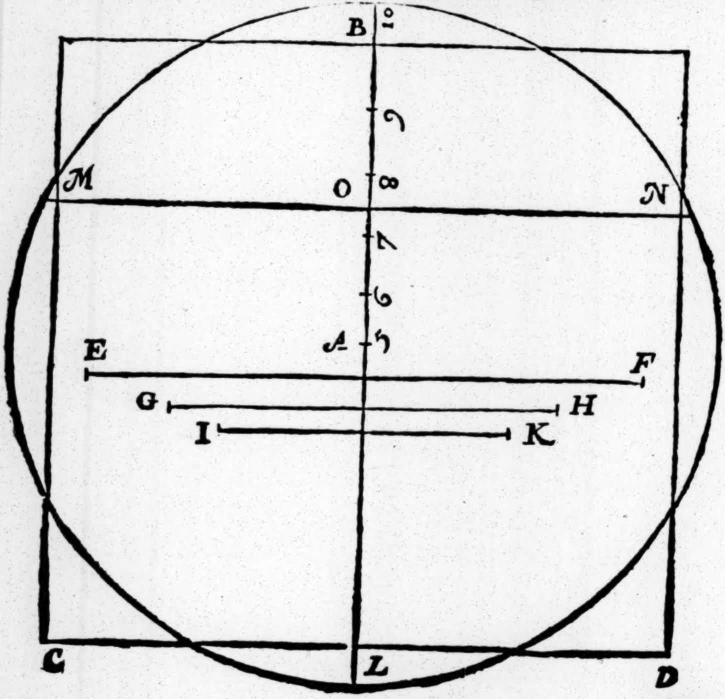
1 To diuide a circle giuen into two ſegments, according to a proportion giuen.

2 To finde a proportion betweene a circle and his ſegments giuen.

Let the *Sector* be opened in the points of an 100, to the diameter of the circle giuen: ſo a parallell taken from the points proportionall to the greater ſegment required, ſhall giue the depth of that greater ſegment.

Or if the ſegments be giuen, let the *Sector* be opened as before; then take the depth of the greater ſegment, and carry it



parallell to the diameter: ſo the number of points wherein they ſtay, ſhall ſhew the proportion to 100.

As if the diameter of the circle giuen were *B L*, the depth of the greater ſegment *L O* being 75, doth ſhew the proportion of the ſegment *O M L N* to the circle to be as 75 to 100. viz. three parts of foure.

Hence I might ſhew, if there were any vſe of it,

To find the ſide of a ſquare, equall to any knowne ſegment of a circle.

The ſide of a ſquare equall to the whole circle, may be found by the former *Cap.* and then hauing the proportion of the ſegment to the circle, we may diminiſh the ſquare in ſuch proportion, by that which hath been ſhewed *Lib. 1. Cap. 3. Prop. 3.*

CHAP. III.

Of the lines of Inſcribed bodies.

THe lines of *inſcribed bodies* are here placed betweene the lines of *lines*, and may be knowne by the letters, *D, S, I, C, O, T*; of which *D* ſignifieth the ſide of a *dodecahedron*, *I* of an *Icoſahedron*, *C* of a *cube*, *O* of an *octahedron*, and *T* of a *tetrahedron*, all inſcribed into the ſame ſphere, whoſe ſemidiameter is here ſignified by the letter *S*.

The vſe of theſe lines may be,

1 *The ſemidiameter of a ſphere being giuen, to find the ſides of the fiue regular bodies, which may be inſcribed in the ſaid ſphere.*

2 *The ſide of any of the fiue regular bodies being giuen, to find the ſemidiameter of a ſphere, that will circumſcribe the ſaid bodie.*

If the ſphere be firſt giuen, take his ſemidiameter, and to it

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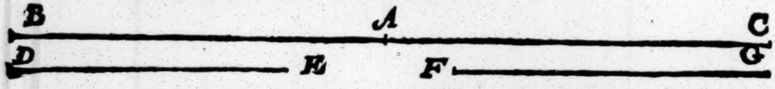
open the *Sector* in the points at *S*: if any of the other bodies be firſt giuen, take the ſide of it, and fit it ouer in his due points: ſo the parallell taken from betweene the points of the other bodies, ſhall be the ſides of thoſe bodies, and may be inſcribed into the ſame ſphere.



So if the ſemidiameter of the ſphere be *A C*, the ſide of the *dodecahedron* inſcribed ſhall be *D E*.

CHAP. IIII.

Of the lines of Equated bodies.

THe lines of *equated bodies* are here placed betweene the lines of *lines* and *ſolids*, noted with theſe letters, *D*, *I*, *C*, *S*, *O*, *T*, of which *D* ſtands for the ſide of a *dodecahedron*, *I* for the ſide of an *Icoſahedron*, *C* for the ſide of a *cube*, *S* for the diameter of a *ſphere*, *O* for the ſide of an *octahedron*, and *T* for the ſide of a *tetrahedron*, all equall one to the other. The vſe of theſe lines may be

1 *The diameter of a ſphere being giuen, to find the ſides of the fiue regular bodies, equall to that ſphere.*

2 *The ſide of any of the fiue regular bodies being giuen, to find the diameter of a ſphere, and the ſides of the other bodies, equall to the firſt body giuen.*

If the ſphere be firſt giuen, take his diameter, and to it open the *Sector* in the points at *S*: if any of the other bodies be firſt giuen, take the ſide of it, and fit it ouer in his due points: ſo the parallels taken from between the points of the other bodies, ſhall be the ſides of thoſe bodies equall to the firſt body giuen.

Thus in the laſt diagram, if the diameter of a ſphere giuen be *B C*, the ſide of the *dodecahedron* equall to this ſphere, would be found to be *FG*.

CHAP. V.

Of the Lines of Mettalls.

THe lines of *Mettalls* are here ioyned with thoſe before of *equated bodies*, and are noted with theſe characters ☉. ☿. ♄. ☽. ♀. ♂. ♃. of which ☉ ſtands for gold, ☿ for quickſiluer, ♄ for leade, ☽ for ſiluer, ♀ for copper, ♂ for iron, and ♃ for tin. The vſe of them is to giue a proportion betweene theſe ſeuerall mettalls, in their magnitude and weight, according to the experiments of *Marinus Ghetaldus*, in his booke called *Promotus Archimedes*.

1 *In like bodies of ſeuerall mettalls and equall weight, hauing the magnitude of the one, to finde the magnitude of the reſt.*

Take the magnitude giuen out of the lines of *Solids*, and to it open the *Sector* in the points belonging to the mettall giuen: ſo the parallells taken from betweene the points of the other mettalls, and meaſured in the lines of *Solids*, ſhall giue the magnitude of their bodies.

Thus hauing cubes or ſpheres of equall weight, but ſeuerall mettalls, we ſhall finde that if thoſe of tin containe 10000 *D*, the others of iron wil contain 9250, thoſe of copper 8222, thoſe of ſiluer 7161, thoſe of lead 6435, thoſe full of quickſiluer 5453, and thoſe of gold 3895.

2 *In like bodies of ſeuerall mettalls and equall magnitude, hauing the weight of one to finde the weight of the reſt.*

This propoſition is the conuerſe of the former, the proportion not direct, but reciprocall, wherefore hauing two like bodies, take the giuen weight of the one out of the lines of *Solids*, and to it open the *Sector* in the points belonging to

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the mettall of the other body : ſo the parallell taken from the points belonging to the body giuen, and meaſured in the lines of *Solids*, ſhall giue the weight of the body required.

As if a cube of gold weighed 38 l. and it were required to know the weight of a cube of lead hauing equal magnitude. Firſt I take 38 l. for the weight of the golden cube, out of the lines of *Solids*, & put it ouer in the points of ♄ belonging to lead: ſo the parallell taken from betweene the points of ☉ ſtanding for gold, and meaſured in the lines of *Solids*, doth giue the weight of the leaden cube required to be 23 l.

Thus if a ſphere of gold ſhall weigh 10000, we ſhall finde that a ſphere of the ſame diameter full of quickſiluer ſhall weigh 7143, a ſphere of lead 6053, a ſphere of ſiluer 5438, a ſphere of copper 4737, a ſphere of iron 4210, and a ſphere of tin 3895.

3 *A bodie being giuen of one mettall, to make another like vnto it, of another mettall, and equall weight.*

Take out one of the ſides of the bodie giuen, and put it ouer in the points belonging to his mettall: ſo the parallell taken from betweene the points belonging to the other mettall, ſhall giue the like ſide, for the bodie required. If it be an irregular bodie, let the other like ſides be found out in the ſame manner.

Let the bodie giuen be a ſphere of lead containing in magnitude 16 D, whoſe diameter is *A*, to which I am to make a ſphere of iron, of equall waight: If I take out the diameter *A*, and put it ouer in the points of ♄ belonging to lead, the parallell taken from betweene the points of ♂ ſtanding for iron, ſhall be *B*, the diameter of the iron ſphere required. And this compared with the other diameter, in the lines of *ſolids* will be found to be 23 *d.* in magnitude.

body being giuen of one mettall, to make another like vnto it of another mettall, according to a weight giuen.

Firſt find the ſides of a like bodie of equall weight, then may we either augment or diminiſh them according to the proportion giuen by that which we ſhewed before in the ſecond and third *Prop.* of *Solids.*

As if the bodie giuen were a ſphere of lead, whoſe diameter is *A*, and it were required to find the diameter of a ſphere of iron, which ſhall weigh three times as much as the ſphere of lead: I take *A*, and put it ouer in the points of ♄, his parallell taken from betweene the points of ♂, ſhall giue me *B* for the diameter of an equall ſphere of iron: if this be augmented in ſuch proportion as 1 vnto 3, it giueth *C* for the diameter required.

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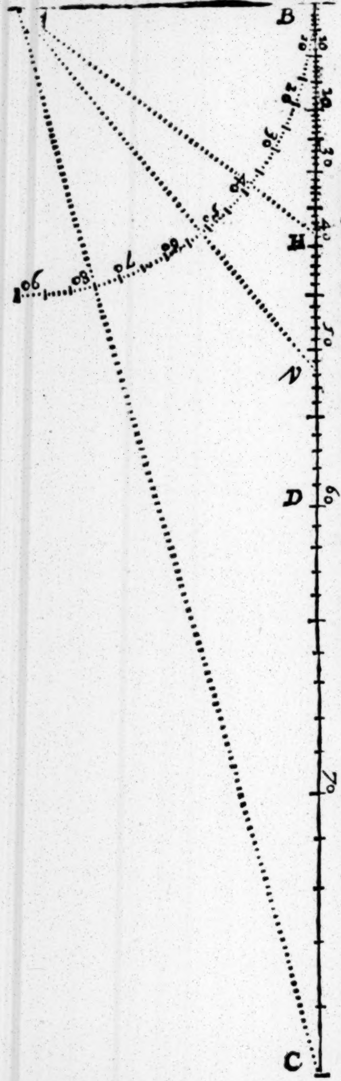

CHAP. VI.

Of the lines on the edges of the Sector.

HAuing ſhewed ſome vſe of the lines on the flat ſides of the *Sector*, there remaine onely thoſe on the edges. And here one halfe of the outward edge is diuided into inches, and numbred according to their diſtance from the ends of the *Sector*. As in the Sector of fourteene inches long, where we find 1 and 13, it ſheweth that diuiſion to be 1 inch from the nearer end, and 13 inches from the farther end of the *Sector*.

The other halfe containeth a line of leſſer *tangents*, to which the gnomon is Radius. They are here continued to 75 *gr.* And if there be need to produce them farther, take 45 out of the number of degrees required, and double the remainder: ſo the *tangent* and *ſecant* of this double remainder being added, ſhall make vp the *tangent* of the degrees required.

As if *A B* being the Radius, and *B C* the tangent line, it were required to find the tangent of 75 *gr.* If we take 45 *gr.* out of 75 *gr.* the remainer is 30 *gr.* and the double 60 *gr.* whoſe tangent is B D, and the ſecant is A D: if then we adde A D to B D, it maketh *B C* the tangent of 75 *gr.* which was required. In like ſort the ſecant of 61 *gr.* added to the tangent of 61 *gr.* giueth the tangent of 75 *gr.* 30 *m.* and the ſecant of 62 *gr.* added to the tangent of 62 *gr.* giueth the tangent of 76 *gr.*

and ſo in the reſt. The vſe of this line may be

To obſerue the altitude of the Sunne.

Hold the *Sector* ſo as the tangent *BC* may be verticall,and the gnomon *B A* parallell to the horizon ; then turne the gnomon toward the Sunne,ſo that it may caſt a ſhadow vpon the tangent,and the end of the ſhadow ſhal ſhew the altitude of the Sunne.So if the end of the gnomon at *A*, do giue a ſhadow vnto *H*, it ſheweth that the altitude is 38 *gr.* $\frac{1}{2}$, if vnto D,then 60 *gr.* and ſo in the reſt.

There is another vſe of this ***tangent*** line, for the drawing of the houre lines vpon any ordinary plane , whereof I will ſet downe theſe propoſitions.

1 *To draw the houre lines vpon an horizontall plane.*
2 *To draw the houre lines vpon a direct verticall plane.*

Firſt draw a right line *AC* for the horizon and the equator, and croſſe it at the point *A* about the middle of the line with *AB* another right line,which may ſerue for the meridian and the houre of 12; then take out 15 *gr.* out of the tangents,and pricke them downe in the equator on both ſides from 12: ſo the one point ſhall ſerue for the houre of 11, and the other for the houre of 1. Againe,take out the tangent of 30 *gr.* and pricke it downe in the equator on both ſides from 12: ſo the one of theſe points ſhall ſerue for the houre of 10, and the other for the houre of 2. In like maner may you prick downe the tangent of 45 *gr.* for the houres of 9 and 3, and the tangent of 60 *gr.* for the houres of 8 and 4, and the tangent of 75 *gr.* for the houres of 7 and 5.

Or if any pleaſe to ſet downe the parts of an houre,he may allow 7 *gr.* 30 *m.* for euery halfe houre,and 3 *gr.* 45 *m.* for euery quarter. This done,you are to conſider the latitude of the place,and the qualitie of the plane: For the *ſecant* of the latitude ſhal be the ſemidiameter in a vertical plane,& the *ſecant* of the complement of the latitude in an horizontall plane.

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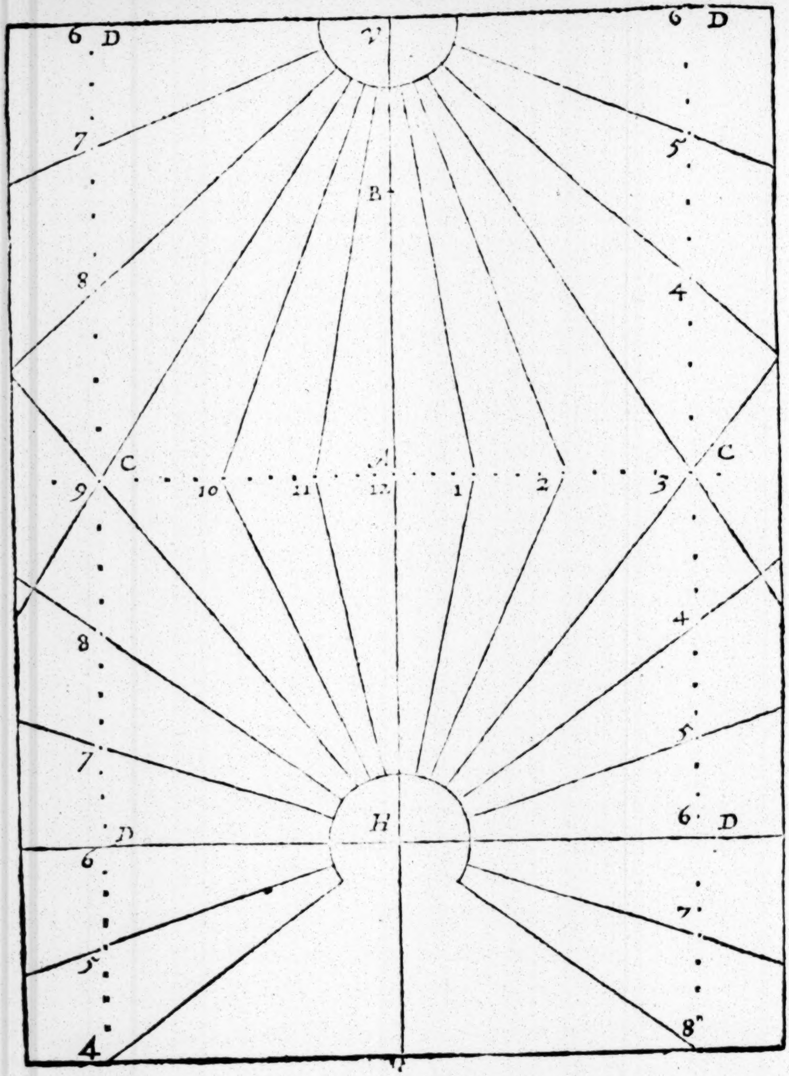
6 D
V
6 D
B
C
A
C
H
6 D
D

For example, about London the latitude is 51 *gr.* 30 *m.* and let the plane be verticall. If you take *A V* the ſecant of 51 *gr.* 30 *m.* out of the *Sector*, and pricke it downe in the meridian line from *A* vnto *V*, the point *V* ſhall be the center: and if you draw right lines from *V* vnto 11, and 10, and the reſt of the houre points, they ſhall be the houre lines required.

But if the plane be horizontall, then you are to take out *A H* the ſecant of 38 *gr.* 30 *m.* for the ſemidiameter, and prick it downe in the meridian line from *A* vnto *H*: ſo the right lines drawne from the center *H* vnto the houre points, ſhall be the houre lines required; only the houre of 6 is wanting, and that muſt alwayes be drawne parallell to the equator, through the center V in a verticall, through the center *H* in an horizontall plane.

3 *To draw the houre lines on a polar plane.*
4 *To draw the houre lines on a meridian plane.*

In a *polar* plane the equator may be alſo the ſame with the horizontall line, and the houre points may be pricked on as before, but the houre lines muſt be drawne parallell to the meridian.

In a meridian plane, the equator will cut the horizontall line with an angle equall to the complement of the latitude of the place; then may you make choiſe of the point *A*, and there croſſe the equator with a right line, which may ſerue for the houre of 6: ſo the tangent of 15 *gr.* being pricked downe in the equator on both ſides from 6, ſhal ſerue for the houres of fiue 5 and 7; and the tangent of 30 *gr.* for the houres of 8 and 4; and the tangent of 45 *gr.* for the houres of 3 and 9; and the tangent of 60 *gr.* for the houres of 2 and 10; and the tangent of 75 *gr.* for the houres of 1 and 11. And if you draw right lines through theſe houre points, croſſing the equator at right angles, they ſhall be the houre-lines required.

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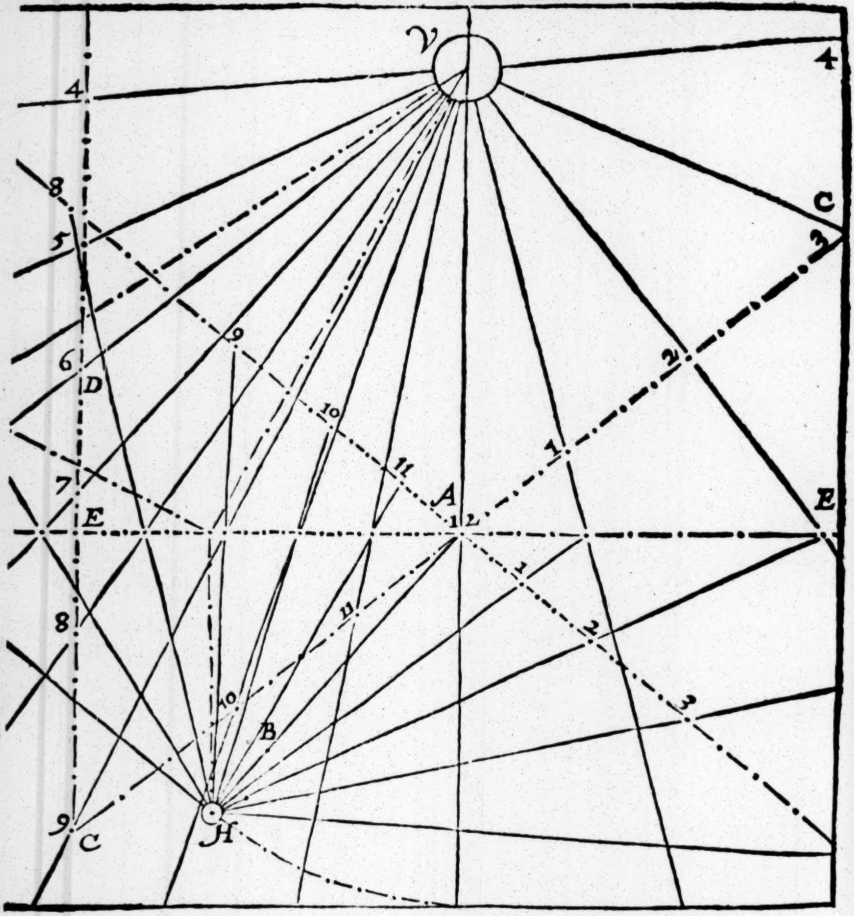

5 *To draw the houre lines in a verticall declining place.*

First, draw *AV* the meridian, and *AE* the horizontal line, croſſing one the other at right angles in the point *A*.

2 Then take out *A V*, the ſecant of the latitude of the place, which you may ſuppoſe to be 51 *gr.* 30 *m.* and prick it downe in the meridian line from *A* vnto *V.*

3 Becauſe it is a declining plane, and you may ſuppoſe it to decline 40 *gr.* Eaſtward, you are to make an angle of the declination vpon the center *A*, below the horizontall line, and to the left hand of the meridian line, becauſe the declination is Eaſtward, for otherwiſe it ſhould haue bin to the right hand, if the declination had bin Weſtward.

4 Take *A H*, the ſecant of the complement of the latitude out of the *Sector*, & pricke it downe in the line of declination from *A* vnto *H*, as you did before for the ſemidiameter in the horizontall plane.

5 Draw a line at full length through the point *A*, which muſt be perpendicular vnto *A H*, and cut the horizontall line according to the angles of declination, and it will be as the equator in the horizontall plane.

6 Take the houre points out of the *Tangent* line in the *Sector*, and pricke them downe in this equator on both ſides from the houre of 12 at *A*.

7 Lay your ruler, & draw right lines through the center *H*, & each of theſe houre points: ſo haue you all the houre lines of an horizontall plane, onely the houre of 6 is wanting, and that may be drawne through *H* perpendicular to *H A*.

Laſtly you are to obſerue and marke the interſections, which theſe houre lines do make with *A E* the horizontall line of the plane: and then if you draw right lines through the center *V*, and each of theſe interſections, they ſhal be the houre lines required.

6 *To pricke downe the houre points another way.*

Hauing drawne a right line for the equator as before, and made choice of the point *A*, for the houre of 12: you may at pleaſure cut of two equal lines *A* 10, and *A* 2. Then vpon the diſtance betweene 10 and 2, make an equilaterall triangle, and you ſhall haue *B* for the center of your equator, and the

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line *A B* ſhall giue the diſtance from *A* to 9, and from A to 3. That done take out the diſtance betweene 9 and 3, and this ſhall giue the diſtance from *B* vnto 8, and from 8 vnto 7, and from 8 vnto 1: and againe from B vnto 4, and from 4 vnto 5, and from 4 vnto 11. So haue you the houre points, and if you take out the diſtance B 1, B 3, B 5. &c. You may finde the points not onely for the halfe houres, but alſo for the quarters.

But if it ſo fall out, that ſome of theſe houre points fall out of your plane, you may helpe your ſelfe by the larger *tangent*, both in the verticall, and horizontall planes.

For if at the houre points of 3 and 9, you draw occult lines parallell to the meridian; the diſtances D C, betweene the houre line of 6, and the houre points of 3 and 9, will be equal to the ſemidiameter A V in a verticall, and AH in a horizontall plane, and if they be diuided in ſuch ſort as the line A C is diuided, you ſhall haue the points of 4, and 5, and 7, and 8, with their halfes and quarters.

As in the horizontall plane, take out the ſemidiameter A H, and make it a parallell Radius by fitting it ouer in the *ſines* of 90 and 90: Then take 15 *gr.* out of the larger *tangent*, and lay them on the lines of *ſines*, where they will reach from the center vnto the ſines of 15 *gr.* 32 *m.* therefore take out the parallell ſine of 15 *gr.* 32 *m.* and it ſhall giue the diſtance from 6 vnto 5, and from 6 vnto 7, in your horizontall plane. That done take out 30 *gr.* out of the larger *tangent*, and lay them on the *ſines*, from the center vnto the ſines of 35 *gr.* 16 *m.* and the parallell ſine of 35 *g.* 16 *m.* ſhall giue you the diſtance from 6 vnto 4, and from 6 vnto 8, in your horizontall plane. The like may be done for the halfe houres and quarters.

So alſo in the verticall declining plane. If you firſt take out the *ſecant* of the declination of the plane, and prick it downe in the horizontall line from A vnto E, and through E draw right lines parallell to the meridian, which will cut the former houre lines of 3 and 9, or one of them in the point C: then take out the ſemidiameter A V, and prick it downe in thoſe parallells from C vnto D, and draw right lines from A vnto C,and from V vnto D ;the line V D ſhall be the houre of 6, and if you diuide theſe lines A C and D C, in ſuch ſort as you diuided the like line D C in the horizontall plane, you ſhall haue all the houre points required.

Or you may find the point D,in the houre of 6, without knowledge either of *H* or *C*. For hauing prickt downe *A V* in the meridian line, and *A E* in the horizontall line, and drawne parallels to the meridian through the points at *E*, you may take the ***tangent*** of the latitude out of the *Sector*,and fit it ouer in the ſines of 90 and 90: ſo the parallell ſine of the declination meaſured in the ſame ***tangent*** line, ſhall there ſhew the complement of the angle DVA, which the houre line of 6 maketh with the meridian; then hauing the point D, take out the ſemidiameter VA, and pricke it downe in thoſe parallels from D vnto C: ſo ſhall you haue the lines D C and A C to be diuided as before.

The like might be vſed for the houre lines vpon all other planes.But I muſt not write all that may be done by the *Sector*.It may ſuffice that I haue wrote ſomething of the vſe of each line,and thereby giuen the ingenuous Reader occaſion to thinke of more.

The concluſion to the Reader.

IT is well knowne to many of you, that this Sector was thus contriued, the moſt part of this booke written in latin, many copies tranſcribed and diſperſed more then ſixteene yeares ſince. I am at the laſt contented to giue way that it come forth in Engliſh. Not that I thinke it worthy either of my labour or the publique view, but partly to ſatisfy their importunity, who not underſtanding the Latin, yet were at the charge to buy the inſtrument, and partly for my owne eaſe. For as it is painefull for others to tranſcribe my copie, ſo it is troubleſome for me to giue ſatisfaction herein to all that deſire it. If I finde this to giue you content, it ſhall incourage me to do the like for my Croſſe-ſtaffe, *and ſome other Inſtruments. In the meane time beare with the Printers faults, and ſo I reſt.*

Greſham Coll. 1. Maij. 1623. *E. G.*

FINIS.

THE
FIRST BOOKE OF THE
CROSSE-STAFFE.

CHAP. I.

Of the deſcription of the Staffe.

THe *Croſſe-Staffe* is an inſtrument wel knowne to our Sea-men, and much vſed by the ancient Aſtronomers and others, ſeruing Aſtronomically for obſeruation of altitude and angles of diſtance in the heauens, Geometrically for perpendicular heights and diſtances on land and ſea.

The deſcription and ſeuerall vſes of it are extant in print, by *Gemma Friſius* in Latin, in Engliſh by *Dr. Hood.* I differ ſomething from them both, in the proiection of this *Staffe*, but ſo, as their rules may be applied vnto it, and all their propoſitions be wrought by it: and therefore referring the Reader to their bookes, I ſhal be briefe in the explanation of that which may be applied from theirs vnto mine, and ſo come to the vſe of thoſe lines which are of my addition, not extant heretofore.

The neceſſary parts of this Inſtrument are fiue: the *Staffe*, the *Croſſe*, and the three *ſights*. The Staffe which I made for my owne vſe, is a full yard in length, that ſo it may ſerue for meaſure.

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The Croſſe belonging to it is 26 inches $\frac{1}{7}$ betweene the two outward ſights. If any would haue it in a greater forme, the proportion betweene the Staffe and the Croſſe, may be ſuch as 360 vnto 262.

The lines inſcribed on the Staffe are of foure ſorts. One of them ſerues for meaſure and protraction: one for obſeruation of angles: one for the Sea-chart; and the foure other for working of proportions in ſeuerall kinds.

The line of meaſure is an *inch line*, and may be knowne by his equall parts. The whole yard being diuided equally into 36 inches, and each inch ſubdiuided, firſt into ten parts, and then each tenth part into halfes.

The line for obſeruation of angles may be knowne by the double numbers ſet on both ſides of the line, beginning at the one ſide at 20, and ending at 90: on the other ſide at 40, and ending at 180: and this being diuided according to the degrees of a quadrant, I call it the *tangent line on the Staffe.*

The next line is the meridian of a Sea-chart, according to *Mercators* proiection from the Equinoctiall to 58 *gr.* of latitude, and may be knowne by the letter *M*, and the numbers 1.2. 3.4. vnto 58.

The lines for working of proportions, may be knowne by their vnequall diuiſions, and the numbers at the end of each line.

1 The line of *numbers* noted with the letter N, diuided vnequally into 1000 parts, and numbred with 1.2.3.4. vnto 10.

2 The line of *artificiall tangents* is noted with the letter *T*, diuided vnequally into 45 degrees, and numbred both ways, for the Tangent and the complement.

3 The line of *artificiall ſines*, noted with the letter *S*, diuided vnequally into 90 degrees, and numbred with 1.2.3.4. vnto 90.

4 The line of *verſed ſines* for more eaſie finding the houre and azimuth, noted with V, diuided vnequally into about 164 *gr.* 50 *m.* numbred backward with 10.20.30. vnto 164.

Thus there are ſeuen lines inſcribed on the Staffe: there are fiue lines more inſcribed on the Croſſe.

1 A Tangent line of 36 *gr.* 3 *m.* numbred by 5.10.15. vnto 35: the midſt whereof is at 20 *gr*; and therefore I call it the *tangent of* 20; and this hath reſpect vnto 20 *gr.* in the Tangent on the Staffe.

2 A Tangent line of 49 *gr.* 6 *m.* numbred by 5.10.15. vnto 45; the midſt whereof is at 30 *gr.* and hath reſpect vnto 30 *gr.* in the Tangent on the Staffe, whereupon I call it the *tangent of* 30.

3 A line of *inches* numbred with 1.2.3. vnto 26; each inch equally ſubdiuided into ten parts, anſwerable to the inch line vpon the Staffe.

4 A line of ſeuerall *chords*, one anſwerable to a circle of twelue inches ſemidiameter, numbred with 10.20.30. vnto 60: another to a ſemidiameter of a circle of ſix inches; and the third to a ſemidiameter of a circle of three inches; both numbred with 10.20.30. vnto 90.

5 A continuation of the *meridian* line from 57 *gr.* of latitude vnto 76 *gr*; and from 76 *gr.* to 84 *gr.*

For the inſcription of theſe Lines. The firſt for meaſure is equally diuided into inches and tenth parts of inches.

The tangent on the Staffe for obſeruation of angles, with the tangent of 20 and the tangent of 30 on the Croſſe, may all three be inſcribed out of the ordinary *table of tangents.* The Staffe being 36 inches in length; the Radius for the tangent on the Staffe will be 13 inches and 103 parts of 1000: ſo the whole line will be a tangent of 70 *gr.* and muſt be numbred by their complements, & the double of their complements, the tangent of 10 *gr.* being numbred with 80 and 160.

The Radius for the tangent of 20 on the Croſſe, will be 36 inches, and the whole line betweene the ſights a tangent of 36 *gr.* 3 *m.* according as it is numbred. The Radius for the tangent of 30 *gr.* on the Croſſe, will be 22 inches and 695 parts of 1000: ſo the whole line betweene the ſights wil containe a tangent of 49 *gr.* 6 *m.* in ſuch ſort, as they are numbred.

The meridian line may be inſcribed out of the Table which I ſet downe for this purpoſe in the vſe of the Sector:

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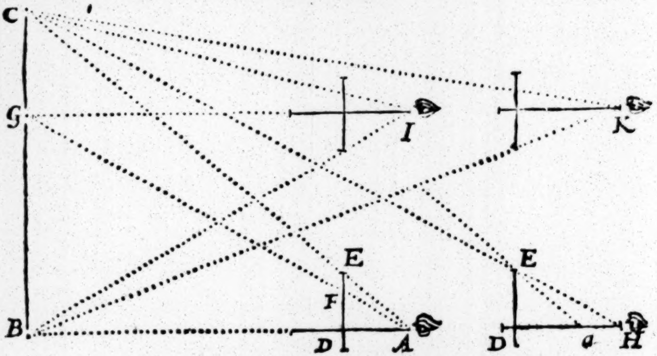
The line of numbers may be inſcribed out of the firſt Chiliad of Mr. *Briggs* Logarithmes: & the reſt of the lines of proportion out of my *Canon of artificiall ſines and tangents*; and in recompence thereof this booke will ſerue as a comment to explane the vſe of my *Canon*.

CHAP. II.

The vſe of the lines of inches *for perpendicular heights and diſtances.*

IN taking of heights and diſtances, the Staffe may be held in ſuch ſort, that it may be euen with the diſtance, and the Croſſe parallell with the height: and then if the eye at the beginning of the Staffe ſhall ſee his marks by the inward ſides of the two firſt ſights, there will be ſuch proportion betweene the diſtance and the height, as is betweene the parts intercepted on the Staffe and the Croſſe. Which may be farther explaned in theſe propoſitions.



1 *To find an height at one ſtation, by knowing the diſtance.*

Set the middle ſight vnto the diſtance vpon the Staffe, the height will be found vpon the Croſſe. For

As the ſegment of the Saffe
vnto the ſegment on the Croſſe:
So is the diſtance giuen,
vnto the height.

As if the diſtance *A B* being knowne to be 256 feete, it were required to find the height *B C*: firſt I place the middle ſight at 25 *inches* and 6 *parts* of 10; then holding the Staffe leuell with the diſtance, I raiſe the Croſſe, parallell vnto the height, in ſuch ſort, as that my eye may ſee from *A* the beginning of the inches on the Staffe by the ſight *E*, at the beginning of the inches on the Croſſe vnto the mark *C*: which being done, if I find 19 inches and 2 parts of 10 intercepted on the Croſſe betweene the ſights at *E* and *D*, I would ſay the height *B C* were 192 feete.

Or if the obſeruation were to be made before the diſtance were meaſured, I would ſet the middle ſight either vnto 10 inches, or 12, or 16, or 20, or 24, or ſome ſuch other number as might beſt be diuided into ſeuerall parts, and then worke by proportion. As if in the former example the middle ſight were at 24 on the Staffe, and 18 on the Croſſe, it ſhould ſeem that the height is ¾ of the diſtance; and therefore the diſtance being 256, the height ſhould be 192.

2 *To finde an height, by knowing ſome part of the ſame height.*

As if the height from *G* to *C* were knowne to be 48, and it were required to find the whole height *B C*: either put the third ſight or ſome other running ſight vpon the Croſſe betweene the eye and the marke *G*. For then

As the difference betweene the ſights,
vnto the whole ſegment of the Croſſe:
So is the part of the height giuen,
vnto the whole height.

If then the difference betweene the ſights *E* and *P*, ſhall

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be 45, and the ſegment of the Croſſe E D 180, the whole height *B C* will be found to be 192.

3 *To find an height at two ſtations, by knowing the difference of the ſame ſtations.*

As the difference of ſegments on the Staffe,
vnto the difference of ſtations:
So is the ſegment of the Croſſe,
vnto the height.

Suppoſe the firſt ſtation being at *H*, the ſegment of the Croſſe *E D* were 180, and the ſegment of the Staffe *H D* 300: then coming 64 feete nearer vnto *B*, in a direct line, vnto a ſecond ſtation at *A*, and making another obſeruation; ſuppoſe the ſegment of the Croſſe *E D* were 180 as before, and the ſegment of the Staffe *A D* 240; take 240 out of 300, the difference of ſegments will be 60 parts. And

As 60 parts vnto 64 the difference of ſtations:
So D E 180 vnto *B C* 192 the height required.

In theſe three *Prop.* there is a regard to be had of the height of the eye. For the height meaſured, is no more then from the leuell of the eye vpward.

4 *To find a diſtance, by knowing the height.*

As the ſegment of the Croſſe,
vnto the ſegment of the Staffe:
So is the height giuen,
vnto the diſtance.

So the ſegment E D being 18, and D A 24, the height *C B* 192, will ſhew the diſtance *A B* to be 256.

5 *To find a diſtance, by knowing part of the height.*

As the difference betweene the ſights,
vnto the ſegment of the Staffe:

So is the part of the height giuen,
vnto the diſtance.

And thus the difference betweene *E* and *F* being 45, and the ſegment D A 240; the part of the height *GC* 48, will giue the diſtance *A* B to be 256.

6 *To finde a diſtance at two ſtations, by knowing the difference of the ſame ſtations.*

As the difference of ſegments on the Staffe,
vnto the difference of ſtations:
So is the whole ſegment,
vnto the diſtance.

And thus the ſegment of the Croſſe being 180, the ſegment of the Staffe at the firſt ſtation 240, at the ſecond 300, the difference of the ſegments 60,& the difference of ſtations 64, the diſtance A B at the firſt ſtation will be found to be 256, and the diſtance H B at the ſecond ſtation 320.

7 *To find a bredth by knowing the diſtance perpendicular to the bredth.*

This is all one with the firſt *Prop.* For this bredth is but an height turned ſidewayes: and therefore

As the ſegment of the Staffe,
vnto the ſegment of the Croſſe:
So is the diſtance
vnto the bredth.

And thus the ſegment of the Staffe being 24,and the ſegment of the Croſſe 18, the diſtance A B 256, will giue the bredth B C to be 192.

8 *To find a bredth at two ſtations in a line perpendicular to the bredth, by knowing the difference of the ſame ſtations.*

This is alſo the ſame with the third *Prop*: and therefore

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As the difference of ſegments on the Staffe,
vnto the difference of ſtations:
So the ſegment on the Croſſe betweene the two ſights,
vnto the bredth required.

And thus the difference betweene the ſtations at *A* and H being *64* the difference of ſegments on the Staffe *60*, the ſegment of the Croſſe 180, the bredth B C will be found to be 192.

In like maner may we finde the bredth *G C* for hauing found the bredth B C the proportion will hold.

As D E is vnto F E, ſo B *C* vnto *G C*. Or otherwiſe,
As H *A* vnto H A, ſo F E vnto *G C*.

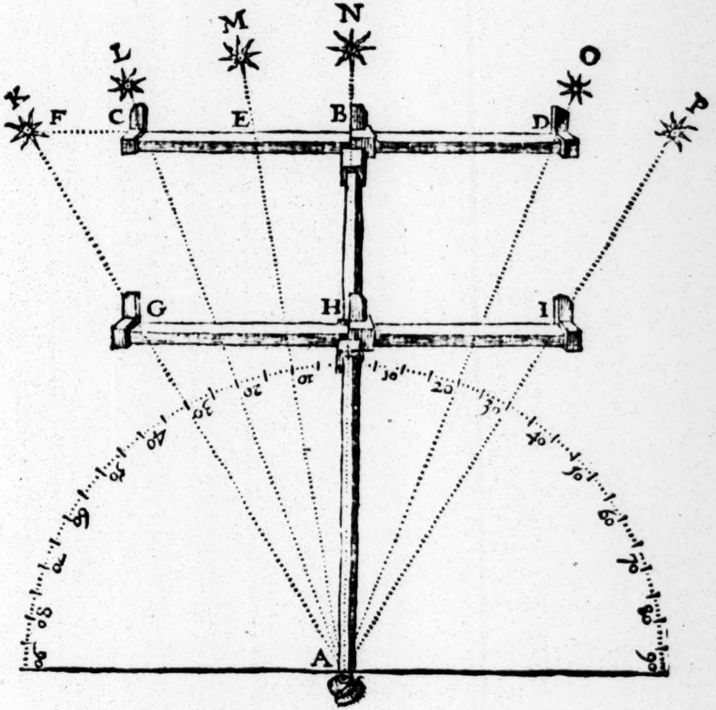
Neither is it materiall whether the two ſtations be choſen at the one end of the bredth propoſed, or without it, or within it, if the line betweene the ſtations be perpendicular vnto the bredth: as may appeare if in ſtead of the ſtations at *A* and H, we make choiſe of the like ſtations at *I* and *K*.

There might be other wayes propoſed to work theſe *Prop.* by holding the Croſſe euen with the diſtance, and the *S*taffe parallell with the height: but theſe would proue more troubleſome, and thoſe which are deliuered are ſufficient, and the ſame with thoſe which others haue ſet downe vnder the name of the *Iacobs ſtaffe.*

CHAP.

CHAP. III.

The vſe of the Tangent lines in taking of Angles.



1 *To find an angle by the Tangent on the Staffe.*

Let the middle ſight be alwayes ſet to the middle of the Croſſe, noted with 20 and 30, and then the Croſſe

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drawne nearer the eye, vntill the marks may be ſeene cloſe within the ſights. For ſo if the eye at *A* (that end of the Staffe which is noted with 90 and 180) beholding the marks *K* and *N*, betweene the two firſt ſights, *C* and *B*, or the marks *K* and *P* betweene the two outward ſights, the Croſſe being drawne downe vnto *H*, ſhall ſtand at 30 and 60, in the Tangent on the Staffe: it ſheweth that the angle *K A N* is 30 *gr.* the angle *K A P* 60 *gr.* the one double to the other; which is the reaſon of the double numbers on this line of the Staffe: and this way wil ſerue for any angle from 20 *gr.* toward 90 *gr.* or from 40 *gr.* toward 180 *gr.* But if the angle be leſſe then 20 *gr.* we muſt then make vſe of the Tangent vpō the Croſſe.

2 *To find an angle by the Tangent of* 20 *vpon the Croſſe.*

Set 20 vnto 20, that is, the middle ſight to the middeſt of the Croſſe at the end of the Staffe, noted with 20: ſo the eye at *A*, beholding the marks *L* and *N*, cloſe betweene the two firſt ſights, *C* and *B*, ſhall ſee them in an angle of 20 *gr.*

If the marks ſhall be nearer together, as are *M* and *N*, then draw in the Croſſe from *C* vnto *E*: if they be farther aſunder, as are *K* and *N*, then draw out the Croſſe from *C* vnto *F*; ſo the quantitie of the angle ſhal be ſtill found in the Croſſe in the Tangent of 20 *gr.* at the end of the Staffe; and this will ſerue for any angle from 0 *gr.* toward 35 *gr.*

3 *To find an angle by the Tangent of* 30 *vpon the Croſſe.*

This Tangent of 30 is here put the rather, that the end of the Staffe reſting at the eye, the hand may more eaſily remoue the Croſſe: for it ſuppoſeth the Radius to be no longer then *A H*, which is from the eye at the end of the Staffe vnto 30 *gr.* about 22 *inches* and 7 *parts.* Wherefore here ſet the middle ſight vnto 30 *gr.* on the Staffe, and then either draw the Croſſe in or out, vntill the marks be ſeene betweene the two firſt ſights ; ſo the quantitie of the angle will be found in the Tangent of 30, which is here repreſented by the line *GH* ; and this will ſerue for any angle from 0 *gr.* toward 48 *gr.*

4 *To obſerue the altitude of the Sunne backward.*

Here it is fit to haue an horizontall ſight ſet to the beginning of the Staffe, and then may you turne your backe toward the Sun, and your Croſſe toward your eye. If the altitude be vnder 45 *gr.* ſet the middle ſight to 30 on the Staffe, and looke by the middle ſight through the horizontall vnto the horizon, mouing the Croſſe vpward or downward, vntill the vpper ſight doe ſhadow the vpper halfe of the horizontall ſight : ſo the altitude will be found in the Tangent of 30.

If the altitude ſhal be more then 45 *gr.* ſet the middle ſight vnto the middeſt of the Croſſe, and look by the inward edge of the lower ſight through the horizontall to the horizon, mouing the middle ſight in or out, vntill the vpper ſight do ſhadow the vpper halfe of the horizontall ſight : ſo the altitude will be found in the degrees on the Staffe betweene 40 and 180.

5 *To ſet the Staffe to any angle giuen.*

This is the conuerſe of the former *Prop.* For if the middle ſight be ſet to his place and degree, the eye looking cloſe by the ſights as before, cannot but ſee his obiect in the angle giuen.

6 *To obſerue the altitude of the Sunne another way.*

Set the middle ſight to the middle of the Croſſe, and hold the horizontall ſight downward, ſo as the Croſſe may be parallell to the horizon, then is the Staffe verticall ; and if the outward ſight of the Croſſe do ſhadow the horizontall ſight,

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the complement of the altitude wil be found in the tangent on the Staffe.

7 *To obſerue an altitude by thread and plummet.*

Let the middle ſight be ſet to the middeſt of the Croſſe, and to that end of the Staffe which is noted with 90 and 180; then hauing a thread and a plummet at the beginning of the Croſſe, and turning the Croſſe vpward, and the Staffe toward the Sunne, the thread will fall on the complement of the altitude aboue the horizon. And this may be applied to other purpoſes.

8 *To apply the lines of inches to the taking of angles.*

If the angles be obſerued betweene the two firſt ſights, there wil be ſuch proportion between the parts of the Staffe and the parts of the Croſſe, as betweene the Radius and the Tangent of the angle.

As if the parts intercepted on the Staffe were 20 inches, the parts on the Croſſe 9 inches. Then by proportion as 20 vnto 9, ſo 100000 vnto 45000 the tangent of 24 *gr.* 14 *m.*

But if the angle ſhall be obſerued betweene the two outward ſights, the parts being 20 and 9 as before, the angle will be 48 *gr.* 28 *m.* double vnto the former.

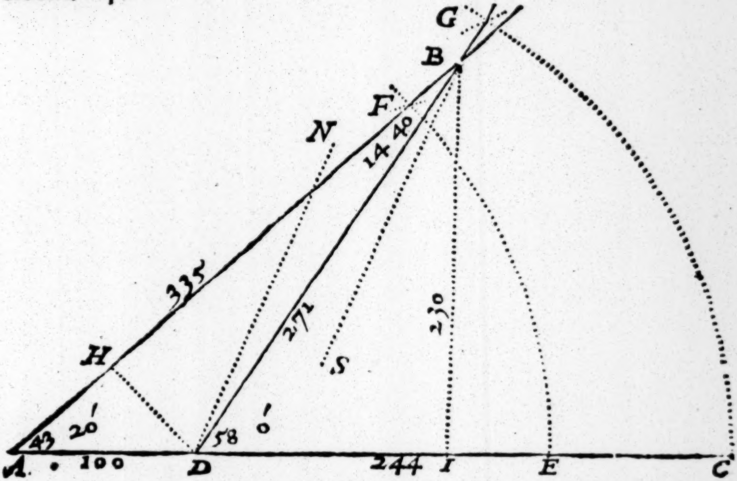
In all theſe there is a regard to be had to the parallax of the eye, and his height aboue the Horizon in obſeruations at Sea; to the Semidiameter of the Sun, his parallax and refraction, as in the vſe of other ſtaues. And ſo this will be as much, or more then that which hath been heretofore performed by the *Croſſe-ſtaffe.*

CHAP.

CHAP. IIII.

The vſe of the lines of equall parts ioyned with the lines of Chords.

THe lines of equall parts do ſerue alſo for protraction, as may appeare by the former *Diagrams*; but being ioyned with the lines of Chords, which I place vpon one ſide of the Croſſe, they will farther ſerue for the protraction and reſolution of right line triangles; whereof I will giue one example in finding of a diſtance at two ſtations otherwiſe then in the ſecond *Cap*.



Let the diſtance required be *A B*. At *A* the firſt ſtatiō I make choiſe of a ſtation line toward *C*, and obſerue the angle *B A C* by the tangent lines, which may be 43 *gr*. 20 *m*; then hauing gon an hundred paces toward *C*, I make my ſecond ſtation at D, where ſuppoſe I find the angle *B D C* to be 58 *gr*. or the

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the angle *BDA* to be 122 *gr*; this being done, I may finde the diſtance *AB* in this maner.

1 I draw a right line *AC*, repreſenting the ſtation line.

2 I take 100 out of the lines of equall parts, and pricke them downe from *A* the firſt ſtation vnto D the ſecond.

3 I open my compaſſes to one of the chords of 60 *gr*. and ſetting one foote in the point *A*, with the other I deſcribe an occult arke of a circle interſecting the ſtation line in *E*.

4 I take out of the ſame line of *chords* a chord of 43 *gr*. 20 *m*. (becauſe ſuch was the angle at the firſt ſtation) and this I inſcribe into that occult arke from *E* vnto *F*, which makes the angle F A D equall to the angle obſerued at the firſt ſtation.

5 I deſcribe another like arke vpon the center D, and inſcribe into it a chord of 58 *gr*. from *C* vnto *G*, and draw the right line DG, which doth meet with the other line *AF* in the point *B*, and makes the angle BDC equall to the angle obſerued at the ſecond ſtation. So the angles in the *Diagram* being equall to the angles in the field, their ſides wil be alſo proportionall: and therefore,

6 I take out the line *AB* with my compaſſes, and meaſuring it in the ſame line of equall parts, from which I tooke A D, I find it to be 335, and ſuch is the diſtance required.

CHAP.

CHAP. V.

The vſe of the Meridian line.

1 THe Meridian line, noted with the letter *M*, may ſerue for the more eaſie diuiſion of the plane ſea-chart, according to *Mercators* proiection. For if you ſhall draw parallell meridians, each degree being halfe an inch diſtant from other, the degrees of this meridian line on the Staffe, ſhall giue the like degrees for the meridians on the chart, from the Equinoctiall toward the Pole: and then if through theſe degrees you draw ſtreight lines perpendicular to the meridians, they ſhall be parallels of latitude.

If any deſire to haue the degrees of his chart larger then thoſe which I haue put on the Staffe, he may take theſe and increaſe them in a double, or treble, or a decuple proportion at his pleaſure.

2 This *meridian* line being ioyned with the line of *chords*, may ſerue for the protraction & reſolution of ſuch right line triangles as concerne latitude, longitude, rumb and diſtance in the practiſe of nauigation. As may appeare by this example.

Suppoſe two places giuen, *A* in the latitude of 50 *gr.* D in the latitude of 52 *gr.* $\frac{1}{2}$, the difference of longitude betweene them being 6 *gr.* and let it be required to know, firſt what Rumb leadeth from the one place to the other; ſecondly how many degrees diſtant they are aſunder.

1 I draw a right line *A E*, repreſenting the parallell of the place from whence I depart.

2 I take 6 *gr.* for the difference of longitude, either out of the line of *inches*, allowing halfe an inch for euery degree, or out of the beginning of the *meridian* line; (for there the meridian degrees differ very little from the equinoctiall degrees) and theſe 6 *gr.* I pricke downe in the parallell from *A* to *E*.

3 In *A* and *E*, I erect two perpendiculars, A M and E D, repreſenting the meridians of both places.

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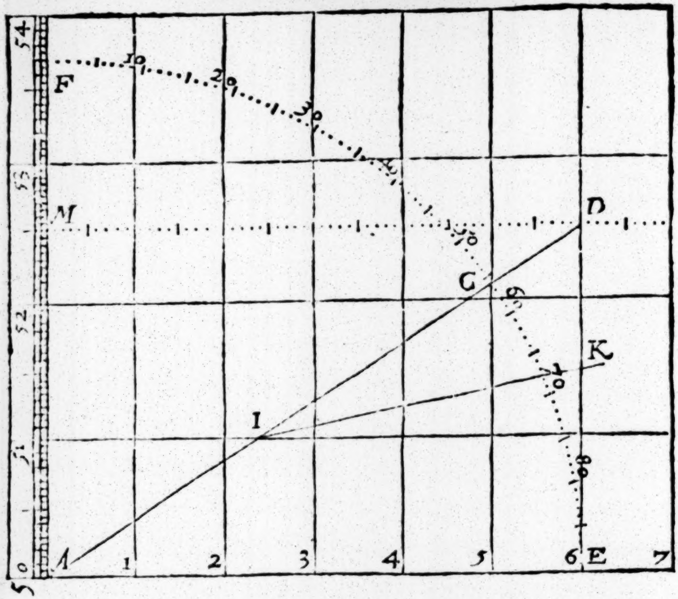

4 I take the difference of latitude from 50 *gr* to 52 *gr.* 30 *m.* out of the *meridian* line, and pricke it downe in the meridians from *A* vnto *M*, and from E to D, and draw the right line M D for the parallell of the ſecond place, and the right line A D for the line of diſtance betweene both places: ſo the angle M A D ſhall giue the Rumb that leadeth from the one place to the other.

5 To finde the quantitie of this angle M A D, I may either make vſe of the Protractor, or elſe of a line of *chords*, and ſo I open my compaſſes vnto one of the chords of 60 *gr.* and ſetting one foote in the point *A*, with the other I deſcribe an occult arke of a circle, interſecting the meridian in *F*, and the line of diſtance in G; then I take the chord F G with my compaſſes, and meaſuring it in the ſame line of *chords* as before, I find it 56 *gr.* $\frac{1}{4}$: and ſuch is the inclination of

the Rumb to the meridian, which is the firſt thing that was required.

6 To find the quantitie of the line of diſtance A D, I take it out with my compaſſes, and meaſuring it in the meridian line, ſetting one foote beneath the leſſer latitude, and the other foote as much aboue the greater latitude, I find about 4 *gr.* ½ intercepted betweene both feet: and ſuch is the diſtance vpõ the Rumb, which is the ſecond thing that was required.

But if this example were protracted according to the common Sea-chart, where the degrees of the equinoctiall and meridian are both alike; the Rumb M A D would be found to be aboue 67 *gr.* and A D the diſtance vpon the Rumb about 6 *gr.* ½.

Suppoſe farther, that hauing ſet forth from *A* toward D, vpon the former Rumb of 56 *gr.* 15 *m.* N *E b E*, after the ſhip had runne 36 leagues, the wind changing, it ran 50 leagues more vpon the ſeuenth Rumb of *E b N*, whoſe inclination to the meridian is 78 *gr.* 45 *m.* And let it be required to know what longitude and latitude the ſhip is in, by pricking down the way thereof vpon the Chart.

Hauing drawne a blanke chart as before, with meridians & parallels, according to the latitude of the places propoſed.

1 I would make an angle M A D of 56 *gr.* 15 *m.* for the Rumb of *N E b E*, which is done after this maner: I open my compaſſes to one of the *chords* of 60 *gr.* and ſetting one foote in the point *A*, with the other I deſcribe an occult ark of a circle, interſecting the meridian in *F*; then I take 56 *gr.* 15 *m.* out of the ſame line of *chords*, and pricke them downe from *F* vnto *G*: ſo the right line *A G* ſhall be the Rumb of *N E b E*.

2 I would take 36 leagues out of the *meridian line*, extending my compaſſes from 50 *gr.* to 51 *gr.* 48 *m.* or rather from as much below 50 as aboue 51, and prick them downe vpon the Rumb from *A* vnto *I*; ſo the point *I*, ſhal repreſent the place wherein the ſhip was when the wind changed. And this is in the latitude of 51 *gr.* 0 *m.* and in the longitude of 2 *gr.* 21 *m.* Eaſtward from the meridian *A M*.

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3 By the ſame reaſon, I may draw the right line *I K* for the Rumb of *E b N*, and pricke downe the diſtance of 50 leagues from *I* vnto *K*: ſo the point *K* ſhal repreſent the place whither the ſhip came, after the running of theſe 50 leagues: and this is in the latitude of 51 *gr.* 30 *m.* and in longitude 6 *gr.* 16 *m.* Eaſtward from the firſt meridian *A M*, and therefore 16 *m.* Eaſtward from the ſecond meridian E D.

But if theſe two courſes were to be pricked downe by the common ſea-chart, the point *I* would fall in the latitude of 51 *gr.* 0 *m.* and the point *K* in the latitude of 51 *gr.* 30 *m.* But the longitude of *I* would be onely 1 *gr.* 30 *m.* and the longitude of *K* onely 3 *gr.* 57 *m.* which is 33 *m* Weſtward from the meridian of the place to which the ſhip was bound.

Such is the difference betweene both theſe charts.

CHAP. VI.

The vſe of the line of Numbers.

1 ***Hauing two numbers giuen to find a third in continuall proportion, a fourth, a fift, and ſo forward.***

EXtend the compaſſes from the firſt number vnto the ſecond; then may you turne them, from the ſecond to the third, and from the third to the fourth, and ſo forward.

Let the two numbers giuen be 2 and 4. Extend the compaſſes from 2 to 4, then may you turne them from 4 to 8, and from 8 to 16, and from 16 to 32, and from 32 to 64, and from 64 to 128.

Or if the one foote of the compaſſes being ſet to 64, the other fall out of the line, you may ſet it to another 64 nearer the beginning of the line, and there the other foot will reach to 128, and from 128 you may turne them to 256, and ſo forward.

Or if the two firſt numbers giuen were 10 and 9: extend the compaſſes from 10 at the end of the line, backe vnto 9, then may you turne them from 9 vnto 8.1, and from 8.1

vnto 7.29. And ſo if the two firſt numbers giuen were 1 and 9, the third would be found to be 81, the fourth 729, with the ſame extent of the compaſſes.

In the ſame maner, if the two firſt numbers were 10 and 12, you may finde the third proportionall to be 14.4, the fourth 17.28. And with the ſame extent of the compaſſes, if the two firſt numbers were 1 and 12, the third would be found to be 144, and the fourth to be 1728.

2 *Hauing two extreme numbers giuen, to find a meane proportionall betweene them.*

Diuide the ſpace betweene the extreme numbers into two equall parts, and the foote of the compaſſes will ſtay at the meane proportionall. So the extreme numbers giuen being 8 and 32, the meane betweene them will be found to be 16, which may be proued by the former *Prop.* where it was ſhewed, that as 8 to 16, ſo are 16 to 32.

3 *To find the ſquare roote of any number giuen.*

The ſquare roote is alwayes the meane proportionall betweene 1 and the number giuen, and therefore to be found by diuiding the ſpace betweene them into two equall parts. So the roote of 9 is 3, and the roote of 81 is 9, and the roote of 144 is 12.

4 *Hauing two extreme numbers giuen, to find two meane proportionals between them.*

Diuide the ſpace betweene the two extreme numbers giuen, into three equall parts. As if the extreme numbers giuen were 8 and 27, diuide the ſpace betweene them into three equall parts, the feet of the compaſſes will ſtand in 12 and 18.

5 *To find the cubique roote of a number giuen.*

The cubique roote is alwayes the firſt of two meane pro-

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portionals betweene 1 and the number giuen, and therefore to be found by diuiding the ſpace betweene them into three equall parts.

So the roote of 1728 will be found to be 12. The roote of 17280 is almoſt 26: and the roote of 172800 is almoſt 56.

6 *To multiply one number by another.*

Extend the compaſſes from 1 to the multiplicator; the ſame extent applied the ſame way, ſhall reach from the multiplicand to the product.

As if the numbers to be multiplied were 25 and 30: either extend the compaſſes from 1 to 25, and the ſame extent will giue the diſtance from 30 to 750; or extend them from 1 to 30, and the ſame extent ſhall reach from 25 to 750.

7. *To diuide one number by another.*

Extend the compaſſes from the diuiſor to 1, the ſame extent ſhall reach from the diuidend to the quotient.

So if 750 were to be diuided by 25, the quotient would be found to be 30.

8 *Three numbers being giuen to find a fourth proportionall.*

This golden rule, the moſt vſefull of all others, is performed with like eaſe. For extend the compaſſes from the firſt number to the ſecond, the ſame extent ſhall giue the diſtance from the third to the fourth.

As for example, the proportion betweene the diameter and the circumference, is ſaid to be ſuch as 7 to 22: if the diameter be 14, how much is the circumference? Extend the compaſſes from 7 to 22, the ſame extent ſhall giue the diſtance from 14 to 44: or extend them from 7 to 14, and the ſame extent ſhall reach from 22 to 44.

Either of theſe wayes may be tried on ſeuerall places of this line ; but that place is beſt, where the feete of the compaſſes may ſtand neareſt together.

9 *Three numbers being giuen to finde a fourth in a duplicated proportion.*

This propoſition concernes queſtions of proportion betweene *lines* and *ſuperficies*; where if the denomination be of lines, extend the compaſſes from the firſt to the ſecond number of the ſame denomination : ſo the ſame extent being doubled, ſhall giue the diſtance from the third number vnto the fourth.

The diameter being 14, the content of the circle is 154: the diameter being 28, what may the content be? Extend the compaſſes from 14 to 28, the ſame extent doubled will reach from 154 to *616*. For firſt it reacheth from 154 vnto 308; and turning the compaſſes once more, it reacheth from 308 vnto *616*: and this is the content required.

But if the firſt denomination be of the ſuperficiall content, extend the compaſſes vnto the halfe of the diſtance, betweene the firſt number and the ſecond of the ſame denomination : ſo the ſame extent ſhall giue the diſtance from the third to the fourth.

The content of a circle being 154, the diameter is *14*: the content being *616*, what may the diameter be? Diuide the diſtance betweene 154 and *616* into two equall parts, then ſet one foote in 14, the other will reach to 28 the diameter required.

10 *Three numbers being giuen to finde a fourth in a triplicated proportion.*

This propoſition concerneth queſtions of proportion betweene *lines* and *ſolids*; where if the firſt denomination be of lines, extend the compaſſes from the firſt number to the ſecond of the ſame denomination: ſo the extent being tripled, ſhall giue the diſtance frō the third number vnto the fourth.

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Suppoſe the diameter of an iron bullet being 4 inches, the weight of it was 9 ℔: the diameter being 8 inches, what may the weight be? Extend the compaſſes from 4 to 8, the ſame extent being tripled, will reach from 9 vnto 72. For firſt it reacheth from 9 vnto 18; then from 18 to 36; thirdly from 36 to 72. And this is the weight required.

But if the firſt denomination ſhall be of the Solid content, or of the weight, extend the compaſſes to a third part of the diſtance betweene the firſt number and the ſecond of the ſame denomination: ſo the ſame extent ſhal giue the diſtance from the third number vnto the fourth.

The weight of a cube being 72 ℔, the ſide of it was 8 inches: the weight being 9 ℔, what may the ſide be? Diuide the diſtance betweene 72 and 9, into three equall parts; then ſet one foote to 8, the other will reach to 4, the ſide required.

CHAP. VII.

The vſe of the lines of artificiall Sines.

THis line of *ſines* hath ſuch vſe in finding a fourth proportionall, as the ordinary *Canon* of *Sines*: and the maner of finding it, is alwayes ſuch as in this example.

As the ſine of 30 *gr.* vnto the ſine of 52 *gr.*
So the ſine of 38 *gr.* to a fourth ſine.

Extend the compaſſes in the line of *ſines* from 30 *gr.* vnto 52 *gr*; the ſame extent ſhall giue the diſtance from 38 *gr.* vnto 76 *gr.* Or extend them from 30 *gr.* vnto 38 *gr.* the ſame extent will reach from 52 *gr.* vnto 76 *gr.* which is the fourth proportionall ſine required.

And thus may the reſt of all ſinical proportions be wrought two wayes. The minutes which are wanting in the firſt degree, may be ſupplied by the line of *Numbers*.

CHAP. VIII.

The vſe of the line of artificiall Tangents.

THis line of ***Tangents*** hath like vſe, but commonly ioyned with the line of ***ſines***: the maner of working by it, may appeare by this example.

As the Tangent of 38 *gr.* 30 *m.*
is to the Tangent of 23 *gr.* 30 *m.*
So the Sine of 90 *gr.*
to a fourth Sine.

This ***Prop.*** and ſuch others vpon two lines, may be wrought two wayes. For extend the compaſſes from the Tangent of 38 *gr.* 30 *m.* to the Tangent of 23 *gr.* 30 *m*; the ſame extent ſhall giue the diſtance from the ſine of 90 *gr.* to the ſine of 33 *gr.* 8 *m.* Or elſe extend them from 38 *gr.* 30 *m.* in the Tangents vnto 90 *gr.* in the line of ***ſines***; the ſame extent from the Tangent of 23 *gr.* 30 *m.* ſhall reach to the ſine of 33 *gr.* 8 *m.* which is the fourth proportionall ſine required.

And this croſſework in many caſes is the better, in regard the tangents which ſhould paſſe on from 40 *gr.* to 50 *gr.* and ſo forward, do turne backe at 45 *gr.* Theſe two lines of ***Sines*** and ***Tangents***, may ſerue for the reſolution of all ſphericall triangles, according to thoſe Canons which I haue ſet downe in the vſe of the ***Sector***.

Or if at any time one meete with a ***ſecant***, let him account the ſine of 80 *gr.* for a ***ſecant*** of 10 *gr.* and the ſine of 70 *gr.* for a ***ſecant*** of 20 *gr.* and ſo take the ſine of the complement in ſtead of the ***ſecant***. As if the propoſition were,

As the Radius to the ſecant of 51 *gr.* 30 *m.*
So the ſine of 23 *gr.* 30 *m.* to a fourth ſine.

Extend the compaſſes from the Radius that is the ſine of 90 *gr.* to the ſine of 38 *gr.* 30 *m.* the ſame extent will giue the diſtance from the ſine of 23 *gr.* 30 *m.* both to the ſine of 14 *gr.*

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22 *m.* and to the ſine of 39 *gr.* 50 *m.* But in this caſe, the ſine of 39 *gr.* 50 *m.* is the fourth required. For the firſt number being leſſe then the ſecond, that is, the Radius leſſe then the ſecant, the ſine of 23 *gr.* 30 *m.* which is the third, muſt alſo be leſſe then the fourth.

CHAP. IX.

The vſe of the line of Sines and Tangents ioyned with the line of numbers.

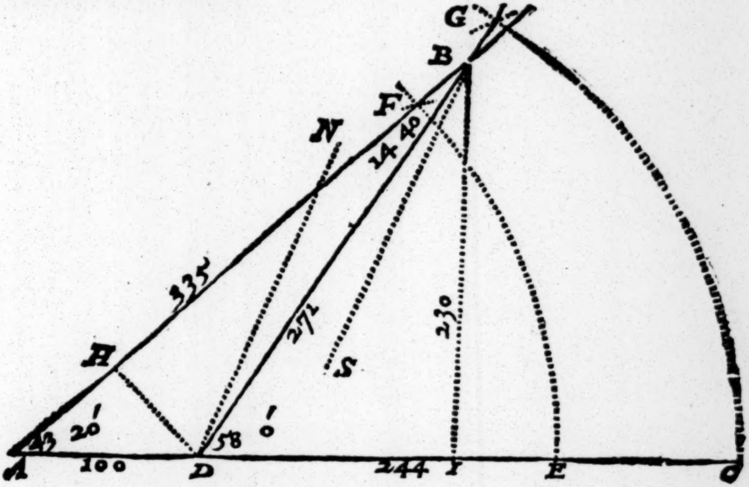
THe lines of *ſines* and *tangents* haue another like vſe ioyned with the line of *numbers*, eſpecially in the reſolution of right line triangles, where the angles are meaſured by degrees and minutes, and the ſides meaſured by abſolute numbers, whereof I will ſet downe theſe propoſitions.

1 *Hauing three angles and one ſide, to find the two other ſides.*

As the ſide of the angle oppoſite to the ſide giuen,
is to the number belonging to that ſide giuen:
So the ſine of the angle oppoſite to the ſide required,
to the number belonging to the ſide required.

As in the example of the fourth *Cap.* of this booke, where knowing the diſtance betweene two ſtations at *A* and *D* to be 100 paces, the angle *BAC* to be 43 *gr.* 20 *m.* and the angle *BDC* to be 58 *gr.* it was required to find the diſtance *AB.*

Firſt hauing theſe two angles, I may find the third angle *ABD* to be 14 *gr.* 40 *m.* either by ſubſtraction or by complement vnto 180. Then in the triangle *BAD*, I haue three angles, and one ſide, whereby I may find both *AB* and *DB*. I know the angle *ABD* oppoſite to the meaſured ſide *AD* to be 14 *gr.* 40 *m.* and the angle *ADB* oppoſite to the ſide required, to be 122 *gr*: wherefore I extend the compaſſes in the line of *ſines* from 14 *gr.* 40 *m.* vnto 122 *gr.* or (which is all one) to 58 *gr.* (for after 90 *gr.* the ſine of 80 *gr.* is alſo the ſine of 100 *gr.* and the ſine of 70 *gr.* the ſine of 110 *gr.* and ſo in the reſt) ſo ſhall I find the ſame extent to reach in the line of *numbers*, from 100 vnto 335. And ſuch is the diſtance required betweene *A* and *B*.



In like maner if I extend my compaſſes from the ſine of 14 *gr.* 40 *m.* to the ſine of 43 *gr.* 20 *m.* the ſame extent will reach in the line of *numbers* from 100 to 271. And ſuch is the diſtance betweene D and B.

Or in croſſe worke, I may extend the compaſſes from 14 *gr.* 40 *m.* in the *ſines*, vnto 100 parts in the line of *numbers*: ſo the ſame extent will giue the diſtance from 58 *gr.* to 335 parts, and from 43 *gr.* 20 *m.* to 271 parts.

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2 *Hauing two ſides giuen, and one angle oppoſite to either of theſe ſides, to find the other two angles and the third ſide.*

As the ſide oppoſite to the angle giuen,
  is to the ſine of the angle giuen:
So the other ſide giuen,
  to the ſine of that angle to which it is oppoſite.

So in the former triangle, hauing the two ſides *AB* 335 paces, and A D 100 paces, and knowing the angle A D B, which is oppoſite to the ſide *AB*, to be 122 *gr.* I may find the angle A B D, which is oppoſite to the other ſide A D. For if I extend the compaſſes from 335 to 100 in the line of *numbers*, I ſhall finde the ſame extent to reach in the line of *ſines* from 122 *gr.* to 14 *gr.* 40 *m*; and therefore ſuch is the angle A B D.

Then knowing theſe two angles ABD and ADB, I may find the third angle B A D either by ſubtraction or by complement to 180, to be 43 *gr.* 20 *m*; and hauing three angles and two ſides, I may well find the third ſide DB, by the former *Prop.*

This may be done more readily by croſſe worke. For if I extend the compaſſes from 335 parts, in the line of *numbers*, to the ſine of 122 *gr.* the ſame extent will reach from 100 parts to the ſine of 14 *gr.* 40 *m.* and backe from 43 *gr.* 20 *m.* to 271 parts; and ſuch is the third ſide DB.

3 *Hauing two ſides and the angle between them, to find the two other angles and the third ſide.*

If the angle contained betweene the two ſides be a right angle, the other two angles will be found readily by this canon.

As the greater ſide giuen,
  is to the leſſer ſide:

So the tangent of 45 *gr.*
to the tangent of the leſſer angle.

So in the rectangle triangle *AIB*, knowing the ſide *AI* to be 244, and the ſide *IB* to be 230: if I extend the compaſſes from 244 to 230 in the line of *numbers*, the ſame extent will reach from 45 *gr.* to about 43 *gr.* 20 *m.* in the line of *tangents*; and ſuch is the leſſer angle *BAI*, and the complement 46 *gr.* 40 *m.* ſhewes the greater angle *ABI*. The angles being knowne, the third ſide AB may be found by the firſt *Prop.*

So likewiſe in the example of the third *Cap.* of this booke, concerning taking of angles by the line of *inches*, where the parts intercepted on the Staffe being 20 *inches*, and the parts on the Croſſe 9 *inches*, it was required to find the angle of altitude. For I may extend the compaſſes in the line of *numbers*, from 20 vnto 9, the ſame extent will reach in the line of *tangents*, from 45 *gr.* to 24 *gr.* 14 *m.* Or in the croſſe worke, I may extend the compaſſes from 20 parts in the line of *numbers* to the tangent of 45 *gr*; the ſame extent ſhall giue the diſtance from 9 parts vnto the tangent of 24 *gr.* 14 *m.* And ſuch is the angle of altitude required.

But if it be an oblique angle that is contained betweene the two ſides giuen, the triangle may be reduced into two rectangle triangles, and then reſolued as before.

As in the triangle ADB, where the ſide AB is 335, and the ſide AD 100, and the angle BAD 43 *gr.* 20 *m*: if I let downe the perpendicular DH vpon the ſide AB, I ſhal haue two rectangle triangles, AHD, DHB; and in the rectangle AHD, the angle at *A* being 43 *gr.* 20 *m.* the other angle ADH will be 46 *gr.* 40 *m*; and with theſe angles and the ſide *AD*, I may find both *AH* and DH, by the firſt *Prop.* Then taking *AH* out of *AB*, there remaines HB for the ſide of the rectangle DHB; and therefore with this ſide HB and the other ſide DH, I may find both the angle at B, and the third ſide DB, as in the former part of this *Prop.*

Or I may find the angles required, without letting downe any perpendicular. For

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As the ſumme of the ſides,
is to the difference of the ſides:
So the tangent of the halfe ſumme of the oppoſite angles,
to the tangent of half the difference between thoſe angles.

As in the former triangle *A D B*, the ſumme of the ſides *A B*, *A D*, is 435, and the difference betweene them 235; the angle contained 43 *gr.* 20 *m*; and therefore the ſumme of the two oppoſite angles 136 *gr.* 40 *m.* and the halfe ſumme 68 *gr* 20 *m.* Hereupon I extend the compaſſes in the line of *numbers* from 455 to 235, and I find them to reach in the line of *tangents* from 68 *gr.* 20 *m.* vnto 53 *gr.* 40 *m*; and ſuch is the halfe difference betweene the oppoſite angles at *B* and *D.* This halfe difference being added to the halfe ſumme, doth giue 122 *gr.* for the greater angle *A D B*: and being ſubtracted, it leaueth 14 *gr.* 40 *m.* for the leſſer angle *A B D.* Then the three angles being knowne, the third ſide B D may be found by the firſt *Prop.*

4 *Hauing the three ſides of a right line triangle, to find the perpendicular and the three angles.*

Let one of the three ſides giuen be the baſe, but rather the greater ſide, that the perpendicular may fall within the triangle; then gather the ſumme, and the difference of the two other ſides, and the proportion will hold.

As the baſe of the triangle,
is to the ſumme of the ſides:
So the difference of the ſides
to a fourth, which being taken forth of the baſe, the perpendicular ſhal fall on the middle of the remainder.

As in the former triangle ADB, where the baſe AB is 335, the ſumme of the ſides AD and DB 371, and the difference of them 171. If I extend the compaſſes in the line of *numbers* from 335 vnto 371, I ſhall find the ſame extent to reach from 171 vnto 189.4. This fourth number I take out of the baſe 335.0, and the remainder is 145.6, the halfe whereof is 72.8, and doth ſhew the place *H*, where the perpendicular ſhall fall, from the angle D, vpon the baſe *A B*, diuiding the former triangle ADB into two right angle triangles, DHA and DHB, in which the angles may be found by the former part of the third *Prop.* And this may ſuffice for right line triangles. But for the more eaſie protraction of theſe triangles, I will ſet downe one propoſition more concerning *chords*.

5 *Hauing the ſemidiameter of a circle, to find the chords of euery arke.*

As the ſine of 30 *gr.*
 to the ſine of halfe the arke propoſed:
So is the ſemidiameter of the circle giuen,
 to the chord of the ſame arke.

As if in protracting the former triangle ADB, it were required to find the length of a chord of 43 *gr.* 20 *m.* agreeing to the ſemidiameter *A E*, which is knowne to be 3 *inches*. The halfe of 43 *gr.* 20 *m.* is 21 *gr.* 40 *m*; wherefore I extend the compaſſes from the ſine of 30 *gr.* to the ſine of 21 *gr.* 40 *m.* and I finde the ſame extent to reach in the line of *numbers* from 3.000 *parts* to 2.215; which ſhewes, that the ſemidiameter being 3 inches, the chord of 43 *gr.* 20 *m.* will be 2 *inches* and 215 *parts* of 1000.

In like maner the chord of 58 *gr.* agreeing to the ſame ſemidiameter, would be found to be 2 *inches* and 909 *parts*. For the halfe of 58 being 29; if I extend the compaſſes in the line of *ſines* from 30 *gr.* to 29 *gr.* the ſame extent will reach in the line of *numbers* from 3.000. vnto 2.909.

Or in croſſe worke, if I extend the compaſſes from the *ſine* of 30 *gr.* to 3.000 in the line of *numbers*, I ſhall find the ſame extent to reach from 21 *gr.* 40 *m.* to 2.215 *parts*, and from 29 *gr.* to 2.909 *parts*, and from 7 *gr.* 20 *m.* to 765 *parts*; for the chord of 14 *gr.* 40 *m.* for the third angle ABD.

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CHAP. X.

The vſe of the line of verſed ſines.

THis line of *verſed ſines* is no neceſſary line. For all triangles, both right lined and ſphericall, may be reſolued by the three former lines of ***numbers***, *ſines* and ***tangents***; yet I thought good to put it on the Staffe for the more eaſie finding of an angle hauing three ſides, or a ſide hauing three angles of a ſphericall triangle giuen.

Suppoſe the three ſides to be, one of them 110 *gr.* the other 78 *gr.* and the third 38 *gr.* 30 *m.* and let it be required to find the angle, whoſe baſe is 110 *gr.*

I firſt adde them together, and from halfe the ſumme ſubtract the baſe, noting the difference after this maner.

The baſe	110 *gr.*	0 *m.*
The one ſide	78	0
The other ſide	38	30
The ſumme of all three	226	30
The halfe ſumme	113	15
The difference	3	15

This done, I come to the Staffe, and extend the compaſſes from the ſine of 90 *gr.* to the ſine of 78 *gr.* which is one of the ſides; and applying this extent from the ſine of the other ſide 38 *gr.* 30 *m.* I find it to reach to a fourth ſine, about 37 *gr.* 30 *m.* From this fourth ſine of 37 *gr.* 30 *m.* I extend the compaſſes again, to the ſine of the halfe ſumme 113 *gr.* 15 *m.* (which is all one with the ſine of 66 *gr.* 45 *m.*) and this ſecond extent wil reach from the ſine of the difference 3 *gr.* 15 *m.* to the ſine of 4 *gr.* 54 *m.* Ouer againſt this ſine you ſhal find 146 *gr.* in the line of *verſed ſines*; and ſuch is the angle required.

THE SECOND BOOKE.

Of the vſe of the former lines of proportiō, more particularly exemplified in ſeuerall kinds.

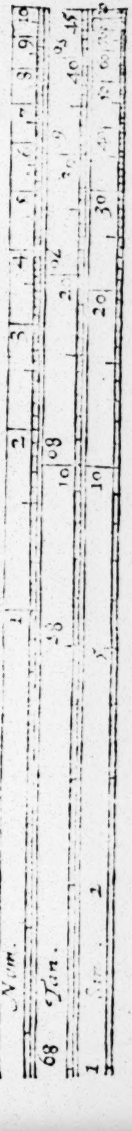
THe former booke containing the generall vſe of each line of proportion, may be ſufficient for all thoſe which know the rule of *Three*, and the doctrine of triangles.

But for others, I ſuppoſe it would be more difficult to find either the declination of the Sunne, or his amplitude, or the like, by that which hath been ſaid in the vſe of the line of *ſines*, vnleſſe they may haue the particular proportions, by which ſuch propoſitions are to be wrought. And therefore for their ſakes I haue adioyned this ſecond booke, containing ſeuerall proportions for propoſitions of ordinary vſe, and ſet them downe in ſuch order, that the Reader conſidering which is the firſt of the three numbers giuen, may eaſily apply them to the Sector, and alſo reſolue them by Arithmetique, beginning with thoſe which require help onely of the line of *numbers*.

CHAP. I.

The vſe of the line of Numbers in broade meaſure, ſuch as boord, glaſſe, and the like.

THe ordinary meaſure for bredth and length are feete and inches, each foote diuided into 12 inches, and euery inch into halues & quarters,



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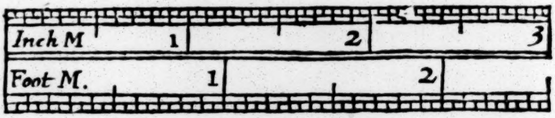
ters, which being parts of ſeueral denominations, doth breed much trouble both in arithmetique and the vſe of inſtruments.

For the auoiding whereof, where I may preuaile I giue this counſell, that ſuch as are delighted in meaſure would vſe ſeuerall lines, firſt a line of inch meaſure, wherein euery inch may be diuided into 10 or 100 parts; ſecondly a line of foote meaſure, wherein euery foote may be diuided into 100 or 1000 parts, both which lines may be ſet on the ſame ſide of a two foote ruler, after this or the like maner.



Then if they be to giue the content of any ſuperficies or ſolid in inches, they may meaſure the ſides of it by the line of inches and parts of inches; but if they be to giue the content in feete, it would be more eaſie for them to meaſure thoſe ſides by the foote line and his parts.

For example, let the length of a plane be 30 inches, and the bredth 21 inches and $\frac{6}{10}$ of an inch; this length multiplied into the bredth, would giue the content to be 648 inches: but if I were to find the content of the ſame plane in feet, I would meaſure the ſides of it by the foote line and his parts; ſo the length would proue to be 2 feete $\frac{50}{100}$, and the bredth 1 foote $\frac{80}{100}$, and the length multiplied by the bredth, cutting off the foure laſt figures, for the foure figures of the parts, would giue the content to be 4.5000, which is 4 foote and 5000 parts, of a foot being diuided into 10000 parts.

21.6
30.0
648.00

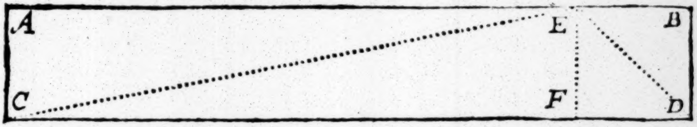
2.50
1.80
20000
250
4.5000

The like reaſon holdeth for yards and elnes, and all other meaſures diuided into 10, 100, or 1000 parts.

This being preſuppoſed, the worke will be more eaſie both by arithmetique and the line of ***numbers***, as may appeare by theſe propoſitions.

1 *Hauing the bredth and length of any oblong ſuperficies giuen in inch-meaſure, to finde the content in inches.*

As 1 inch vnto the bredth in inches:
So the length in inches vnto the content in inches.



Suppoſe in the plane AD, the bredth *A C* to be 30 inches, and the length *A B* to be 183 inches; extend the compaſſes from 1 vnto 30, the ſame extent will reach from 183 vnto 5490; or extend them from 1 vnto 183, the ſame extent will reach from 30 vnto 5490. So both wayes the content required is found to be 5490 inches.

As 1 vnto 30: ſo are 183 vnto 5490.

2 *Hauing the length and bredth of any oblong ſuperficies giuen in inches, to finde the content in feete.*

As 144 inches vnto the bredth in inches:
So the length in inches vnto the content in feete.

And thus in the former plane A D, working as before, the content will be found to be 38.125, which is 38 foote and $\frac{1}{8}$ of a foote.

As 144 vnto 30: ſo are 183 vnto 38.125.

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3 *Hauing the length and bredth of any oblong ſuperficies giuen in foote meaſure, to finde the content in feete.*

As 1 foote vnto the bredth in foote meaſure:
So the length in feete vnto the content in feete.

And thus in the former plane AD, the bredth will be 2 foote 50 parts, and the length 15 foot 25 parts; then working as before, the content will be found to be 38.125.

As 1 vnto 2.50 : ſo are 15.25 vnto 38.125.

4 *Hauing the bredth of any oblong ſuperficies giuen in inches, and the length in foote meaſure, to find the content in feet.*

As 12 inches to the bredth in inches:
So the length in feet to the content in feet.

So alſo in the former plane, the content will be found to be 38.125.

As 12 vnto 30 : ſo are 15.25 vnto 38.125.

5 *Hauing the bredth of an oblong ſuperficies giuen in inches, to find the length of a foot ſuperficiall in inch meaſure.*

As the bredth in inches, vnto 144 inches:
So 1 foot vnto the length in inch meaſure.

So the bredth being 30 inches, the length of a foot wil be found to be 4 inches 80 parts.

As 30 vnto 144 : ſo are 1 vnto 4.80.

6 *Hauing the bredth and length of an oblong ſuperficies giuen in feet, to find the length of a foot ſuperficiall in foot meaſure.*

As the bredth in foot meaſure to 1 foot:
So the number of feet to the length in foot meaſure.

So the bredth being 2 foot 50 parts, the length of a foot will be found to be 40 parts, the length of 2 feete 80 parts, and the length of 3 feet 1 foot 20 parts,&c.

As 250 vnto 1: ſo are 1 vnto 0.40.

7 *Hauing the length and bredth of an oblong ſuperficies, to find the ſide of a ſquare equall to the oblong.*

Diuide the ſpace betweene the length and the bredth into two equall parts, and the foot of the compaſſes will ſtay at the ſide of the ſquare.

So the length being 183 inches,and the bredth 30 inches, the ſide of the ſquare will be found to be almoſt 74 inches and 10 parts of 100.

Or the bredth being 2 foot and 50 parts,the length 15 foot and 25 parts, the ſide of the ſquare wil be found to be about 6 feet and 17 parts.

As 30 vnto 74.10: ſo are 74.10 vnto 183.027.

And as 2.50 vnto 6.174: ſo are 6.174 vnto 15.247.

8 *Hauing the diameter of a circle, to find the ſide of a ſquare equall to that circle.*

As 10000 to the diameter:
So 8862 vnto the ſide of the ſquare.

So the diameter of a circle being 15 inches,the ſide of the ſquare will be found about 13 inches and 29 parts.

As 10000 vnto 8862: ſo are 15 vnto 13.29.

9 *Hauing the circumference of a circle to find the ſide of a ſquare equall to the ſame circle.*

As 10000 to the circumference:
So 2821 to the ſide of the ſquare.

So the circumference of a circle being 47 inches 13 parts, the ſide of the ſquare will be about 13 inches 29 parts.

As 10000 vnto 2821: ſo are 47.13 vnto 13.29.

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10 *Hauing the diameter of a circle, to find the circumference.*

11 *Hauing the circumference of a circle, to find the diameter.*

As 1000 to the diameter:
So 3142 to the circumference.

So the diameter being 15 inches, the circumference will be found about 47 inches 13 parts: or the circumference being 47.13, the diameter will be 15.

CHAP. II.

The vſe of the line of Numbers in the meaſure of land by perches and acres.

1 *Hauing the bredth and length of an oblong ſuperficies giuen in perches, to find the content in perches.*

As 1 perch to the bredth in perches:
So the length in perches to the content in perches.

So in the former plane *A D*, if the bredth *A C* be 30 perches, and the length *A B* 183 perches, the content will be found to be 5490 perches.

2 *Hauing the length and bredth of an oblong ſuperficies giuen in perches, to find the content in acres.*

As 160 to the bredth in perches:
So the length in perches to the content in acres.

So in the former plane *A D*, the content will be found to be 34 acres, and 31 centeſms or parts of an 100.

As 160 vnto 30: ſo are 183 vnto 34.31.

3 *Hauing*

3 *Hauing the length and bredth of an oblong ſuperficies giuen in chaines, to find the content in acres.*

It being troubleſome to diuide the content in perches by 160, we may meaſure the length and bredth by chaines, each chaine being 4 perches in length, and diuided into 100 links, then will the worke be more eaſie in arithmetique. For

As 10 to the content in chaines:
So the length in chaines to the content in acres.

And thus in the former plane AD, the bredth *AC* will be 7 chaines 50 links, and the length *AB* 45 chaines 75 links; then working as before, the content will be found as before, 34 acres 31 parts.

4 *Hauing the perpendicular and baſe of a triangle giuen in perches, to find the content in acres.*

If the perpendicular go for the bredth, and the baſe for the length, the triangle will be the halfe of the oblong. As the triangle CED is the halfe of the oblong AD, whoſe content was found in the former *Prop.* Or without halfing,

As 320 to the perpendicular:
So the baſe to the content in acres.

So in the triangle CED, the perpendicular being 30, and the baſe 183, the content will be found to be about 17 acres and 15 parts.

5 *Hauing the perpendicular and baſe of a triangle giuen in chaines, to find the content in acres.*

As 20 to the perpendicular:
So the baſe to the content in acres.

And ſo in the triangle CED, the perpendicular *EF* be-

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ing 7.50, and the baſe C D 45.75, the content will be as before to be about 17 acres 15 parts.

6 *Hauing the content of a ſuperficies after one kind of perch, to find the content of the ſame ſuperficies according to another kind of perch.*

As the length of the ſecond perch
to the length of the firſt perch:
So the content in acres to a fourth number;
and that fourth to the content in acres required.

Suppoſe the plane A D meaſured with a chaine of *66* feet, or with a perch of 16 feete and an halfe, contained 34 acres 31 parts; and it were demanded how many acres it would containe if it were meaſured with a chaine of 18 foot to the perch: theſe kind of propoſitions are wrought by the backward rule of *three*, after a duplicated proportion. Wherefore I extend the compaſſes from 16.5 vnto 18.0, and the ſame extent doth reach backward, firſt from 34.31 to 31.45, and then from 31.45 to 28.84, which ſhewes the content to be 28 acres 84 parts.

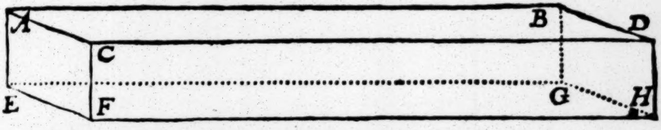
7 *Hauing the plot of a plane with the content in acres, to find the ſcale by which it was plotted.*

Suppoſe the plane A D contained 34 acres 31 centeſmes; if I ſhould meaſure it with a ſcale of 10 in the inch, the length AB would be 38 chaines and about 12 centeſimes, and the bredth *AC* 6 chaines and 25 centeſimes; and the content would be found by the third *Prop.* of this Chapter, to be about 23 acres 82 parts, whereas it ſhould be 34 acres 31 parts.

Wherefore I diuide the diſtance betweene 23.82, and 34.31, vpon the line of *numbers* into two equall parts; then ſetting one foote of the compaſſes vpon 10, my ſuppoſed ſcale, I find the other to extend to 12, which is the ſcale required.

CHAP. III.

The vſe of the line of Numbers in ſolid meaſure, ſuch as ſtone, timber, and the like.



1 *Hauing the ſide of a ſquare equall to the baſe of any ſolid giuen in inch meaſure, to find the length of a foot ſolid in inch meaſure.*

THe ſide of a ſquare equall to the baſe of a ſolid, may be found by diuiding the ſpace betweene the length and bredth into two equall parts, as in the 7 *Prop.* of broad meaſure. Then

As the ſide of the ſquare in inches to 41.57:
So is 1 foot to a fourth number;
and that fourth to the length in inches.

So in the ſolid *A H*, the ſide of the ſquare equall to the baſe *E C*, being about 25 inches 45 parts, the length of a foot ſolid will be found about 2 inches 67 parts, and the length of two foot ſolid 5 inches 33 parts.

As 25.45 vnto 41.57 : ſo 1.00 vnto 1.63:
and ſo are 1.63 vnto 2.67.

2 *Hauing the ſide of a ſquare equall to the baſe of any ſolid giuen in foot meaſure, to find the length of a foot ſolid in foot meaſure.*

As the ſide of the ſquare in feet vnto 1:
So is 1 vnto a fourth number;
And that fourth to the length in foot meaſure.

So in the ſolid *A H*, the ſide of the ſquare equall to the

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baſe *E C*, being about 2 foote 120 parts, the length of a foot ſolid will be found about 222 parts of a foot.

As 2.120 vnto 1.000: ſo 1.000 vnto 0.471:
and ſo are 471 vnto 222.

3 *Hauing the bredth and depth of a ſquared ſolid giuen in foot meaſure, to find the length of a foot ſolid in foot meaſure.*

As 1 vnto the bredth in foot meaſure:
So the depth in feet to a fourth number;
which is the content of the baſe in foot meaſure. Then
As this fourth number vnto 1:
So 1 vnto the length in foot meaſure.

So in the ſolid *A H*, the bredth being 2 foot 50 parts, the depth 1 foot 80 parts, the content of the baſe *E C* will be found 4 foot 50 parts, and the length of one foot ſolid about 222 parts, the length of two foot ſolid about 444 parts of 1000.

As 1.00 vnto 2.50: ſo are 1.80 vnto 4.50.
As 4.50 vnto 1.00: ſo 1.000 vnto 0.222.

4 *Hauing the bredth and depth of a ſquared ſolid giuen in inches, to find the length of a foot ſolid in inch meaſure.*

As 1 hath to the bredth in inches:
So the depth in inches to a fourth number;
which is the content of the baſe in inches. Then
As this fourth number vnto 1728:
So 1 vnto the length of a foot in inch meaſure.

So in the ſolid *A H*, the bredth *A C* being 30 inches, and the depth *A E* 21 inches 60 parts, the content of the baſe *E C* will be found to be 648 inches, and the length of a foot ſolid about 2 inches 67 parts.

As 1 vnto 21.6: ſo 30 vnto 648:
As 648 vnto 1728: ſo 1 vnto 2.667.

Or as 12 to the bredth in inches:
So the depth in inches to a fourth number.

As this fourth number to 144:
So 1 vnto the length of a foot ſolid in inch meaſure.

So in the ſolid *AH*, the bredth being 30 inches, the depth 21 inches 6 parts, the fourth number will be found to be 54: and the depth of a foot ſolid 2 inches 67 parts.

As 12 vnto 21.6: ſo 30 vnto 54.
As 54 vnto 144: ſo 1 vnto 2.667.

5 *Hauing the ſide of a ſquare equall to the baſe of any ſolid, and the length thereof giuen in inch meaſure, to find the content thereof in feet.*

As 41.57 to the ſide of the ſquare in inches:
So the length in inches to a fourth number;
and that fourth to the content in foot meaſure.

So in the ſolid *AH*, the length *AB* being 183 inches, and the ſide of the ſquare equall to the baſe *EC* about 25 inches 45 parts, the fourth number will be found about 112, and the whole ſolid content about 68 feet 62 parts.

As 41.57 vnto 25.45: ſo 183 vnto 112:
and ſo are 112 vnto 68.62.

6 *Hauing the ſide of a ſquare equall to the baſe of any ſolid, and the length thereof giuen in foot meaſure, to find the content thereof in feet.*

As 1 to the ſide of the ſquare in foot meaſure:
So the length in feet to a fourth number;
and that fourth to the content in foot meaſure.

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So in the former ſolid *A H*, the ſide of the ſquare equall to the baſe *A E*, being about 2 foot 12 parts, and the length *A B* 15 foot 25 parts, the conent will be found to be about 68 foot 62 parts.

As 1 vnto 2.12: ſo 15.25 vnto 32.35:
and ſo are 32.35 vnto 68.62.

7 *Hauing the ſide of a ſquare equall to the baſe of any ſolid giuen in inch meaſure, & the length of the ſolid giuen in foot meaſure, to find the content thereof in feete.*

As 12 to the ſide of the ſquare giuen in inches:
So the length in feet to a fourth number;
and that fourth to the content in foot meaſure.

So in the former ſolid *A H*, the ſide of the equall ſquare being 25 inches 45 parts, the content will be found to be aabout 68 feet 62 parts.

As 12 vnto 25.45: ſo 15.25 vnto 32.35:
and ſo are 32.35 vnto 68.62.

8 *Hauing the length, bredth and depth of a ſquared ſolid giuen in inches, to find the content in inches.*

As 1 vnto the bredth in inches:
So the depth in inches vnto the baſe in inches. Then
As 1 vnto the baſe:
So the length in inches vnto the ſolid content in inches.

So in the ſolid *A H*, whoſe bredth *A C* is 30 inches, the depth *A E* 21 inches and 6 parts of 10, and length *A B* 183, the content of the baſe *E C* wil be found 648 inches, and the whole ſolid content about 118500 inches.

As 1 vnto 21.6: ſo are 30 vnto 648:
As 1 vnto 648: ſo are 183 to 118584.

9 *Hauing the length,bredth and depth of a ſquared ſolid giuen in inches, to find the content in feet.*

As 1 to the bredth in inches:
So the depth in inches to the baſe in inches.

As 1728 to that baſe:
So the length in inches to the content in feet.

So in the ſolid *AH*, the content will be found to be about *68* feet *62* parts.

As 1 vnto 21.*6*: ſo 30 vnto *648*:
As 1728 vnto 648: ſo 183 to 68.*62*.

Or as 12 to the bredth in inches:
So the depth in inches to a fourth number.
As 144 to that fourth number:
So the length in inches to the content in feet.

And ſo alſo in the ſame ſolid AH, the content will be found to be about 68 feet 62 parts.

As 12 vnto 21.*6*: ſo 30 vnto 54:
As 144 vnto 54: ſo 183 vnto *68.62*.

10 *Hauing the length,bredth and depth of a ſquared ſolid giuen in foot meaſure, to finde the content in feet.*

As 1 vnto the bredth in foot meaſure:
So the depth in feet to the baſe in feet.

As 1 vnto that baſe:
So the length in feet to the content in feet.

And thus in the former ſolid AH, the bredth *AC* will be 2 foot 50 parts, the depth *AE* 1 foot 80 parts, and the length *AB* 15 foot 25 parts; then working as before, the content of the baſe *AF* will be found 4 feet 50 parts, and the whole ſolid content about 68 foot 62 parts, which of all others may

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very eaſily be tried by arithmetique.

As 1 vnto 2.50: ſo 1.80 vnto 4.50.
As 1 vnto 4.50: ſo 15.25 vnto 68.625.

11 *Hauing the bredth and depth of a ſquared ſolid giuen in inches, and the length in foot meaſure, to find the content thereof in feet.*

As 1 vnto the bredth in inches:
So the depth in inches vnto a fourth number:
which is the content of the baſe in inches.

As 144 hath vnto that fourth number:
So the length in feet to the content in feet.

And ſo in the ſame ſolid A H, the content will be found to be about 68 feet 62 parts.

As 1 vnto 21.6: ſo 30 vnto 648.
As 144 vnto 15.25: ſo 648 vnto 68.62.

Or as 144 vnto the bredth in inches:
So the depth in inches vnto a fourth number:
which is the content of the baſe in feet.

As 1 hath vnto that fourth number:
So the length in feet to the content in feet.

And ſo in the ſame ſolid AH, the content will be found to be about 68 feet 62 parts.

As 144 vnto 21.6: ſo 30 vnto 4.50.
As 1 vnto 4.50: ſo 15.25 vnto 68.62.

Or as 12 vnto the bredth in inches:
So the depth in inches vnto a fourth number.

As 12 vnto this fourth number:
So the length in feet to the content in feet.

And ſo alſo in the ſame ſolid AH, the content wil be found to be about 68 feet 62 parts.

As 12 vnto 21.6: ſo 30 vnto 54.
As 12 vnto 54: ſo 15.25 vnto 68.62.

All theſe varieties (and ſuch like not here mentioned) do follow vpon making of the baſe of the ſolid, to be *E C*; there would be as many more if any ſhall begin with the baſe E H, and ſo likewiſe if they make the baſe to be F D.

12 *Hauing the diameter of a cylinder giuen in inch meaſure, to find the length of a foot ſolid in inches.*

As the diameter in inches vnto 46.90:
So is 1 vnto a fourth number:
and that fourth to the length in inches.

So the diameter of a cylinder being 15 inches, the fourth number will be about 3.12, and the length of a foote ſolid 9 inches 78 parts.

As 15 vnto 46.90: ſo 1 vnto 3.127:
and ſo are 3.127 vnto 9.778.

13 *Hauing the diameter of a cylinder giuen in foot meaſure, to find the length of a foot ſolid in foot meaſure.*

As the diameter in feet vnto 1.128:
So is 1 vnto a fourth number;
and that fourth to the length in foot meaſure.

So the diameter being 1 foot 25 parts, the length of a foot ſolid wil be found about 8.14 parts of 1000.

As 1.25 vnto 1.128: ſo 1.00 to 0.9027:
and ſo are 9027 vnto 8148.

14 *Hauing the circumference of a cylinder giuen in inches, to find the length of a foot ſolid in inch meaſure.*

As the circumference in inches to 147.36:
So is 1 to a fourth number;
and that fourth to the length in inches.

So the circumference being 47 inches 13 parts, the length of a foot ſolid will be found about 9 inches 78 parts.

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As 47.13 vnto 147.36: ſo 1.00 to 3.13:
and ſo are 3.13 vnto 9.78.

15 *Hauing the circumference of a cylinder giuen in foot meaſure, to find the length of a foot ſolid in foot meaſure.*

As the circumference in feete to 3.545:
So is 1 to a fourth number;
and that fourth to the length in foot meaſure.

So the circumference being 3 foot 927 parts, the length of a foot ſolid will be found to be about 815 parts.

As 3.927 vnto 3.545: ſo 1.000 vnto 0.903:
and ſo are 903 vnto 815.

16 *Hauing the ſide of a ſquare equall to the baſe of a cylinder, to find the length of a foot ſolid.*

The ſide of a ſquare equall to the circle, may be found by the eighth *Prop.* of broad meaſure, and then this *Prop.* may be wrought by the firſt and ſecond *Prop.* of ſolid meaſure.

17 *Hauing the diameter of a cylinder, and the length giuen in inches, to find the content in inches.*

As 1.128 vnto the diameter in inches:
So the length in inches to a fourth number;
and that fourth number to the content in inches.

So the diameter being 15 inches, and the length 105, the content of the cylinder will be found to be about 18560 inches.

As 1.1284 vnto 15: ſo are 105 vnto 1395.87:
and ſo are 1395.87 vnto 18555.34.

18 *Hauing the diameter and length of a cylinder in foot meaſure, to find the content in feet.*

As 1.128 to the diameter in feet:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the diameter being 1 foote 25 parts, and the length 8 foot and 75 parts, the content of the cylinder will be found about 10 foot 74 parts.

As 1.128 vnto 1.25: ſo 8.75 vnto *9.69*:
and ſo are *9.69* vnto 10.737.

19 *Hauing the diameter of a cylinder, and the length giuen in inches, to find the content in feet.*

As 46.90 to the diameter in inches:
So the length in inches to a fourth number;
and that fourth to the content in feet.

So the diameter being 15 inches, and the length 105, the content will be found about 10 foot 74 parts.

As 46.906 vnto 15: ſo 105 vnto 33.58:
and ſo are 33.58 vnto 10.737.

20 *Hauing the diameter of a cylinder giuen in inches and the length in feet, to find the content in feet.*

As 13.54 to the diameter in inches:
So the length in feet to a fourth number;
and that fourth to the content in feete.

So the diameter being 15 inches, and the length 8 foote 75 parts, the content will be found about 10 foot 74 parts.

As 13.54 vnto 15: ſo 8.75 vnto *9.69*:
and ſo are *9.69* vnto 10.74.

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21 *Hauing the circumference and the length of a cylinder giuen in inches, to find the content in inches.*

As 3.545 to the circumference in inches:
So the length in inches to a fourth number;
and that fourth to the content in inches.

So the circumference being 47 inches 13 parts, and the length 105 inches, the content will be found about 18560 inches.

As 3.545 vnto 47.13: ſo 105 vnto 1396:
and ſo are 1396 vnto 18555.

22 *Hauing the circumference and length of a cylinder giuen in inches, to find the content in feet.*

As 147.36 to the circumference in inches:
So the length in inches to a fourth number;
and that fourth to the content in feet.

So the circumference being 47 inches 13 parts, and the length 105 inches, the content will be found about 10 foote 74 parts.

As 147.36 vnto 47.13: ſo 105 vnto 33.58:
and ſo are 33.58 vnto 10.74.

23 *Hauing the circumference and length of a cylinder giuen in foot meaſure, to find the content in feet.*

As 3.545 to the circumference in feet:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the circumference being 3 foote 927 parts, and the length 8 foot 75 parts, the content wil be found to be 10 foot 74 parts.

As 3.545 vnto 3.927: ſo 8.75 vnto 9.69:
and ſo are 9.69 vnto 10.74.

24 *Ha-*

24 *Hauing the circumference of a cylinder giuen in inches and the length in foot meaſure, to find the content in feete.*

As 42.54 to the circumference in inches;
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the circumference being 47 inches 13 parts, and the length 8 foot 75 parts, the content will be found as before, 10 foot 74 parts.

As 42.54 vnto 47.13: ſo 8.75 vnto 9.69:
and ſo are 9.69 vnto 10.74.

CHAP. IIII.

The vſe of the line of Numbers in gaugeing of veſſels.

THe veſſels which are here meaſured, are ſuppoſed to be cylinders, or reduced vnto cylinders, by taking the mean betweene the diameter at the head and the diameter at the bongue, after the vſuall maner.

1 *Hauing the diameter and the length of a veſſell with the content thereof, to find the gauge point.*

Extend the compaſſes in the line of *numbers* to halfe the diſtance betweene the content and the length of the veſſell, the ſame extent will reach from the diameter to the gauge point.

I put this propoſition firſt, becauſe theſe kind of meaſures are not alike in all places. Here at London it is ſaid that a wine veſſell being 66 inches in length, and 38 inches the diameter, would containe 324 gallons: which if it be true, we

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may diuide the ſpace betweene 324 and 66 into two equall parts, and the middle will fall about 146, and the ſame extent which reacheth from 324 to 146, wil reach from the diameter 38 vnto 17.15 the gauge point for a gallon of wine or oyle after London meaſure. The like reaſon holdeth for the like meaſures in all other places.

2 *Hauing the meane diameter and the length of a veſſell, to find the content.*

Extend the compaſſes from the gauge point to the meane diameter, the ſame extent being doubled, ſhall giue the diſtance from the length to the content.

So the meane diameter of a wine veſſell being 20 inches, and the length 25 inches, the content will be found to be 34 gallons after Londō meaſure. For extend the compaſſes from 17.25 vnto 20, the ſame extent wil reach from 25 vnto 29.15, and from 29.15 vnto 34.

In like maner if the meane diameter were 16 inches, and the length 23, the content would be found to be about 20 gallons. For the ſame extent which reacheth back from 17.15 vnto 16, will reach from 23 to 21.45, and from 21.45 vnto 20.

So that if the meane diameter ſhall be 17 inches and 15 centeſmes or parts of 100, the number of inches in the length of the veſſell, will giue the number of gallons contained in the ſame veſſell: if the diameter ſhall be more or leſſe then 17.25, the content in gallons will be accordingly more or leſſe then the length in inches.

3 *Hauing the diameter and content, to find the length.*

Extend the compaſſes from the diameter to the gauge point, the ſame extent being doubled ſhall giue the diſtance from the content to the length of the veſſell.

So the gauge point ſtanding as before, if the diameter ſhal be 38 inches, and the content 324 gallons wine meaſure, the length of the veſſels will be found about 66 inches.

4 *Hauing the length of a veſſell and the content, ta find the diameter.*

Extend the compaſſes to halfe the diſtance betweene the length and the content, the ſame extent ſhall reach from the gauge point to the diameter.

So the length being 66 inches, and the content 324 gallons wine meaſure, the gauge point ſtanding as before, the diameter of the veſſell will be found to be about 38 inches.

CHAP. V.

Containing ſuch Aſtronomicall propoſitions as are of ordinary vſe in the practiſe of Nauigation.

1 *To find the altitude of the Sunne by the ſhadow of a gnomon ſet perpendicular to the horizon.*

As the parts of the ſhadow
  are to the parts of the gnomon:
So the tangent of 45 *gr.*
  to the tangent of the altitude.

Extend the compaſſes in the line of *numbers*, from the parts of the ſhadow to the parts of the *gnomon*; the ſame extent wil giue the diſtance from the tangent of 45 *gr.* to the tangent of the Sunnes altitude.

So the *gnomon* being 36, and the ſhadow 27, the altitude will be found to be 36 *gr.* 52 *m.* Or the *gnomon* being 27, and the ſhadow 36, the altitude will be found to be 53 *gr.* 8 *m.* Or the ſhadow being 20, and the gnomon 9, the altitude will be found to be 25 *gr*, 14 *m.* as in the eighth *Prop.* of the vſe of the *tangent* line.

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2 *Hauing the diſtance of the Sunne, from the next equinoctiall point, to find his declination.*

As the Radius is in proportion
to the ſine of the Sunnes greateſt declination:
So the ſine of the Suns diſtance from the next equinoctiall point,
to the ſine of the declination required.

Extend the compaſſes in the line of *ſines*, from 90 *gr.* to 23 *gr.* 30 *m.* the ſame extent will giue the diſtance from the Sunnes place vnto his declination.

So the Sunne being either in 29 *gr.* of ♉, or 1 *gr.* of ♒, or 1 *gr.* of ♌, or 29 *gr.* of ♏, that is 59 *gr.* diſtant from the next equinoctiall point, the declination will be found about 20 *gr.*

If the Sunne be ſo neare the equinoctiall point that his declination fall to be vnder 1 *gr.* it may be found by the line of *numbers*. As if the Sunne were in 2 *gr.* 5 *m.* of ♈, that is, 125 *m.* from the equinoctiall point, the former extent of the compaſſes from the ſine of 90 *gr.* to the ſine of 23 *gr.* 30 *m.* will reach in the line of *numbers* from 125 vnto 50, which ſhewes the declination to be about 50 *m.*

3 *Hauing the latitude of the place, and the declination of the Sun, to find the time of the Suns riſing and ſetting.*

As the cotangent of the latitude
to the tangent of the *S*uns declination:
So is the Radius
to the line of the aſcenſionall difference betweene the houre of 6 and the time of the *S*uns riſing or ſetting.

Extend the compaſſes from the tangent of the complement of the latitude, to the tangent of the declination: the ſame extent wil reach from the ſine of 90 *gr.* to the ſine of the aſcenſionall difference.

Or extend the compaſſes from the cotangent of the latitude to the ſine of 90 *gr*, the ſame extent will reach from the tangent of the declination to the ſine of the aſcenſionall difference.

So the latitude being 51 *gr*. 30 *m*. Northward, and the declination 20 *gr*. the difference of aſcenſion wil be found to be 27 *gr*. 14 *m*. which reſolued into houres and minutes, doth giue 1 houre and almoſt 49 *m*. for the difference betweene the *S*unnes riſing or ſetting, and the houre of 6, according to the time of the yeare.

4 *Hauing the latitude of the place, and the diſtance of the Sun from the next equinoctiall point, to find his amplitude.*

As the coſine of the latitude
to the ſine of the Suns greateſt declination:
So the ſine of the place of the Sun,
to the ſine of the amplitude.

So the latitude being 51 *gr*. 30 *m*. and the place of the Sun in 1 *gr*. of ♒, that is 59 *gr*. diſtant from the next equinoctiall point, the amplitude will be found about 33 *gr*. 20 *m*. For extend the compaſſes in the line of *ſines*, from 38 *gr*. 30 *m*. the ſine of the complement of the latitude, vnto 23 *gr*. 30 *m*. the ſine of the Suns greateſt declination; the ſame extent will reach from 59 *gr*. vnto 33 *gr*. 20 *m*. Or extend them from 38 *gr*. 30 *m*. vnto 59 *gr*. the ſame extent will reach from 23 *gr*. 30 *m*. vnto 33 *gr*. 20 *m*. as before.

5 *Hauing the latitude of the place, and the declination of the Sun, to find his amplitude.*

As the coſine of the latitude
is to the Radius:
So the ſine of the declination,
to the ſine of the amplitude.

Extend the compaſſes from the coſine of the latitude to

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the ſine of **90 *gr.*** the ſame extent will reach from the ſine of the Suns declination to the ſine of the amplitude.

Or extend them from the coſine of the latitude to the ſine of the declination, the ſame extent will reach from the ſine of *90 gr.* to the ſine of the amplitude.

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the amplitude will be found to be 33 *gr.* 20 *m.*

6 *Hauing the latitude of the place, and the declination of the Sun, to find the time when the Sun cometh to be due Eaſt or Weſt.*

As the tangent of the latitude,
    is to the tangent of the declination:
So the Radius
    to the coſine of the houre from the meridian.

Extend the compaſſes from the tangent of the latitude to the tangent of the declination; the ſame extent will reach from the ſine of *90 gr.* to the ſine of the complement of the houre.

Or extend them from the tangent of the latitude to the ſine of *90 gr*; the ſame extent will reach from the tangent of the declination to the ſine of the complement of the houre.

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the Sunne will be 73 *gr.* 10 *m*: that is 4 houres and 53 *m.* frō the meridian, when he cometh to be in the Eaſt or Weſt.

7 *Hauing the latitude of the place, and the declination of the Sun, to find what altitude the Sun ſhall haue, when he cometh to be due Eaſt or weſt.*

As the ſine of the latitude
    is to the ſine of the declination:
So the Radius
    to the ſine of the altitude.

Extend the compaſſes in the line of *lines* from the latitude to the ſine of the declination , the ſame extent will reach from the ſine of 90 *gr.* to the ſine of the altitude.

Or extend them from the ſine of the latitude to the ſine of 90 *gr*; the ſame extent will reach from the ſine of the declination to the ſine of the altitude.

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the altitude will be found about 25 *gr.* 55 *m.*

8 *Hauing the latitude of the place, and the declination of the Sun, to find what altitude the Sun ſhall haue at the houre of ſix.*

As the Radius is in proportion
  to the ſine of the ſine of the declination:
So the ſine of the latitude
  to the ſine of the altitude.

Extend the compaſſes in the line of *ſines*, from 90 *gr.* to the declination; the ſame extent will reach from the latitude to the altitude.

Or extend them from 90 *gr.* to the latitude , the ſame extent will hold from the declination to the altitude.

So the latitude being 51 *gr.* 30 *m.* and the declination of the Sunne 20 *gr.* the altitude of the Sun will be found to be about 15 *gr.* 30 *m.*

9 *Hauing the latitude of the place, and the declination of the Sun, to find what azimuth the Sun ſhall haue at the houre of ſix.*

As the coſine of the latitude
  is to the Radius:
So the cotangent of the Suns declination,
  to the tangent of the azimuth from the North part of the meridian,

So

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So the latitude being 51 *gr.*30 *m.* and the declination 20 *gr.* the azimuth will be found to be 77 *gr.*14 *m.* For extend the compaſſes in the line of *ſines*, from 38 *gr.*30 *m.* to 90 *gr.* the ſame extent will reach from the tangent of 70 *gr.* to the tangent of 77 *gr.*14 *m.*

10 *Hauing the latitude of the place, and the declination of the Sun, and the altitude of the Sun, to find the azimuth.*

First conſider the declination of the Sunne, whether it be toward the North or the South, ſo haue you his diſtance from your pole: then adde this diſtance, the complement of his altitude, and the complement of your latitude, all three together, and from halfe the ſumme ſubtract the diſtance from the pole, and note the difference.

1 As the Radius is in proportion
to the coſine of the altitude:
So the coſine of the latitude,
to a fourth ſine.

2 As this fourth ſine
is to the ſine of the halfe ſumme:
So the ſine of the difference,
to a ſeuenth ſine.

Then find a meane proportionall betweene this ſeuenth ſine and the Radius, this meane ſhall be the ſine of the complement of halfe the azimuth from the North part of the meridian.]

Suppoſe the declination of the Sun being knowne by the time of the yeare to be 20 *gr.* Southward, the altitude aboue the horizon found by obſeruation 12 *gr.* and the latitude Northwards 51 *gr.*30 *m.* it were required to find the azimuth. The declination is Southward, and therefore the diſtance from the pole 110 *gr*; then turning the altitude and latitude vnto their complements, I adde them all three together, and from halfe the ſumme ſubtract the diſtance from the pole, noting the difference after this maner.

Declin.South	20 *gr.*	0 *m.*	The diſtance	110 *gr.*	0 *m.*
Altitude	12	0	The complement	78	0
Latitude N.	51	30	The complement	38	30
			The ſumme of all three	226	30
			The halfe ſumme	113	15
			The difference	3	15

This done, I come to the Staffe, and extend the compaſſes from the ſine of 90 *gr.* to the ſine of 78 *gr.* and find the ſame extent to reach from the ſine of 38 *gr.* 30 *m.* vnto 37 *gr.* 30 *m.* Or if I extend them from 90 *gr.* to 38 *gr.* 30 *m.* the ſame extent doth reach from 78 *gr.* vnto 37 *gr.* 30 *m.* which is the fourth ſine required.

Then I extend the compaſſes againe, from this fourth ſine of 37 *gr.* 30 *m.* vnto the ſine of the halfe ſumme 113 *gr.* 15 *m.* that is to the ſine of 66 *gr.* 45 *m.* (for after 90 *gr.* the ſine of 80 *gr.* doth ſtand for a ſine of 100 *gr.* and the ſine of 70 *gr.* for a ſine of 110 *gr.* and ſo the reſt for thoſe which are their complements to 180 *gr.*) and this ſecond extent doth reach from the ſine of the difference 3 *gr.* 15 *m.* to the ſine of 4 *gr.* 54 *m.* Or if I extend them from the fourth ſine of 37 *gr.* 30 *m.* to the ſine of the difference 3 *gr.* 15 *m.* the ſame extent will reach from the ſine of the halfe ſumme 113 *gr.* 15 *m.* vnto 4 *gr.* 54 *m.* which is the ſeuenth ſine required.

Laſtly, I diuide the ſpace betweene this ſeuenth ſine of 4 *gr.* 54 *m.* and the ſine of 90 *gr.* into two equall parts, and I find the meane proportionall ſine to fall on 17 *gr.* whoſe complement is 73 *gr*; the double of 73 *gr.* is 146 *gr.* and ſuch is the azimuth required.

Or hauing found the ſeuenth ſine to be 4 *gr.* 54 *m* I might looke ouer againſt it, in the line of *verſed ſines,* and there I ſhould find 146 *gr.* for the azimuth from the North part of the meridian; and the complement of 146 *gr.* to a ſemicircle being 34 *gr.* will giue the azimuth from the South part of the meridian.

But if it were required to find the azimuth in the ſame latitude of 51 *gr.* 30 *m.* Northward, with the ſame altitude of

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12 *gr.* and like declination of 20 *gr.* to the Northward, it would be found to be onely 72 *gr.* 52 *m.* though the maner of worke be the ſame as before.

Declin. North	20 *gr.*	0 *m*	The diſtance is	70 *gr.*	0 *m.*
Altitude	12	0	The complement	78	0
Latitud. North	51	30	The complement	38	30
			The ſumme of all three	186	30
			The halfe ſumme	93	15
			The difference	23	15

Here as the Radius is to the ſine of 78 *gr*: ſo the ſine of 38 *gr.* 30 *m.* to the ſine of 37 *gr.* 30 *m.* which is the fourth ſine, and the ſame as before.

Then as this fourth ſine of 37 *gr.* 30 *m.* is to the ſine of 93 *gr.* 15 *m*: ſo the ſine of 23 *gr.* 15 *m.* to the ſine of 40 *gr.* 20 *m.* which is the ſeuenth ſine.

The halfe way betweene this ſeuenth ſine and the ſine of 90 *gr.* doth fall at 53 *gr.* 34 *m.* whoſe complement is 36 *gr.* 26 *m*; and the double of that is 72 *gr.* 52 *m.* the azimuth required.

Or I may find this ſame azimuth in the line of *verſed ſines*, ouer againſt the ſeuenth ſine of 40 *gr.* 20 *m.*

11 *Hauing the latitude of the place, the declination of the Sun, and the altitude of the Sun, to find the houre of the day.*

Adde the complement of the Suns altitude, and the diſtance of the Sun from the pole, and the complement of your latitude, all three together, and from halfe the ſumme ſubtract the complement of the altitude, and note the difference.

1 As the Radius is in proportion
to the ſine of the Suns diſtance from the pole:
So the ſine of the complement of the latitude,
to a fourth ſine.

2. As this fourth ſine
is to the ſine of the halfe ſumme:
So the ſine of the difference
to a ſeuenth ſine.

The meane proportionall betweene this ſeuenth ſine and the ſine of *90 gr.* will be the ſine of the complement of halfe the houre from the meridian.

Thus in our latitude of 51 *gr.* 30 *m.* the declination of the Sun being 20 *gr.* Northward, and the altitude 12 *gr.* I might find the Sun to be 95 *gr.* 52 *m.* from the meridian.

Altitude	12 *gr.*	0 *m.*	The complement is	78 *gr.*	0 *m.*
Declin. North	20	0	the diſt. from the pole	70	0
Latitude	51	30	the complement is	38	30
			The ſumme of all three	186	30
			The halfe ſumme	93	15
			The difference	15	15

Here as the Radius is to the ſine of 70 *gr.*
So the ſine of 38 *gr.* 30 *m.* to the ſine of 35 *gr.* 48 *m.*

As this ſine of 35 *gr.* 48 *m.* is to the ſine of 93 *gr.* 15 *m.*
So the ſine of 15 *gr.* 15 *m.* to the ſine of 26 *gr.* 40 *m.*

The halfe may between this ſeuenth ſine of 26 *gr.* 40 *m.* and the ſine of 90 *gr.* doth fall at 42 *gr.* 4 *m.* whoſe complement is 47 *gr.* 56 *m.* and the double of that, 95 *gr.* 52 *m.* which conuerted into houres, doth giue 6 houres and almoſt 24 *m.* from the meridian.

Or I might find theſe 95 *gr* 52 *m.* in the line of *verſed ſines*, ouer againſt the ſeuenth ſine of 26 *gr.* 40 *m.*

12 *Hauing the azimuth, the Suns altitude, and the declination, to find the houre of the day.*

As the coſine of the declination
is to the ſine of the azimuth:
So the coſine of the altitude
to the ſine of the houre.

Thus the declination being 20 *gr.* Southward, the altitude 12 *gr.* and the azimuth found by the tenth *Prop.* 146 *gr.* I might find the time to be 35 *gr.* 36 *m.* that is 2 houres 21 *m.* from the meridian.

13 *Hauing the houre of the day, the Sunnes altitude, and the declination, to find the azimuth.*

As the coſine of the altitude
is to the ſine of the houre:
So the coſine of the declination,
to the ſine of the azimuth.

So the altitude of the Sun being 12 *gr.* and the declination 20 *gr.* Southward, and the angle of the houre 35 *gr.* 36 *m.* I ſhould find the azimuth to be 34 *gr.* And ſo it is if it be reckoned from the South; but 146 *gr.* if it be taken from the North part of the meridian.

14 *Hauing the diſtance of the Sun from the next equinoctiall point, to find his right aſcenſion.*

As the Radius
to the coſine of the greateſt declination:
So the tangent of the diſtance,
to the tangent of the right aſcenſion.

So the Sun being in the firſt degree of ♒, that is 59 *gr.* diſtant from the next equinoctiall point, and the greateſt declination 23 *gr.* 30 *m.* the right aſcenſion will be found to be 56 *gr.* 46 *m.* ſhort of the beginning of ♈, and therefore 303 *gr.* 14 *m.*

15 *Hauing the declination of the Sun, to find his right aſcenſion.*

As the tangent of the greateſt declination
is to the tangent of the declination giuen:

So the Radius
to the ſine of the right aſcenſion.

So the greateſt declination being 23 *gr.* 30 *m.* and the declination of the Sun giuen 20 *gr.* the right aſcenſion will be found about 56 *gr.* 50 *m.*

Theſe are ſuch Aſtronomicall propoſitions as I take to be vſefull for Sea-men. For the firſt and ſecond will help them to find their latitude; the third to find the Suns riſing and ſetting; the 4.5.6.7.8.9.10.13. *Prop.* to finde the variation of their compaſſe; the 11 and 12 *Prop.* to find the houre of the day; and the two laſt toward the finding of the houre of the night. For hauing the latitude of the place, with the declination and altitude of any ſtarre, they may find the houre of the ſtarre from the meridian, as in the 11 *Prop.* Then comparing the right aſcenſion of the ſtarre with the right aſcenſion of the Sunne, they may haue the houre of the night.

All theſe propoſitions and ſuch others may be wrought alſo by the tables of *ſines* and *tangents*. For where foure numbers do hold in proportion; as the firſt to the ſecond, ſo the third to the fourth; there if we multiply the ſecond into the third, and diuide the product by the firſt, the quotient will giue the fourth required. As in the example of the laſt *Prop.* where the delination being giuen, it was required to find the right aſcenſion. The tangent of 20 *gr.* the declination giuen is 3639702, which being multiplied by the Radius, the product is 36397020000000, and this diuided by 4348124 the tangent of 23 *gr.* 30 *m.* the quotient is 8370741 the ſine of 56 *gr.* 50 *m.* for the right aſcenſion required.

Or if any will vſe my tables of *artificiall ſines* and *tangents*, they may adde the ſecond and the third together, and from the ſumme ſubtract the firſt, the remainder will giue the fourth required. And ſo my tangent of 20 *gr.* is 9561.0658, which being added to the Radius, makes 19561.0658; from this if they ſubtract 9638.3019 the tangent of 23 *gr.* 30 *m.* they ſhall find the remainder to be 9922.7639, which is my

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Canon is the ſine of 56 *gr.* 49 *m.* 56 *ſecõds*; and ſuch is the right aſcenſion required, if it be reckoned from the next equinoctiall point.

The like reaſon holdeth for all other Aſtronomicall propoſitions, as I will farther ſhew by thoſe two examples which I gaue before for the finding of the azimuth in the 10 *Prop.* becauſe they are thought to be harder then the reſt, and require three operations.

In the firſt example.

Declin. South	20 *gr.*	0 *m.*	The diſtance	110 *gr.*	0 *m.*
Altitude	12	0	the complement	78	0
Latitude Nor.	51	30	the complement	38	30
			The ſumme of all three	226	30
			The halfe ſumme	113	15
			The difference	3	15

The firſt operation will be to finde the fourth ſine; and that is done by adding the ſine of the complement of the altitude to the ſine of the complement of the latitude, and ſubtracting the Radius: ſo adding 9990.4044 the ſine of 78 *gr.* vnto 9794.1495 the ſine of 38 *gr.* 30 *m.* the ſumme will be 19784.5539. And the Radius being ſubtracted, the remainder 9784.5539 is the fourth ſine, and belongeth to 37 *gr.* 30 *m.*

The ſecond operation will be to find the ſeuenth ſine; and that is done by adding the ſine of the halfe ſumme to the ſine of the difference, and ſubtracting the fourth ſine. So the halfe ſumme being 113 *gr.* 15 *m.* I take his complement to a ſemicircle, and ſo find his ſine to be 9963.2168, to which I adde 8753.5278, the ſine of the difference 3 *gr.* 15 *m*; and the ſumme is 18716.7446. From this I take the fourth ſine 9784 5539, and the remainder will be 8932.1907, which is the ſeuenth ſine, and belongeth to 4 *gr.* 54 *m.*

The third operation will be to finde the meane proportionall ſine betweene the ſeuenth ſine and the Radius. This in common arithmetique is done by multiplying the two extremes, and taking the ſquare roote of the product. As in finding a meane proportionall betweene 4 and 9, we multiply 4 into 9, and the product is 36, whoſe ſquare root is 6, the meane proportionall betweene 4 and 9. But here it is done by adding the ſine and the Radius, and taking the halfe of them. So the ſumme of the laſt ſeuenth ſine and the Radius is 18932.1907 and the halfe of that 9466.0953, which is the meane proportionall ſine required, and belongeth to 17 *gr.* whoſe complement is 73 *gr.* and the double of that 146 *gr.* the ſame azimuth as before.

In the ſecond example.

Declin.North	20 *gr.*	0 *m.*	The diſtance	70 *gr.*	0 *m.*
Altitude	12	0	the complement	78	0
Latitud.North	51	30	the complement	38	30
			The ſumme of all three	186	30
			The halfe ſumme	93	15
			The difference	23	15

The firſt operation will be to finde the fourth ſine; and that is here 9784.5539, as in the former example.

The ſecond operation wil be to find the ſeuenth ſine; and ſo here the ſine of the halfe ſumme 93 *gr.* 15 *m.* being the ſame with the ſine of 86 *gr.* 45 *m.* his complement to 180 *gr.* I find it to be 9999.3009, to which I adde 9596.3153 the ſine of the difference 23 *gr.* 15 *m.* and the ſumme is 19595.6162. From this I take the fourth ſine 9784.5539, and the remainder will be 9811.0623 for the ſeuenth ſine, and belongeth to 40 *gr.* 20 *m.*

The third operation will be to find the meane proportionall ſine betweene the ſeuenth ſine and the Radius. And ſo here the Radius being added to the ſeuenth ſine, the ſumme will be 19811.0623, and the halfe of that 9905.5311 doth giue the meane proportionall ſine belonging to about 53 *gr.* 34 *m.* whoſe complement is 36 *gr.* 26 *m.* & the double of that 72 *gr.* 52 *m.* the ſame azimuth as before.

I haue ſet downe theſe three examples thus particularly, that I might ſhew the agreement betweene the *Staffe* and the *Canon*. But otherwiſe I might deliuer both the precept and

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the worke, for the two laſt, more compendiouſly. For generally in all ſphericall triangles, where three ſides are knowne, and an angle required, make that ſide which is oppoſite to the angle required, to be the baſe; and gather the ſumme, the halfe ſumme, and the difference as before.

As the rectangle contained vnder the ſines of the ſides,
is to the ſquare of the whole ſine:
So the rectangle contained vnder the ſines of the halfe ſumme and the difference,
to the ſquare of the coſine of halfe the angle.

Then for the worke, we may for the moſt part leaue out the two laſt figures; and if they be aboue 50, put an vnitie to the ſixt place, after this maner.

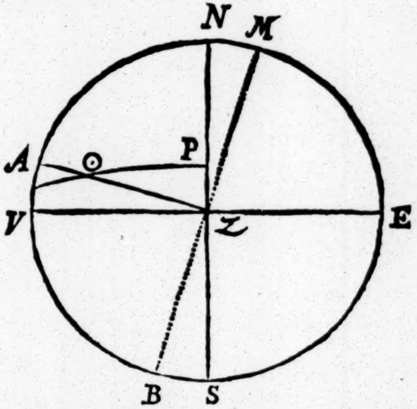
The ſecond example.

70 *gr.*	0 *m*				
78	0	9990	40		
38	30	9794	15		
186	30	19784	55		
93	15	9999	30		
23	15	9596	32		
		20000	00		
		39595	62		
		19811	07		
36	26	9905	53	53 *gr.*	34 *m.*
72	52			107	8

Or for ſuch numbers as are to be ſubtracted, I may take them out of the Radius, and write downe the reſidue, and then adde them together with the reſt. As in the ſame ſecond example, the ſines of 78 *gr.* and of 38 *gr.* 30 *m.* being the numbers to be ſubtracted; if I take 9990.4044 the ſine of 78 *gr.* out of the Radius 10000 0000, the reſidue is 9.5956: and ſo the reſidue of 9794.1495 is 205.8505. Wherefore in ſtead of ſubtracting thoſe ſines, I may adde theſe reſidues after this maner:

70 *gr.*	0 *m.*				
78	0	9	59		
38	30	205	85		
186	30				
93	15	9999	30		
23	15	9596	32		
		19811	06		
36	26	9905	53	53 *gr.*	34 *m.*
72	52			107	8

Hauing theſe meanes to find the Sunnes azimuth, we may compare it with the magneticall azimuth, and ſo finde the variation of the needle.



For let the circle *A M B*, drawne on the center *Z*, be a plane, parallell to the horizon; *A* the point whereon the Sun beareth from vs, *M* the North point of the magneticall needle, and the angle *A Z M* the magneticall azimuth. If we find the Sunnes azimuth as before, to be 72 *gr.* 52 *m.* from the North to the Weſtward, we may allow ſo many degrees from *A* vnto *N*, and ſo we haue the true North point from the meridian, and conſequently the Eaſt, South, and Weſt points of the horizon; and the diſtance betweene *N* and *M* ſhall be the variation of the needle. So that if the magneticall azimuth *AZM* ſhall be 84 *gr.7 m.* and the Suns azimuth *AZN* 72 *gr.52 m.* then muſt *NZM* the difference betweene the two meridians, giue the variation to be 11 *gr.15 m.* as Mr. *Bourough* heretofore found it by his obſeruations at *Limehouſe* in the yeare 1580. But if the magneticall azimuth *AZM* ſhall be 79 *gr.7 m.* and the Suns azimuth *AZN* 72 *gr.52 m.* then ſhall the variation *NZM* be only 6 *gr.15 m.* as I haue ſometimes found it of late. Herevpon I enquired after the place where Mr. *Bourough* obſerued, and went to *Limehouſe* with ſome of my friends, and tooke with vs a quadrant of 3 foote ſemidiameter, and two needles, the one aboue 6 inches, and the other 10 inches long, where I made the ſemidiameter of my horizontall plane *AZ* 12 inches: and toward night the 13 of Iune 1622, I made obſeruation in ſeuerall parts of the ground, and found as followeth.

Alt. ☉		*AZM*		*AZN*		*Variat*	
Gr.	*M.*	*Gr.*	*M.*	*Gr.*	*M.*	*Gr.*	*M.*
19	0	82	2	75	52	6	10
18	5	80	50	74	44	6	6
17	34	80	0	74	6	5	54
17	0	79	15	73	20	5	55
16	18	78	12	72	32	5	40
16	0	77	50	72	10	5	40
10	10	71	2	64	49	6	13
9	52	70	12	64	25	5	47

CHAP.

CHAP. VI.

Containing ſuch nauticall queſtions, as are of ordinary vſe, concerning longitude, latitude, Rumb, and diſtance.

1 *To keep an account of the ſhips way.*

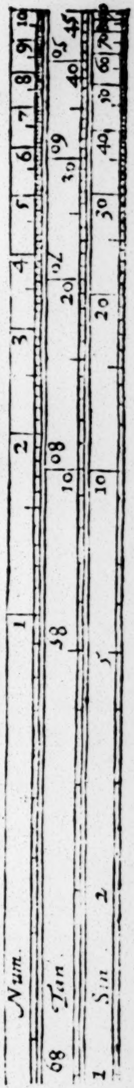
THe way that the ſhip maketh, may be knowne to an old ſea-man by experience, by others it may be found for ſome ſmall portion of time, either by the log line, or by the diſtance of two knowne markes on the ſhips ſide. The time in which it maketh this way, may be meaſured by a watch, or by a glaſſe. Then as long as the wind continueth at the ſame ſtay, it followeth by proportion,

As the time giuen is to an houre:
So the way made, to an houres way.

Suppoſe the time to be 15 ſeconds, which make a quarter of a minute, and the way of the ſhip 88 feet: then becauſe there are 3600 ſeconds in an houre, I may extend the compaſſes in the line of ***numbers***, from 15 vnto 3600, and the ſame extent will reach from 88 vnto 21120.

Or I may extend them from 15 vnto 88, and this extent will reach from 3600 vnto 21120; which ſhewes that an houres way came to 21120 feete.

But this were an vnneceſſary buſineſſe, to hearken after feet or fadoms. It ſufficeth our ſea-men to find the way of their ſhip in leagues or miles. And they ſay that there are 5 feet in a pace, 1000 paces in a mile, and 60 miles in a degree, and therefore 300000 feete in a degree. Yet compa-



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ring ſeuerall obſeruations, and their meaſures with our feete vſuall about *London*, I find that we may allow 352000 feete to a degree; and then if I extend the compaſſes in the line of *numbers* from 352000 vnto 21120, I ſhall find the ſame extent to reach from 20 leagues the meaſure of one degree, to 1.2, and from 60 miles to 3.6; which ſhewes the houres way to be 1 league and 2 tenths of a league, or 3 miles and 6 tenths of a mile.

But to auoid theſe fractions and other tedious reductions, I ſuppoſe it would be more eaſie to keep this account of the ſhips way (as alſo of the difference of latitude, and the difference of longitude) by degrees and parts of degrees, allowing 100 parts to each degree, which we may therefore call by the name of *centeſmes*. Neither would this be hard to conceiue. For if 100 ſuch parts do make a degree, then ſhall 50 parts be equall to 30 minutes, as 30 minutes are equall to 10 leagues. And 5 parts ſhall be equall to 3 minutes, as 3 minutes are equall to 1 league. And ſo the ſame extent as before, will reach from 100 parts vnto 6; which ſhewes that the houres way required is 6 *cent.* ſuch as 100 do make a degree, and 5 do make an ordinary league.

This might alſo be done at one operation. For vpon theſe ſuppoſitions, diuide 44 feet into 45 lengths, and ſet as many of them as you may conueniently betweene two markes on the ſhips ſide, and note the ſeconds of time in which the ſhip goeth theſe lengths: ſo the lengths diuided by the time, ſhall giue the *cent.* which the ſhip goeth in an houre.

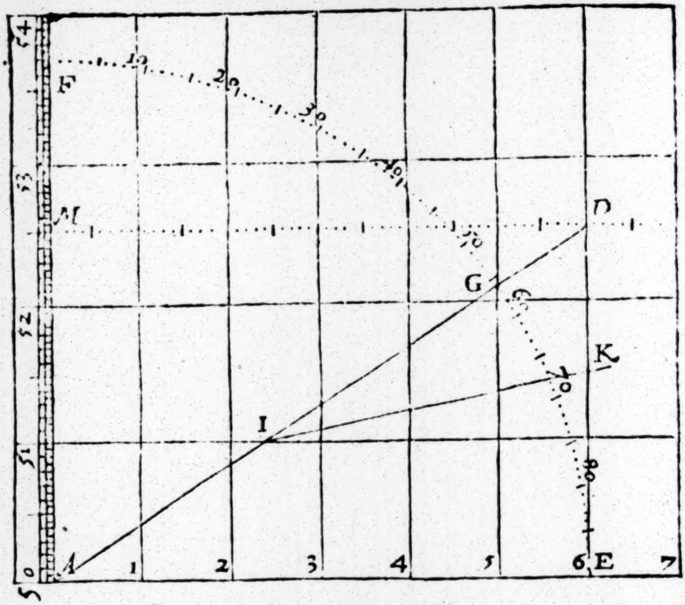
Suppoſe the diſtance betweene the two markes to be 60 lengths (which are 58 feet and 8 inches) and let the time be 12 ſeconds: extend the compaſſes from 12 to 1, in the line of *numbers*; ſo the ſame extent will reach from 60 vnto 5. Or extend them from 12 vnto 60, and the ſame extent will reach from 1 vnto 5. This ſhewes that the ſhips way is according to 5 *cent.* in an houre.

This may be found yet more eaſily, if the log line ſhall be fitted to the time. As if the time be 45 ſeconds, the log line may haue a knot at the end of euery 44 feete; then doth the ſhip run ſo many *cent.* in an houre, as there are knots vered out in the ſpace of 45 ſeconds. If 30 ſeconds do ſeeme to be a more conuenient time, the log line may haue a knot at the end of euery 29 feet and 4 inches; and then alſo the *centeſmes* will be as many as the knots. Or if the knots be made to any ſet number of feet, the time may be fitted vnto the diſtance. As if the knots be made at the end of euery 24 feet, the glaſſe may be made 24 ſeconds and ſomewhat more then an halfe of a ſecond; and ſo theſe knots will ſhew the *cent.* If there be 5 knots vered out in a glaſſe, then 5 *cent*; if 6 knots, then the ſhip goeth 6 *cent.* in the ſpace of an houre; and ſo in the reſt. For vpon this ſuppoſition, the proportion between the time and the feet will be as 45 vnto 44. But according to the common ſuppoſition it ſhould ſeeme to be as 45 vnto 37 $\frac{1}{2}$, or in leſſer termes as 6 vnto 5. Thoſe which are vpon the place, may make proofe of both, and follow that which agrees beſt with their experience.

2 *By the latitude and difference of longitude, to find the diſtance vpon a courſe of Eaſt and Weſt.*

Extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of the complement of the latitude; the ſame extent ſhal reach in the line of *numbers* from the difference of longitude to the diſtance.

So the meaſure of one degree in the equator, being 100 *cent.* the diſtance belonging to one degree of longitude in the latitude of 51 *gr.* 30 *m.* will be found about 62 *cent.* and $\frac{1}{4}$. Or if the meaſure of a degree be 60 miles, the diſtance will be found about 37 miles and $\frac{1}{3}$. If the meaſure be 20 leagues, then almoſt 12 leagues and $\frac{1}{2}$. If the meaſure be 17 $\frac{1}{2}$, as in the Spaniſh charts, then ſomewhat leſſe then 11 leagues ſailing vpon this parallell, will giue an alteration of one degree of longitude.



3 *By the latitude and diſtance vpon a courſe of Eaſt or weſt, to find the difference of longitude.*

Extend the compaſſes from the ſine of the complement of the latitude, to the ſine of 90 *gr*; the ſame extent wil reach in the line of *numbers* from the diſtance to the difference of longitude.

So the diſtance vpon a courſe of Eaſt or Weſt, in the latitude of 51 *gr*. 30 *m*. being 100 *cent*. the difference of longitude will be found 1.60, which make one degree and 60 *centeſmes* or 1 *gr*. 36 *m*.

Or if it be 60 miles, the difference of longitude wil be 96, which alſo make 1 *gr*. 36 *m*. as before.

4 *The longitude and latitude of two places being giuen, to find the Rumb leading from the one to the other.*

Extend the compaſſes in the line of ***numbers*** from the difference of latitudes to the difference of longitudes; the ſame extent will giue the diſtance from the tangent of 45 *gr.* vnto the tangent of the Rumb, according to the proiection of the common ſea-chart.

So the latitude of the firſt place being 50 *gr.* the latitude of the ſecond 52 *gr.* 30 *m.* and the difference of longitude 6 *gr.* the Rumb will be found to be about 67 *gr.* 23 *m.* which is neare the inclination of the ſixth Rumb to the meridian. But this Rumb ſo found, is alwayes greater then it ſhould be, and therefore to be limited; which may be done ſufficiently for the Sea-mans vſe, after this maner:

Extend the compaſſes either from the ſine of 90 *gr.* vnto the ſine of the complement of the middle latitude, the ſame extent will reach frō the tangent of the Rumb before found, to the tangent of the Rumb limited.

Or elſe extend them from the ſine of 90 *gr.* vnto the tangent of the Rumb before found; the ſame extent will reach from the ſine of the complement of the middle latitude, vnto the tangent of the Rumb limited.

So the middle latitude between 50 *gr.* and 52 *gr.* 30 *m.* being 51 *gr.* 15 *m.* and the Rumb before found 67 *gr.* 23 *m.* the Rumb limited will be found to be about 56 *gr.* 20 *m.* which is but fiue minutes more then the inclination of the fift Rumb to the meridian.

2 This Rumb may be found by the help of the ***meridian line*** vpon the Staffe. For if I take the difference of latitude out of the ***meridian line*** from 50 *gr.* vnto 52 *gr.* 30 *m.* and meaſure it in his equinoctiall, or at the beginning of the ***meridian line***, I ſhall find it there to be equal to 4 *gr.* Wherefore I work as if the difference of latitude were 4 *gr.* and extend the compaſſes in the line of ***numbers*** from 4 vnto 6: ſo ſhall I finde

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the ſame extent to reach from the tangent of 45 *gr.* vnto the tangent of 56 *gr.* 20 *m.* and this is the inclination of the Rumb required.

5 *By the Rumb and both latitudes, to find the diſtance vpon the Rumb.*

Extend the compaſſes from the ſine of the complement of the Rumb, vnto the ſine of 90 *gr.* the ſame extent in the line of *numbers* ſhall reach from the difference of latitude vnto the diſtance vpon the Rumb.

So the latitude of the firſt place being 50 *gr.* the latitude of the ſecond 52 *gr.* 30 *m.* and the Rumb the fift from the meridian. If I extend the compaſſes from 33 *gr.* 45 *m.* vnto the ſine of 90 *gr.* I ſhall find the ſame extent in the line of *numbers* to reach from 2 *gr.* 50 *cent.* to 4 *gr.* 50 *cent.* and ſuch is the diſtance required.

6 *By the diſtance and both latitudes to find the Rumb.*

Extend the compaſſes in the line of *numbers* from the diſtance vnto the difference of latitudes; the ſame extent will reach in the line of *ſines*, from 90 *gr.* vnto the complement of the Rumb.

So the one place being in the latitude of 50 *gr.* the other in the latitude of 52 *gr.* 30 *m.* and the diſtance between them 4 *gr.* 50 *cent.* If I extend the compaſſes from 4.50 vnto 2.50 in the line of *numbers*, I ſhall find the ſame extent to reach from the ſine of 90 *gr.* vnto the complement of 56 *gr.* 15 *m.* and ſuch is the inclination of the Rumb required.

7 *By one latitude, Rumb, and diſtance, to find the difference of latitudes.*

Extend the compaſſes in the line of *ſines*, from 90 *gr.* vnto the complement of the Rumb; the ſame extent in the line of *numbers*, will reach from the diſtance, vnto the difference of latitudes.

So the leſſer altitude being 50 *gr.* and the diſtance 4 *gr.* 50 *cent.* vpon the fifth Rumb from the meridian: if I extend the compaſſes from the ſine of 90 *gr.* to 33 *gr.* 45 *m.* I ſhall finde the ſame extent to reach from 4.50 in the line of *numbers*, vnto 2.50; and therefore the ſecond latitude to be 52 *gr.* 30 *m.*

8 *By the Rumb and both latitudes, to find the difference of longitude.*

Extend the compaſſes from the tangent of 45 *gr.* vnto the tangent of the Rumb; the ſame extent will reach in the line of *numbers* from the difference of latitudes vnto the difference of longitude, according to the proiection of the common ſea-chart.

So the firſt latitude being 50 *gr.* and the ſecond 52 *gr.* 30 *m.* and the Rumb the fifth from the meridian: if I extend the compaſſes from the tangent of 45 *gr.* vnto 56 *gr.* 15 *m.* I ſhall find the ſame extent to reach from 2.50 in the line of *numbers* to about 3.75, which make 3 *gr.* 45 *m.* But this difference of longitude ſo found, is alwayes leſſer then it ſhould be, and therefore to be enlarged, which may be done ſufficiently for the ſea-mens vſe, after this maner:

Extend the compaſſes from the ſine of the complement of the middle latitude, vnto the ſine of 90 *gr.* the ſame extent will reach in the line of *numbers* from the difference of longitude before found, vnto the difference of longitude inlarged.

So the middle latitude in this example being 51 *gr.* 15 *m.* and the difference of longitude before found 3 *gr.* 75 *cent.* the difference of longitude inlarged will be found about 5 *gr.* 99 *cent.* which are neare 6 *gr.*

2 This difference of longitude may be found by help of the *meridian line* vpon the Staffe. For if I take the proper difference of latitude out of the meridian line, and meaſure it in his equinoctiall, or at the beginning of the meridian line,

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I ſhall find it to be equall to foure of thoſe degrees. Wherefore hauing extended the compaſſes as before from the tangent of 45 *gr.* vnto the tangent of 56 *gr.* 15 *m.* the ſame extent will reach from 4.00 in the line of *numbers*, vnto 5.99: which ſhewes the difference of longitude to be about 5 *gr.* 99 *cent.* or about halfe a minute ſhort of ſix degrees.

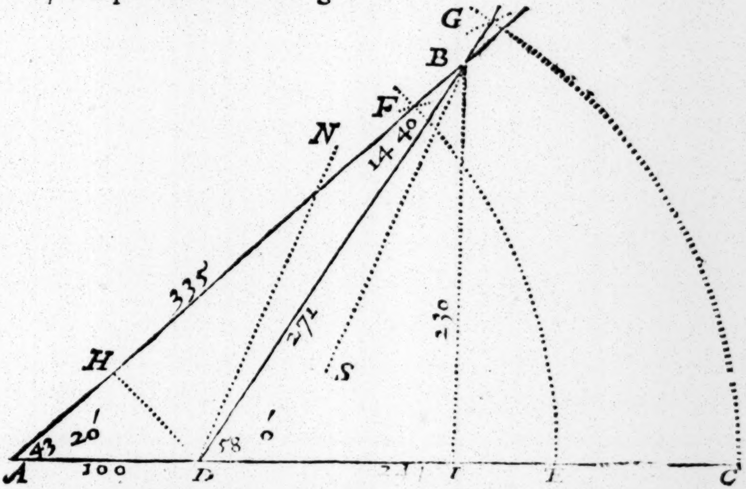
9 *By the Rumb and both latitudes, to finde the diſtance belonging to the chart of* Mercators *projection.*

Take the proper difference of latitudes out of the meridian line of the chart, and meaſure it in his equinoctiall, or one of the parallels, and it will there giue the difference of latitudes inlarged. Then extend the compaſſes from the ſine of the complement of the Rumb vnto the ſine of 90 *gr.* the ſame extent will reach in the line of *numbers*, from the latitude inlarged, vnto the diſtance required. Or extend them from the complement of the Rumb to the latitude inlarged, the ſame extent will reach from 90 *gr.* vnto the diſtance.

For example, let the place giuen be *A* in the latitude of 50 *gr.* *D* in the latitude of 52 *gr.* 30 *m.* *A M* the difference of latitudes, and the Rumb *M A D* the fifth from the meridian. Firſt I take out *A M* the difference of latitudes, and meaſure it in *A E* one of the parallels of the equinoctiall; I find it to be very neare 4 *gr.* this is the difference of latitudes inlarged. Then if I extend the compaſſes from the ſine of 33 *gr.* 45 *m.* the complement of the fifth Rumb vnto the ſine 90 *gr.* I ſhall find the ſame extent to reach in the line of *numbers* from 4.00 vnto 7.20. And this is the diſtance belonging to the chart. Wherefore I take out theſe 7 *gr.* 20 *cent.* out of the ſcale of the parallell *A E*, and pricke it downe vpon the Rumb from *A* vnto *D*, where it meeteth with the parallell of the ſecond latitude. Laſtly, I meaſure it in the *meridian line*, ſetting one foote of the compaſſes as much below the leſſer latitude as the other aboue the greater latitude, and find it to be 4 *gr.* 50 *cent.* which is the ſame diſtance that I found before in the 5. *Prop.*

10 By the way of the ſhip, and two angles of poſition, to finde the diſtance betweene the ſhip and the land.

The way of the ſhip may be knowne as in the firſt *Prop.* The angles may be obſerued either by the Staffe, or by a needle ſet on the Staffe. For example, ſuppoſe that being at *A*, I had ſight of the land at *B*, the ſhip going Eaſt Northeaſt from *A* toward *C*, and the angle of the ſhips poſition *BAC* being 43 *gr.* 20 *m:* and after that the ſhip had made 10 *cent.* or 2 leagues of way from *A* vnto *D*, I obſerued againe, and found the ſecond angle of the ſhips poſition *BDC* to be 58 *gr.* or the inward angle *BDA* to be 122 *gr.* then may I finde the third angle *A B D* to be 14 *gr.* 40 *m.* either by ſubtraction or by complement vnto 180 *gr.*



In this and the like caſes, I haue a right line triangle, in which there is one ſide and three angles knowne, and it is required to finde the other two ſides and the *Canon* for it, is this:

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As the ſine of the angle oppoſite to the knowne ſide,
is to that knowne ſide:
So the ſine of the angle oppoſite to the ſide required,
is to the ſide required.

Wherefore I extend the compaſſes from 14 *gr*. 40 *m*. in the *ſines*, to 10 in line of *numbers*, and this extent doth reach from 58 *gr*. to 33 $\frac{1}{3}$, and ſuch is the diſtance between *A* and *B*, and it reacheth from 43 *gr*. 20 *m*. vnto 27 in the line of *numbers*; and ſuch is the diſtance from *D* to *B*.

Theſe two diſtances being knowne, I may ſet out the land vpon the chart. For hauing ſet downe the way of the ſhip from *A* to *D* by that which I ſhewed before in the vſe of the *meridian line*, I may by the ſame reaſon ſet off the diſtance *A B* and *DB*, which meeting in the point *B*, ſhall there reſemble the land required.

11 *By knowing the diſtance between two places on the land, and how they beare one from the other, and hauing the angles of poſition at the ſhip to find the diſtance betweene the ſhip and the land.*

If it may be conueniently, let the angle of poſition be obſerued at ſuch time as the ſhip cometh to be right ouer againſt one of the places. As if the places be Eaſt and Weſt, ſeeke to bring one of them South or North from you, and then obſerue the angle of poſition: ſo ſhall you haue a right line triangle, with one ſide and three angles, whereby to find the two other ſides. Firſt you haue the angle of poſition at the ſhip; then a right angle at the place that is ouer againſt you; and the third angle at the other place is the complement to the angle of poſition. Wherefore

As the ſine of the angle of poſition,
is to the diſtance betweene the two places:
So the coſine of the angle of poſition,
to the diſtance betweene the ſhip and the nearer place

And ſo is the ſine of 90 *gr.*
to the diſtance from the ſhip to the farther place.

So the places being 15 *cent.* or three leagues one from the other, and the angle of poſition 29 *gr*; the nearer diſtance will be found about 27 *cent.* and the farther diſtance about 31 *cent.*

Or howſoeuer the angle of poſition were obſerued, the diſtance betweene the ſhip and the land may be found generally as in this example:

Suppoſe *A* and *D* were two head lands knowne to be Eaſt Northeaſt, and Weſt Southweſt, 10 *cent.* or two leagues one from the other; and that the ſhip being at *B*, I obſerued the angle of the ſhips poſition *DBA*, and found it to be 14 *gr.* 40 *m.* and that D did beare 9 *gr.* 30 *m.* and *A* 24 *gr.* 10 *m.* from the meridian *BS*, this example would be like the former. For if the angle *SBD* be 9 *gr.* 30 *m.* from the South to the Weſtward, then ſhall *NDB* be 9 *gr.* 30 *m.* from the North to the Eaſtward. Take theſe 9 *gr.* 30 *m.* out of the angle *NDE* which is 67 *gr.* 30 *m.* becauſe the two head lands lie Eaſt Northeaſt, and there will remaine 58 *gr.* for the angle *BDE*, and the inward angle BDA ſhall be 122 *gr.* Take theſe two angles A B D and B D A out of 180 *gr.* and there wil remaine 43 *gr* 20 *m.* for the third angle BAD. Wherefore here alſo are three angles and one ſide, by which I may find the two other ſides, as in the laſt *Prop.*

Theſe propoſitions thus wrought by the Staffe, are ſuch as I thought to be vſefull for ſea-men, and thoſe that are skilfull may apply the example to many others. Thoſe that begin, and are willing to practiſe, may buſie themſelues with this which followeth.

Suppoſe foure ports, *L*, *N*, *O*, *P*; of which *L* is in the latitude of 50 *gr.* *N* is North from *L* 200 leagues or 1000 *centeſmes*; *O* Weſt from *L* 1000 *cent.* and *P* Weſt from *N* 1000 *cent*: ſo that *L* and *O* will be in the ſame latitude of 50 *gr.* *N* and *P* both in the latitude of 60 *gr.* Then let two ſhips depart from *L*, the one to touch at *O*, the other at *N*,

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and then both to meet at *P*, there to lade, and from thence to returne the neareſt way vnto *L*. Here many queſtions may be propoſed.

1 What is the longitude of the port at *O*?

2 What is the longitude of *P*? And why *O* and *P* ſhould not be in the ſame longitude?

3 What is the Rumb from *O* vnto *P*?

4 What is the diſtance from *O* vnto *P*? And why the way ſhould be more from *L* vnto *P*, going by *O*, then by *N*?

5 What is the Rumb from *P* vnto *L*?

6 What is the diſtance from *P* vnto *L*?

7 What is the Rumb from *N* vnto *O*?

8 What is the diſtance from *N* vnto *O*? And why it ſhould not be the like Rumb and diſtance from *N* vnto *O*, as from *P* vnto *L*?

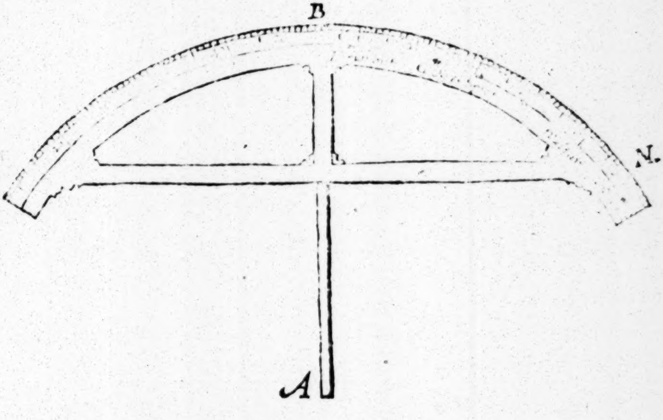
Theſe queſtions well conſidered, and either reſolued by the Staffe, or pricked downe on the chart, and compared with the globe and the common Sea-chart, will giue ſome light to the direction of a courſe, and reduction of places to their due longitude, which are now fouly diſtorted in the common Sea-charts.

An Appendix concerning

The deſcription and vſe of an inſtrument, made in forme of a Croſſe-bow, *for the more eaſie finding of the latitude at Sea.*

THe former *Prop.* ſuppoſe the latitude to be knowne, I will here ſhew how it may be eaſily obſerued.

Vpon the center *A*, and ſemidiameter *A B*, deſcribe an ark of a circle *S B N*. The ſame ſemidiameter will ſet of *60 gr.* from *B* vnto *S* for the South end, and other *60 gr.* from *B* vnto *N* for the North end of the Bow: ſo the whole Bow will containe 120 *gr.* the third part of a circle. Let it therefore be diuided into ſo many degrees, and each degree ſubdiuided into ſix parts, that each part may be ten minutes: but let the numbers ſet to it be 5. 10. 15. vnto *90 gr.* and then againe 5. 10 15. vnto 25. that 55 may fall in the middle, as in this figure.



The Bow being thus diuided and numbred, you may ſet the moneths and dayes of each moneth vpon the backe, and ſuch ſtarres as are fit for obſeruation vpon the ſide of the Bow.

If you deſire to make vſe of it in North latitude, you may number 23 *gr.*30 *m.* from 90 towards the end of the Bowe at N, and there place the tenth day of Iune. And 23 *gr.*30 *m.* from 90 toward *S*; and there at 66 *gr.*30 *m.* place the tenth day of December. And ſo the reſt of the dayes of the yeare, according to the declination of the Sunne at the ſame dayes. The ſtarres may be placed in like maner according to their declinations.

Arcturus at	21 *gr.*	10 *m.*
The Buls eye	15	42
The Lions heart	13	45
The Vultures heart	7	58
The little dog	6	9

from 90 toward the North end of the Bow at N. Then for Southerne ſtarres, you may number their declination from 90 toward the South end of the Bow at *S*. As firſt the three ſtarres in *Orions* girdle,

The firſt at	0 *gr.*	37 *m.*
The ſecond	1	28
The third	2	11
The Hydra's heart	7	5
The virgins ſpike	9	10
The great dog	16	12
The Scorpions heart	25	30
Fomahant	31	30

And ſo the South crowne, the triangle, the clouds, the croſiers, or what other ſtarres you think fit for obſeruation. This I call the fore ſide of the Bow.

If you deſire to make vſe of it in South latitude, you may turne the Bow, and diuide the backe ſide of it, and number it in like maner; and then put on the moneths and dayes of the yeare, placing the tenth of December at the South end, and the tenth of Iune toward the middle of the Bow, and the reſt of the dayes according to the Sunnes declination as before.

The chiefeſt of the Northerne ſtarres may here be placed in like maner according to their declination, Anno 1625.

The pole ſtarre at		87 *gr.*	20 *m.*
The firſt guard		75	45
The ſecond guard		73	25
The great Beares backe		63	45
In the great Beares taile	firſt	58	2
	ſecond	57	55
	third	51	15
The ſide of Perſeus		48	28
The goate		45	33
The taile of the ſwan		44	0
The head of Meduſa		39	30
The harp		38	30
Caſtor		32	38
Pollux		28	52
The North crowne		28	0
TheRams head		21	40
Arcturus		21	10
The Buls eye		15	42
The Lions heart		13	45
The Vultures heart		7	58
Orions right ſhoulder		7	17
Orions left ſhoulder		5	57

And ſo any other ſtarre, whoſe declination is knowne vnto you, which being done. The vſe of this Bow may be

1 *The day of the moneth being knowne, to finde the declination of the Sunne.*

2 *The declination being giuen, to finde the day of the moneth.*

Theſe two *Prop.* depend on the making of the Bow. If the day be knowne, looke it out in the backe of the Bow: ſo the declination will appeare in the ſide. Or if the declination be knowne, the day of the moneth is ſet ouer againſt it. As if the day of the moneth were the 14 of Iuly: looke for

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this day in the backe of the Bow,and you ſhall find it ouer againſt 20 *gr.* of North declination. If the declination giuen be 20 *gr.* to the Southward,you ſhal find the day to be either the eleuenth of Nouember,or the eleuenth of Ianuary.

3 *To find the altitude of the Sunne or ſtarres.*

Here it is fit to haue two running ſights, which may be eaſily moued on the backe of the Bow.The vpper ſight may be ſet either to 60 *gr.* or to 70 *gr.* or to 80 *gr.* as you ſhall find to be moſt conuenient: the other ſight may be ſet on,to any place betweene the middle and the other end of the Bow. Then with the one hand hold the center of the Bow to your eye,ſo as you may ſee the Sunne or ſtarre by the vpper ſight, and with the other hand moue the lower ſight vp or downe vntill you haue brought one of the edges of it to be euen with the horizon (as when you obſerue with the Croſſe-ſtaffe:) ſo the degrees contained betweene that edge and the vpper ſight,ſhall ſhew the altitude required.

Thus if the vpper ſight ſhal be at 80 *gr.* and the lower ſight at 50 *gr.* the altitude required is 30 *gr.*

4 *To find any North latitude, by knowing either the day of the moneth, or the declination of the Sunne.*

As oft as you are to obſerue in North latitude, place both the ſights on the fore ſide of the Bow,the vpper ſight to the declination of the Sunne, or the day of the moneth at the North end, and the lower ſight toward the South end.Then when the Sunne cometh to the meridian,turne your face to the South,and with the one hand hold the center of the Bow to your eye, ſo as you may ſee the Sunne by the vpper ſight; with the other hand moue the lower ſight, vntill you haue brought one of the edges of it to be euen with the horizon: ſo that edge of the lower ſight ſhall ſhew the latitude of the place in the fore ſide of the Bow.

Thus being in North latitude vpon the ninth of October if I ſet the vpper ſight to this day, at the fore ſide and North end of the Bow, I ſhall find it to fall to the Southward of 90 vpon 80 *gr.* and therefore at 10 *gr.* of South declination. Then the Sunne coming to the meridian, I may ſet the center of the Bow to mine eye, as if I went to find the altitude of the Sunne, holding the North end of the Bow vpward, with the vpper ſight betweene mine eye and the Sunne, and mouing the lower ſight, vntill it come to be euen with the horizon. If here the lower ſight ſhall ſtay at 50 *gr.* I may well ſay, that the latitude is 50 *gr.* For the meridian altitude of the Sunne is 30 *gr.* by the laſt *Prop.* and the Sun hauing 10 *gr.* of South declination, the meridian altitude of the equator would be 40 *gr*; and therefore the obſeruation was made in 50 *gr.* of North latitude.

By the ſame reaſon, if the lower ſide had ſtayed at 51 *gr.* 30 *m.* the latitude muſt haue been 51 *gr.* 30 *m.* and ſo in the reſt.

5 *To find any North latitude, by the meridian altitude of the ſtarres to the Southward.*

Let the vpper ſight be ſet to the ſtarre, which you intend to obſerue, here placed in the fore ſide of the Bow. Then hold the North end of the Bow vpward, and turning your face to the South, obſerue the meridian altitude as before: ſo the lower ſight ſhall ſhew the latitude of the place in the fore ſide of the Bow.

Thus if in obſeruing the meridian altitude of the great Dog-ſtarre, the lower ſight ſhall ſtay at 50 *gr.* it would ſhew the latitude to be 50 *gr.* For this ſtarre being here placed at 73 *gr.* 48 *m.* if we take thence 50 *gr.* his meridian altitude would be 23 *gr.* 48 *m.* to this if we adde 16 *gr.* 12 *m.* for the South declination of this ſtarre, it would ſhew the meridian altitude of the equator to be 40 *gr.* and therefore the latitude to be 50 *gr.*

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6 *To find any North latitude, by the meridian altitude of the ſtarres to the Northward.*

Let the vpper ſight be ſet to the ſtarre which you intend to obſerue, here placed on the backe ſide of the Bow. Then hold the North end of the Bow vpward, and turning your face to the North, obſerue the altitude of the ſtarre when he cometh to be in the meridian and vnder the pole: ſo the lower ſight ſhall ſhew the altitude of the pole in the back ſide of the Bow.

Thus the former guard coming to be in the meridian vnder the pole, if you obſerue and find the lower ſight to ſtay at 50 *gr.* ſuch is the eleuation of the pole, and the latitude of the place to the Northward. For the diſtance betweene the two ſights will ſhew the altitude to be 35 *gr.* 45 *m.* & the ſtar is 14 *gr.* 15 *m.* diſtant from the North pole. Theſe two doe make vp 50 *gr.* for the eleuation of the North pole, and therefore ſuch is the North latitude.

7 *To find any South latitude, by knowing either the day of the moneth, or the declination of the Sunne.*

When you are come into South latitude, turne both your ſights to the backſide of the Bow: the vpper ſight to the declination of the Sun, or the day of the moneth at the South end, and the lower ſight toward the North end of the Bow. Then the Sunne coming to the meridian, turne your face to the North, and holding the South end of the Bow vpward, obſerue the meridian altitude as before: ſo the lower ſight ſhall ſhew the latitude of the place in the backe ſide of the Bow.

Thus being in South latitude, vpon the tenth of May if you obſerue and finde the lower ſight to ſtay at 30 *gr.* on the backe ſide of the Bow, ſuch is the latitude. For the declination is 20 *gr.* Northward, the altitude of the Sunne betweene the two ſights 40 *gr.* the altitude of the equator 60 *gr.* and therefore the latitude 30 *gr.*

8 *To find any South latitude, by the meridian altitude of the ſtarres to the Northward.*

Let the vpper ſight be ſet to the ſtarre which you intend to obſerue, here placed on the backe ſide of the Bow. Then hold the South end of the Bow vpward , and turning your face to the North, obſerue the meridian altitude as before: ſo the lower ſight ſhall ſhew the latitude of the place in the back ſide of the Bow.

Thus being in South latitude, and the former guard comming to be in the meridian ouer the pole. If you obſerue and finde the lower ſight to ſtay at 5 *gr.* ſuch is the latitude. For this ſtarre is 14 *gr.* 15 *m.* from the North pole, the altitude of the ſtarre betweene the two ſights 9 *gr.* 15 *m.* the North pole depreſſed 5 *gr.* and therefore the latitude 5 *gr.* to the Southward.

9 *To obſerue the altitude of the Sunne backward.*

Set the vpper ſight either to 60, or 70, or 80 *gr.* as you ſhall find it to be moſt conuenient, the lower ſight on any place betweene the middle and the other end of the Bow, and haue an horizontall ſight to be ſet to the center. Then may you turne your backe to the Sunne, and the back of the Bow toward your ſelfe, looking by the lower ſight through the horizontall ſight, and mouing the lower ſight vp & downe, vntill the vpper ſight doe caſt a ſhadow vpon the middle of the horizontall ſight: ſo the degrees contained betweene the two ſights on the Bow, ſhall giue the altitude required.

Thus if the vpper ſight ſhall be at 80 *gr.* and the lower ſight at 50 *gr.* the altitude required is 30 *gr.* as in the third *Prop.*

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10 *To find any North latitude by a backe obſeruation, knowing either the day of the moneth, or the declination of the Sunne.*

When you obſerue in North latitude, place your three ſights on the fore ſide of the Bow: the vpper ſight to the declination of the Sun, or the day of the moneth, at the North end; the lower ſight toward the South end of the *B*ow; and the horizontall ſight to the center. Then the Sunne coming to the meridian, turne your face to the North, & holding the North end of the *B*ow vpward, the South end downeward, with the back of it toward your ſelfe, obſerue the ſhadow of the vpper ſight as in the former *Prop.* ſo the lower ſight ſhall ſhew the latitude of the place in the fore ſide of the *B*ow.

Thus being in North latitude vpon the ninth of October, if you obſerue and find the lower ſight to ſtay at 50 *gr.* on the fore ſide of the Bow, ſuch is the latitude. For the declination is 10 *gr.* Southward, and the altitude of the Sunne betweene the two ſights 30 *gr.* the altitude of the equator 40 *gr.* and therefore the latitude 50 *gr.* as in the fourth *Prop.*

11 *To find any South latitude by a back obſeruation, knowing either the day of the moneth, or the declination of the Sunne.*

When you obſerue in South latitude, place your three ſights on the backe ſide of the Bow: the vpper ſight to the declination of the Sunne, or the day of the moneth at the South end; the lower ſight toward the North end of the Bow, and the horizontall ſight to the center. Then the Sun coming to the meridian, turne your face to the South, and holding the South end of the Bow vpward, with the backe of it toward your ſelfe, obſerue the ſhadow of the vpper ſight as before: ſo the lower ſight ſhal ſhew the latitude of the place in the back ſide of the Bow.

Thus being in the South latitude vpon the tenth of May,

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if you obſerue and find the lower ſight to ſtay at 30 *gr.* on the backe of the Bow, ſuch is the altitude. For the declination is 20 *gr.* Northward, the altitude of the Sunne betweene the two ſights 40 *gr.* the altitude of the equator 60 *gr.* and therefore the latitude 30 *gr.* as in the ſeuenth *Prop.*

12 *To find the day of the moneth, by knowing the latitude of the place, and obſeruing the meridian altitude of the Sunne.*

Place your three ſights according to your latitude; the horizontall ſight to the center, the lower ſight to the latitude, and the vpper ſight among the moneths. Then when the Sunne cometh to the meridian, obſerue the altitude, looking by the lower ſight through the horizontall, and keeping the lower ſight ſtill at the latitude, but mouing the vpper ſight vntil it giue ſhadow vpon the middle of the horizontal ſight: ſo the vpper ſight ſhall ſhew the day of the moneth required.

Thus in our latitude if you ſet the lower ſight to 51 *gr.* 30 *m.* and obſeruing finde the altitude of the Sunne betweene that and the vpper ſight to be 28 *gr.* 30 *m.* this vpper ſight will fall vpon the ninth of October, and the twelfth of Februarie. And if yet you doubt which of them two is the day, you may expect another meridian altitude; and then if you find the vpper ſight vpon the tenth of October, and the eleuenth of Februarie, the queſtion will be ſoone reſolued.

13 *To find the declination of any vnknowne ſtarre, and ſo to place it on the* Bow, *by knowing the latitude of the place, and obſeruing the Meridian altitude of the Starre.*

When you find a ſtarre in the Meridian that is fit for obſeruation. Set the center of the Bow to your eye, the lower ſight to the latitude, and moue the vpper ſight vp or downe vntill you ſee the horizon by the lower ſight, and the ſtarre

by

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by the vpper ſight, then will the vpper ſight ſtay at the declination and place of the ſtarre.

Thus being in 20 *gr.* of North latitude, if you obſerue and find the meridian altitude of the head of the Croſier to be 14 *gr.* 50 *m.* The vpper ſight will ſtay at 34 *gr.* 50 *m.* and there may you place this ſtarre. For by this obſeruatiõ the diſtance of this ſtarre from the South pole ſhould be 34 *gr.* 50 *m.* and the declination from the equator 55 *gr.* 10 *m.* And ſo for the reſt.

The ſtarres which I mentioned before, do come to the meridian in this order, after the firſt point of *Aries.*

	Ho.	*Mi.*		*Ho.*	*Mi.*
The pole ſtarre at	0	29	The lions hart	9	48
The rams head	1	46	The great beares backe	10	40
The head of Meduſa	2	44	Firſt in gr. beares taile	12	37
The ſide of Perſeus	2	58	The Virgins ſpike	13	5
The Buls eye.	4	15	Second in gr. bea. taile	13	9
The goate	4	49	Third in gr. beares taile	13	33
Orions left ſhoulder	5	5	Arcturus	13	58
Orions girdle the firſt	5	13	The formoſt guard	14	52
the ſecond	5	17	The North crowne	15	19
the third.	5	22	The hindmoſt guard	15	25
Orions right ſhoulder	5	35	Scorpions hart	16	7
The great dog	6	29	The harpe	18	24
Caſtor	7	10	Vulturs hart	19	33
The little dog	7	20	Swans taile	20	29
Pollux	7	22	Fomahant	22	36
The Hydra's hart	9	9			

The end of the ſecond Booke.

THE THIRD BOOKE.

Of the vſe of the lines of Numbers, Sines and Tangents for the drawing of Houre-lines on all ſorts of Planes.

THere are ten ſeuerall ſorts of Planes, which take their denomination frō thoſe great circles to which they are parallels, and may ſufficiently for our vſe be repreſented in this one fundamentall Diagram deſcribed before in the vſe of the *Sector*, and be knowne by their horizontall and perpendicular lines, of ſuch as know the latitude of the place, and the circles of the ſphere.

1 An horizontall plane parallell to the horizon, here repreſented by the outward circle *E S W N*.

2 A verticall plane parallell to the prime verticall circle which paſſeth through the zenith and the points of Eaſt and Weſt in the horizon, and is right to the horizon and the meridian; that is,maketh right angles with them both. This is repreſented by *E Z W*.

3 A polar plane parallell to the circle of the houre of 6, which paſſeth through the pole and the points of Eaſt and Weſt, being right to the Equinoctiall and the Meridian, but inclining to the horizon, with an angle equall to the latitude. This is here repreſented by *E P W*.

4 An equinoctiall plane parallell to the Equinoctiall, which paſſeth through the points of Eaſt and Weſt, being right to the Meridian, but inclining to the Horizon, with an angle equall to the complement of the latitude. This is here repreſented by *E A W*.

5 A verticall plane inclining to the horizon, parallell to any great circle, which paſſeth through the points of Eaſt and Weſt, being right to the meridian, but inclining to the horizon, and yet not paſſing through the pole, nor parallell

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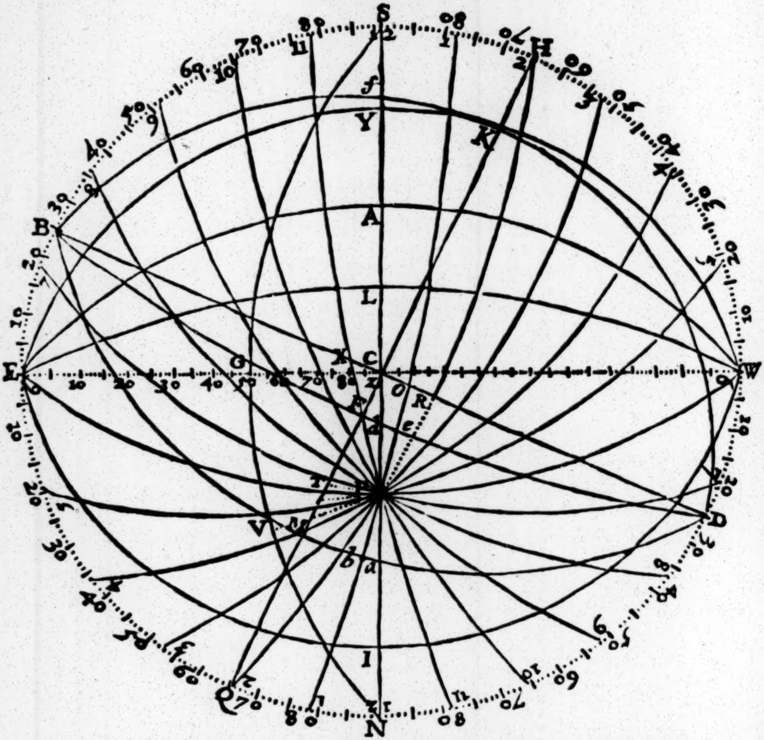

to the equinoctiall. This is here repreſented either by *E I W*, or *E Y W*, or *E L W*.

6 A meridian plane parallell to the meridian, the circle of the houre of 12, which paſſeth through the zenith, the pole, and the points of South and North, being right to the horizon, and the prime verticall. This is here repreſented by *S Z N*.

7 A meridian plane inclining to the horizon, parallell to any great circle, which paſſeth through the points of South and North, being right to the prime verticall, but inclining to the horizon. This is here repreſented by *S G N*.

8 A verticall declining plane,parallell to any great circle, which paſſeth through yͤ zenith,being right to the horizon, but inclining to the meridian. This is repreſented by *BZD*.

9 A polar declining plane,parallell to any great circle, which paſſeth through the pole , being right to the equinoctiall, but inclining to the meridian. This is here repreſented by *H P Q*.

10 A declining inclining plane,parallell to any great circle, which is right to none of the former circles, but declining from the prime verticall, and inclining both to the horizon and the meridian,and all the houre circles . This may be here repreſented either by *B M D*, or *B F D*, or *B K D*, or any ſuch great circle, which paſſeth neither through the South and North,nor Eaſt and Weſt points,nor through the zenith nor the pole.

Each of theſe planes (except the horizontall) hath two faces whereon houre-lines may be drawne; and ſo there are 19 planes in all. The meridian plane hath one face to the Eaſt, and another to the Weſt: the other verticall planes haue one to the South,and another to the North, and the reſt one to the zenith, and another to the nadir: but what is ſaid of the one,may be vnderſtood of the other.

To find the inclination of any Plane.

For the diſtinguiſhing of theſe Planes we may finde whether they be horizontall, or verticall, or inclining to the horizon, and how much they incline,either by the vſuall inclinatorie quadrant, or by fitting a thread and plummet vnto the *Sector*.

For let the *Sector* be opened to a right angle, the lines of *Sines* to an angle of 92 *gr.* the inward edges of the *Sector* to 90 *gr.* and let a thread and plummet be hanged vpon a line parallell to the edges of one of the legs, ſo that leg ſhall be verticall, and the other leg parallell to the horizon.

If the plane ſeeme to be verticall (like the wall of an vp-

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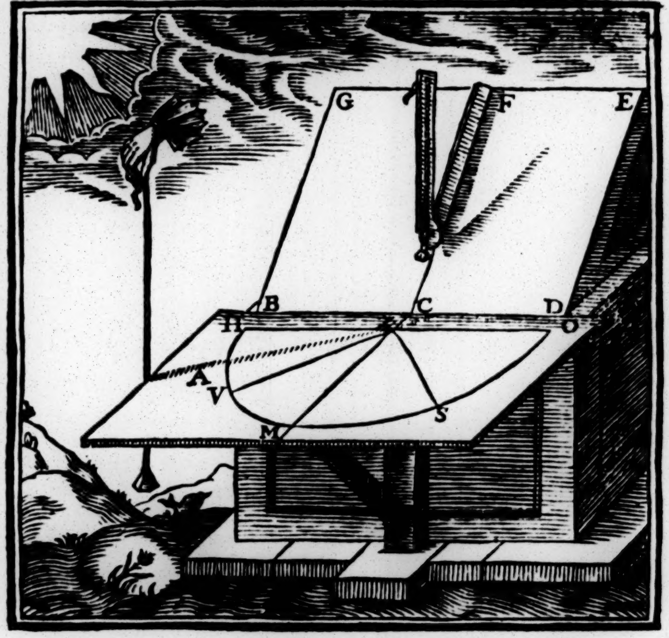

right building) you may trie it by holding the *Sector*, ſo that the thread may fall vpon his plummet line. For then if the verticall edge of the *Sector* ſhall lie cloſe to the plane, the plane is erect, and therefore ſaid to be verticall; and if you draw a line by that edge of the *Sector*, it ſhall be a verticall line.

If the plane ſeeme to be leuell with the horizon, you may trie it by ſetting the horizontall leg of the Sector to the plane, and holding the other leg vpright: for then if the thread ſhall fall on his plummet line, which way ſoeuer you turne the *Sector*, it is an horizontall plane.

If the one end of the plane be higher then the other, and yet not verticall, it is an inclining plane, and you may find the inclination in this maner.

Firſt hold the verticall leg of the Sector vpright, and turne the horizontall leg about, vntill it lie cloſe with the plane, and the thread fall on his plūmet line: ſo the line drawne by the edge of that horizontall leg, ſhall be an horizontall line.

Suppoſe the plane to be *B G E D*, and that *B D* were thus found to be the horizontall line vpon the plane; then may you croſſe the horizontall line at right angles with a perpendicular *C F*: that done, if you ſet one of the legs of the *Sector* vpon the perpendicular line *C F*, and make the other leg with a thread and plummet to become verticall, you ſhal haue the angle betweene the verticall line and the perpendicular on the Plane, as before in the vſe of the *Sector*, pag. 50. and the complement of this angle is the inclination of the plane to the horizon.

To find the declination of a Plane.

The declination of a Plane is alwayes reckoned in the horizon betweene the line of Eaſt and Weſt, and the horizontall line vpon the Plane. As in the fundamentall Diagram, the prime verticall line (which is the line of Eaſt and Weſt) is *E C W*; if the horizontall line of the plane propoſed ſhal be *B C D*, the angle of declination is *E C B*.

But becauſe a Plane may decline diuers wayes, that we may the better diſtinguiſh them, we conſider three lines belonging to euery Plane: the firſt is the horizontall line; the ſecond the perpendicular line, croſſing the horizontall at right angles; the third the axis of the plane, croſſing both the horizontall line, and his perpendicular, and the plane it ſelfe at right angles.

The perpendicular line doth help to find the inclination of the plane as before, the horizontall to finde the declination, the axis to giue denomination vnto the plane.

For example, in a verticall plane here repreſented by *E Z W*, the horizontall line is *E C W*, the ſame with the line of Eaſt and Weſt, and therefore no declination; the perpendicular croſſing it, *C Z* the ſame with the verticall line, drawne

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drawne from the center to the zenith , right vnto the horizon, and therefore no inclination. The axis of the plane is *S C N*, the ſame with the meridian line, drawne from the South to the North ,· and accordingly giues the denomination to the plane. For the plane hauing two faces, and the axis two poles, *S* and *N*; the pole *S* falling directly into the South , doth cauſe that face to which it is next to be called the South face ; and the other pole at *N*, pointing into the North, doth giue the denomination to the other face , and make it to be called the North face of this plane.

In like maner in the declining inclining plane here repreſented by *B F* D, the horizontall line is *B C* D, which croſſeth the prime verticall line *E C W*, and therefore it is called a declining plane, according to the angle of declination *E C B* or *W C* D. The perpendicular to this horizontall line is *C F*, where the point *F* falleth in the plane *Q Z H* perpendicular to the plane propoſed , betweene the zenith and the North part of the horizon, and therefore it is called a plane inclining to the Northward , according to the arke *F Q*, or the angle *F C Q*. The axis of the plane is here repreſented by the line *C K*, where the pole *K* is 90 *gr.* diſtant from the plane , and ſo is as much aboue the horizon at H, and the other pole as much below the horizon at *Q*, as the plane at *F* is diſtant from the zenith : and this pole *K* here falling betweene the meridian and the prime verticall circle into the Southweſt part of the world, this vpper face of the plane is therefore called the Southweſt face , and the lower the Northeaſt face of the plane.

The declination from the prime verticall may be found by the needle in the vſuall inclinatorie Quadrant, or rather by comparing the horizontall line drawne vpon the plane with the azimuth of the Sunne and the meridian line, in ſuch ſort as before we found the variation of the magneticall needle. For take any boord that hath one ſide ſtrait , and draw the line H O parallell to that ſide , and the line *Z M* perpendicular vnto it, and on the center *Z* make a ſemicircle H M O: this done, hold the boord to ÿ plane, ſo as H O may be parallel to B D the horizontall line on the plane and the boord parallell to the horizon; then the Sunne ſhining vpon it, hold out a thread and plummet, ſo as the thread being verticall, the ſhadow of the Sunne may fall on the center *Z*, and draw the line of ſhadow *A Z* repreſenting the common ſection, which the azimuth of the Sunne makes with the plane of the horizon, and let another take the altitude of the Sunne at the ſame inſtant: ſo by reſoluing a triangle, as I ſhewed before pag.*65*. you may find what azimuth the Sun was in when he gaue ſhadow vpon *A Z*.

Suppoſe the azimuth to be (as before pag.*64*.) 72 *gr.* 52 *m.* from the North to the Weſtward, and therefore 17 *gr.* 8 *m.* from the Weſt, we may allow theſe 17 *gr.* 8 *m.* from *A* vnto *V*, and draw the line *Z V*, and ſo we haue the true Weſt point of the prime verticall line: then allowing 90 *gr.* from *V* vnto *S*, we haue the South point of the meridian line *Z S*, and the angle H Z V ſhall giue the declination of the plane from the verticall, and the angle *O Z S* the declination of the plane from the meridian.

Or we may take out onely the angle A Z H, which the line of ſhadow makes with the horizontall line of the plane, and compare it with the angle *A Z V*, which the line of ſhadow makes with the prime verticall. And ſo here if *A Z V* the Sunnes azimuth ſhall be 17 *gr.* 8 *m.* paſt the Weſt, and yet the line of ſhadow *A Z* 7 *gr.* 12 *m.* ſhort of the plane, the declination of the plane ſhall be 24 *gr.* 20 *m.* as may appeare by the ſite of the plane and the circles.

If the altitude of the Sunne be taken at ſuch time as the ſhadow of the thread falleth on B D or H O, and then a triangle reſolued, the declination of the plane will be ſuch as the azimuth of the Sunne from the prime verticall.

If at ſuch a time as the ſhadow falleth on *M Z*, the declination will be ſuch as the azimuth of the Sunne from the meridian.

If it be a faire Summers day you may firſt finde what altitude the Sunne will haue when he cometh to be due Eaſt or Weſt, and then expect vntill he come to that altitude; ſo the

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declination of the plane ſhall be ſuch as the angle contained betweene the line H O and the line of the ſhadow.

Hauing diſtinguiſhed the Planes, the next care will be for the placing of the ſtyle and the drawing of the houre-lines.

The ſtyle will be as the axis of the world, ſometimes parallell to the plane, ſometimes perpendicular, ſometimes cut the plane with oblique angles.

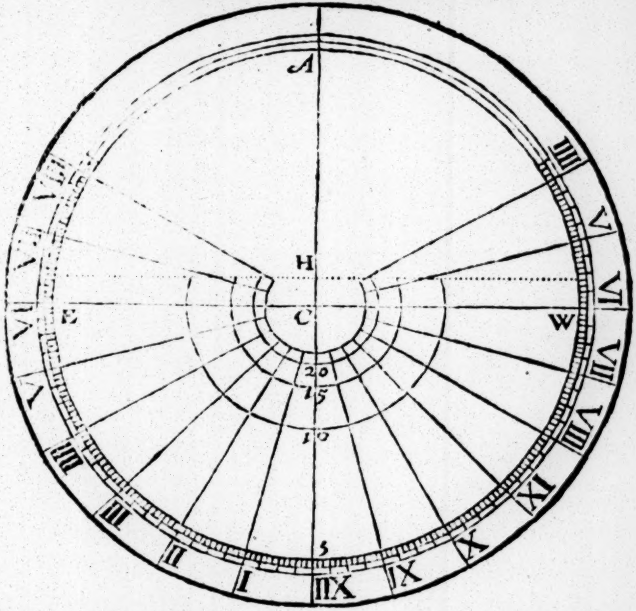
The houre-lines will be either parallell one to the other, or meete in a center with equall angles, or meete with vnequall angles. If the ſtyle be perpendicular to the plane, the angles at the center will be equall; and this falls out onely in the South and North face of an equinoctiall plane: if the ſtyle be parallell to the plane, the houre-lines wil be alſo parallell one to another; and this falls out in all polar planes; as in the Eaſt and Weſt meridian planes parallel to the circle of the houre of 12, in the vpper and lower direct polars parallell to the circles of the houre of 6, and in the vpper and lower declining polars which are parallell to any of the other houre circles.

But in the horizontall and all other planes, the ſtyle will cut the plane with an acute angle, and the houre lines will meet at the root of the ſtyle, and there make vnequall angles.

CHAP. I.

To draw the houre-lines in an equinoctiall Plane.

AN equinoctiall plane is that which is parallell to ẏ equinoctiall circle here repreſented by *E A W*, wherein the ſpaces betweene the houre circles being equall, there is no need of further precept, but onely to draw a circle and to diuide it into 24 equall parts for the 24 houres, and ſubdiuide each houre into halues and quarters, and then to ſet vp the



the ſtyle perpendicular to the plane in the center of the circle. The help which theſe lines of proportion doe here affoord vs, is onely in the diuiſion of the circle, which may be done readily by that which I ſhewed before, *Pag.29.*

For example, ſuppoſe the ſemidiameter of the equinoctiall circle to be ſix inches, and that it were required to know the diſtance of the houre-points each from other: here each houre being *15 gr.* diſtant from other, I extend the compaſſes from the ſine of *30 gr.* vnto the ſine of *7 gr. 30 m.* the halfe of *15 gr.* and I find the ſame extent to reach in the line of numbers from *6.00* vnto *1.56.*

Or in croſſe worke I extend them from the ſine of *30 gr.* vnto *6.00* in the line of numbers, the ſame extent will reach from the ſine of *7 gr. 30 m.* vnto *1.56* in the line of numbers; which ſhewes that in a circle of ſix inches ſemidiameter, the

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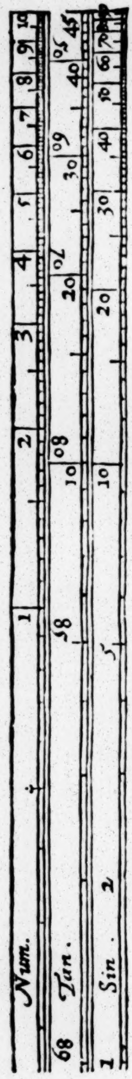


diſtance of the houre-points each from other will be about 1 inch and 56 centeſmes or parts of 100. The like reaſon holds for the inſcribing of all other chords in the *Prop.* following.

CHAP. II.

To draw the houre-lines in a direct polar plane.

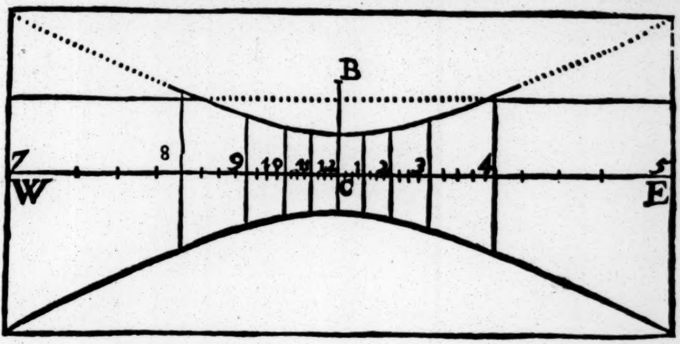
A Direct polar plane is that which is parallell to the houre of 6, here repreſented by *E P W*, wherein the ſtyle will be parallell to the plane, and the houre-lines parallell one to the other, and therefore may be beſt drawne by that which I haue ſhewed in the vſe of the *Sector*. They may be alſo drawne by the helpe of theſe lines of proportion, in this maner.

Firſt draw a right line *W E* for the horizon and the equator, and croſſe it at the point *C*, about the middle of the line with *C B* another right line, which may ſerue for the meridian and the houre of 12, and muſt alſo be the ſubſtylar line wherein the ſtyle ſhall ſtand. Then, to proportion the ſtyle vnto the plane, conſider the length of the horizontall line, and what houre-lines you would haue to fall on your plane.

For the diſtance of any one houre-line from the meridian being knowne, we may finde both the length of the ſtyle and the diſtance of the reſt : becauſe

As the tangent of the houre giuen,
is to the diſtance from the meridian:
So the tangent of 45 *gr.*
to the height of the ſtyle.

Suppoſe



Suppoſe the length of the horizontall line to be 12 inches, and that it were required to put on all the houre-lines from 7 in the morning vnto 5 in the euening. Here we haue 5 houres and 6 inches on either ſide the meridian. Wherefore I allow 15 *gr.* for an houre, and extending the compaſſes the compaſſes from the tangent of 75 *gr.* the meaſure of 5 houres vnto the tangent of 45 *gr.* I find the ſame extent to reach in the line of numbers from 6.00 to about 1.61. This ſhewes both the height of the ſtyle, and the diſtance of the houre-points of 9 & 3 frõ the meridian to be 1 inch, 61 parts.

To find the length of the Tangent betweene the ſubſtylar and the houre-points.

As the tangent of 45 *gr.*
  to the tangent of the houre:
So the height of the ſtyle
  to the length of the tangent line betweene the ſubſtylar and the houre-points.

Thus hauing found the length of the ſtyle in our example to be 1.61, if I exend the compaſſes from the tangent of 45 *gr.* vnto the tangent of 15 *gr.* the meaſure of the firſt houre from the ſubſtylar, I ſhall find the ſame extent to reach in the line of numbers from 1.61 vnto 0.43 for the length of the

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tangent betweene the ſubſtylar and the houre-points of 11 and 1. If I extend them from the tangent of 45 *gr.* vnto the tangent of 75 *gr.* the meaſure of the fift houre, I ſhall finde them to reach in the line of numbers from 1.61 vnto 6.00 for the length of the tangent from the ſubſtylar to the houre-points of 7 and 5. For howſoeuer it be the ſame diſtance in the line of tangents from 45 vnto 75, as from 45 vnto 15; yet becauſe 75 are more, and 15 leſſe then 45, the tangent lines that anſwer to them wil be accordingly more or leſſe then the length of the ſtyle.

Ho.	An. Po		Tang.	
	Gr.	M.	In.	Par
12	0	0	0	0
11. 1	15	00	0	43
10. 2	30	00	0	93
9 3	45	0	1	61
8. 4	60	0	2	79
7. 5	75	0	6	0
6. 6	90	0	Infin	

Againe, if I extend them from 45 *gr.* in the tangents vnto 30 *gr.* the meaſure of the ſecond houre, I ſhall finde them to reach in the line of numbers from 1.61 vnto 0.93 for the houre of 10 and 2: if I extend them from the tangent of 45 *gr.* vnto the tangent of 60 *gr.* for the fourth houre, I ſhal find them to reach in the line of numbers from 1.61 vnto 2 79, and ſuch is the length of the tangent line from the ſubſtylar vnto the houre of 8 and 4. And the like reaſon holdeth for the inſcribing of all other tangent lines in the propoſitions following.

But for ſuch tangents as fall vnder 45 *gr.* I may better vſe croſſe worke, and extend the compaſſes from the tangent of 45 *gr.* vnto 1.61 in the line of numbers, ſo ſhall I finde the ſame extent to reach from 30 *gr.* in the tangents, to 93 parts in the line of numbers, for the diſtance of the ſecond houre, and from 15 *gr.* in the tangents to 43 parts for the diſtance of the firſt houre from the meridian.

Or if this extent from 45 *gr.* backward to 1.61 be too large for the compaſſes, I may extend forward from the tangent of 5 *gr.* 43 *m.* to 1.61 parts in the line of numbers, and the ſame extent ſhall reach from 15 *gr.* in the tangents, to 43 parts in the line of numbers, for the diſtance of the firſt houre; and from 30 *gr.* to 93 parts, for the diſtance of the ſecond houre, as before.

Hauing found the length of the tangent lines in inches and parts of inches, and pricked them in the equator on both ſides of the meridian, from the center *C*; if wee draw right lines through each of thoſe points, croſſing the equator at right angles, they ſhall be the houre-lines required; and if we ſet a ſtyle ouer the meridian, ſo as the edge of it be parallell to the plane, and the height of it be as much aboue the meridian as the diſtance between the meridian and the houre-points of 3 or 9, it ſhall repreſent the axis of the world, and be truly placed for the caſting of the ſhadow vpon the houre-lines in a polar plane.

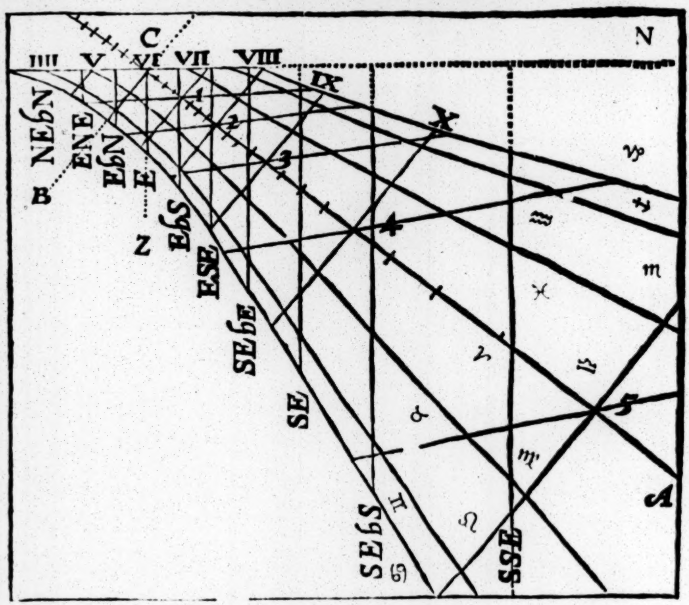
Hora	Ang.Po		Tang	
	Gr.	*M.*	In.	Pa.
6		0		0
	3	45		·65
	7	30	1	32
	11	15	1	99
7	15	0	2	68
	18	45	3	39
	22	30	4	14
	26	15	4	93
8	30	0	5	77
	33	45	6	68
	37	30	7	67
	41	15	8	77
9	45	0	10	00
	48	45	11	40
	52	30	13	03
	56	15	14	97
10	60	0	17	32
	63	45	20	28
	67	30	24	14
	71	15	29	46
11	75	0	37	32
	78	45	50	27
	82	30	75	96
	86	15	152	57
12	90	0	Infin.	

CHAP. III.

To draw the houre-lines in a meridian plane.

A Meridian plane is that which is parallell to the meridian circle here repreſented by *SZN*; it hath two faces, one to the Eaſt, and the other to the Weſt: in each of them the ſtyle will be parallell to the plane, and the houre-lines parallell one to the other, as in a polar plane, the difference being onely in the placing of the equator & in numbring of the houres.

For in theſe meridian planes hauing drawn an occult verticall line *CZ*, and an occult horizontall line *CA*, croſſing one the other at right angles in the point *C*, the equator *AC* wil cut the verticall with an angle *ZCA*, equall to the latitude of the place: then may we croſſe the equator at right angles with the line *CB* for the houre of 6, and from this ſet off the houre-



houre-points in the equator as in the former *Prop.*

For ſuppoſing the length of the ſtyle *C B* to be ten inches, the length of the tangent line belonging to the firſt houre wil be 2 *in.*68 *p.* the length of the ſecond 5 *in.* 77 *p.* as in the Table. Then the tangent of 15 *gr.* being prickt downe in the equator on both ſides from 6, ſhal ſerue for the houres of 5 and 7, and the tangent of 30 *gr.* for the houres of 4 and 8, and ſo in the reſt. This done, if we draw right lines through each of theſe points, croſſing the equator at right angles, they ſhall be the houre-lines required: and if we ſet a ſtyle ouer the houre of 6, ſo as the edge of it may be parallell to the plane, and the height of it may be equall to the diſtance betweene the houres of 6 and 9 in the equator, it ſhall repreſent the axis of the world, and be truly placed for the caſting of the ſhadow vpon the houre-lines in a meridian plane.

CHAP.

CHAP. IIII.

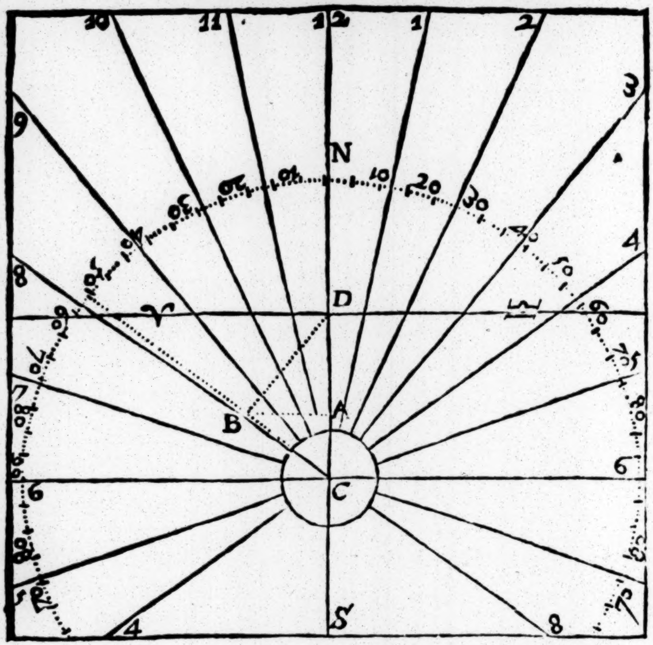
To draw the houre-lines in an horizontall plane.

AN horizontall plane is that which is parallell to the horizon, here repreſented by the outward circle *E S W N*, in which the diameter *S N* drawne from the South to the North, may go both for the meridian line and the meridian circle, *Z* for the zenith, *P* for the pole of the world, and the circles drawne through *P* for the houre-circles of 1.2.3.4.&c. as they are nũbred from the meridian. Theſe houre-circles conſidered with the meridian & the horizon, do make diuers triangles, *PN* 1, *PN* 2, *PN* 3, in which we haue knowne firſt the right angle at *N*, the North interſection of the meridian and the horizon; ſecondly the ſide *PN*, the arke of the meridian between the pole and the horizon, which is alwayes equall to the latitude of the place; thirdly the angles at the pole, made by the meridian and the houre-circles, the angle *N P* 1 being 15 *gr.* *N P* 2 30 *gr.* each houre 15 *gr.* more then other, each halfe houre 7 *gr.* 30 *m.* each quarter 3 *gr.* 45 *m.* And theſe three being known, we may finde the arks of the horizon between the meridian and the houre-circles *N* 1, *N* 2, *N* 3, &c. For

As the ſine of 90 *gr.*
is to the ſine of the latitude:
So the tangent of the houre
to the tangent of the houre-line
from the meridian.

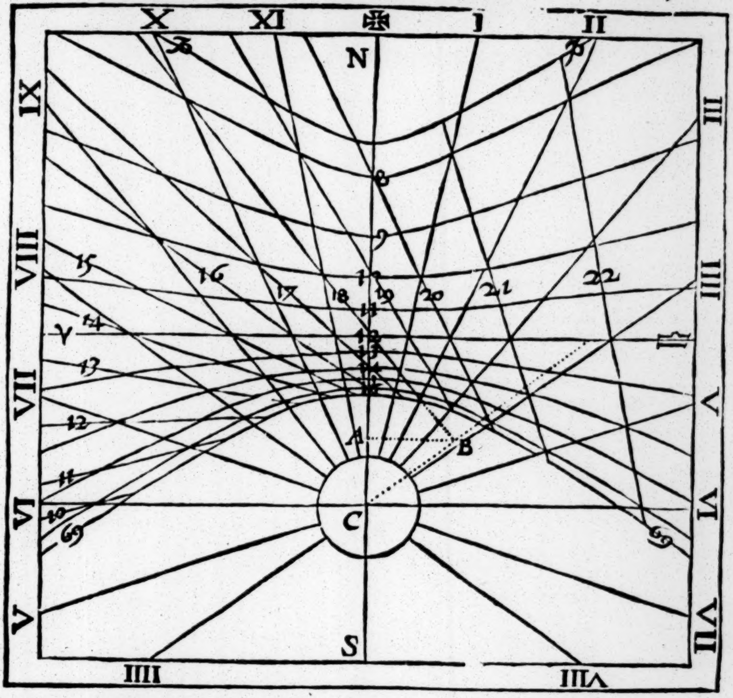
Ho.	Ang. Po		Arc. Pla	
	Gr.	M.	Gr.	M.
12	0	0	0	0
	3	45	2	56
	7	30	5	52
	11	15	8	51
1	15	0	11	50
	18	45	14	52
	22	30	17	57
	26	15	21	6
2	30	0	24	20
	33	45	27	35
	37	30	31	0
	41	15	34	28
3	45	0	38	3
	48	45	41	45
	52	30	45	34
	56	15	49	30
4	60	0	53	35
	63	45	57	47
	67	30	62	6
	71	15	66	33
5	75	0	71	6
	78	45	75	45
	82	30	80	25
	86	15	85	13
6	90	0	90	0

Extend



Extend the compaſſes from the ſine of 90 *gr.* to the ſine of the latitude, ſo the ſame extent ſhall reach from the tangent of the houre, to the tangent of the houre-line from the meridian. Thus the latitude being 51 *gr.* 30 *m.* I extend the compaſſes from the ſine of 90 *gr.* to the ſine of 51 *gr.* 30 *m.* & find the ſame extent to reach from the tangent of 3 *gr.* 45 *m.* vnto the tangent of 2 *gr.* 56 *m.* for the diſtance of the firſt quarter from the meridian; and from the tangent of 7 *gr.* 30 *m.* vnto the tangent of 5 *gr.* 52 *m.* for the halfe houre; and from the tangent of 11 *gr.* 15 *m.* to the tangent of 8 *gr.* 51 *m.* for the third quarter; and from the tangent of 15 *gr.* 0 *m.* vnto 11 *gr.* 50 *m.* for the firſt houre: and ſo in the reſt.

Only when I come to ſet one foote of the compaſſes to



48 *gr.*45 *m.*for the finding of a quarter paſt 3,the other foote will fall out of the line , and then I may either take out ſo much as is out of the line beyond 45 *gr.* and turne it backe into the line, and it will reach from 45 *gr.*to 41 *gr.*45 *m:* or I may vſe croſſe worke,extending the compaſſes from the ſine of 90 *gr.*to the tangent of 48 *gr.*45 *m.* ſo the ſame extent wil reach from the ſine of 51 *gr.*30 *m.* to the tangent of 41 *gr.* 45 *m.* And ſuch is the diſtance of the line of 3 ho.¼ from the meridian,as in this table of the Arks of the plane.

This done, I come to the Plane, and there according as the lines do fall in the fundamentall diagram,

1 I draw a right line *S N* ſeruing for the meridian , the

houre of **12** and the ſubſtylar.

2 In this meridian I make choice of a center at ***C***, and there deſcribe an occult circle repreſenting the horizon.

3 I find a chord of **11** *gr.* 50 *m* and inſcribe it into this circle on either ſide of the meridian for the houres of **11** & **1**; in like manner a chord of 24 *gr.* 20 *m.* for the houres of **10** and **2**; and a chord of 38 *gr.* 3 *m.* for the houres of *9* and *3*; and ſo for the reſt of the houres, their halues and quarters.

4 I draw right lines through the center and the termes of theſe chords, and theſe lines ſo drawne are the houre-lines required.

Laſtly I ſet vp the ſtyle ouer the meridian, ſo as it may cut the plane in the center, and there make an angle with the meridian equall to the latitude of the place, ſo it ſhall repreſent the axis of the world, and be truly placed for caſting of the ſhadow vpon the houre-lines in an horizontall plane.

CHAP. V.

To draw the houre-lines in a verticall plane.

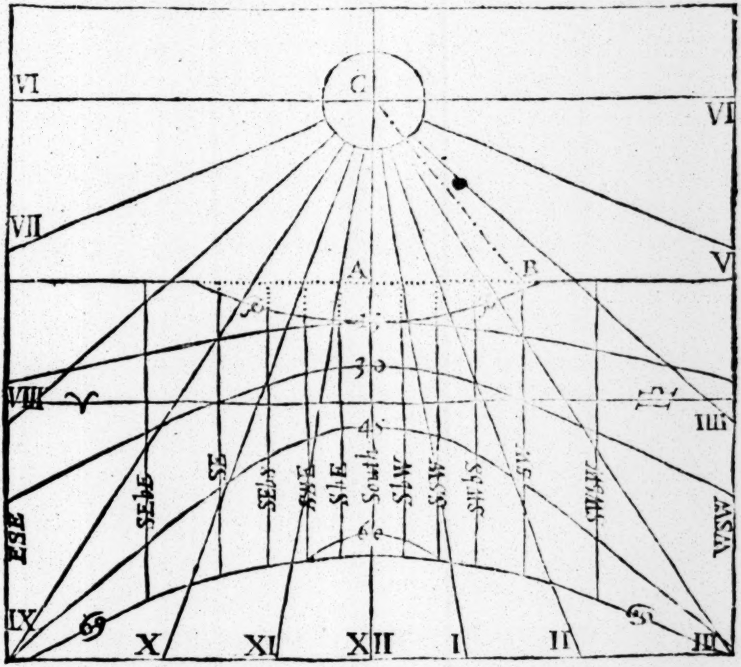
A Verticall plane is that which is parallel to the prime verticall circle here repreſented by *E Z W*. It hath two faces, one to the North, the other to the South; in each of them the ſubſtylar will be the ſame with the meridian line, and the angle of the ſtyle aboue the plane will be equall to *Z P* the complement of the latitude.

The triangles here conſidered are made by the verticall, the meridian, and the houre-circles, in which we know the ſide *Z P*, the angles at the pole, and the right angle at the zenith, and therefore may find the arkes of the verticall, between the meridian and the houre circles after this manner:

As the ſine of *90 gr.*
  is to the coſine of the latitude:
So the tangent of the houre
  to the tangent of the houre-line from the meridian.

Extend the compaſſes from the ſine of 90 *gr.* to the ſine of the complement of the latitude, ſo the ſame extent ſhall reach from the tangent of the houre, to the tangent of the houre-line from the meridian.

Thus in the latitude of 51 *gr* 30 *m* I extend the compaſſes from the ſine of 90 *gr.* to the ſine of 38 *gr.* 30 *m.* and find the ſame extent to reach from the tangent of 15 *gr.* to the tangent of 9 *gr.* 28 *m* for the diſtance of the firſt houre from the meridian: and from the tangent of 75 *gr.* vnto the tangent of 66 *gr* 42 *m.* for the fift houre: and ſo in the reſt as in the Table following.



Theſe arks being knowne, I may come to the plane, and there by help of a thread and plummet draw a verticall line

ſeruing

ſeruing both for the meridian and the houre of 12, and the ſubſtylar; then may I draw an occult verticall circle, and therein inſcribe the chords of thoſe former arks, and draw the houre-lines, and ſet vp the ſtyle, as before in the horizontall plane.

If it be the South face of the plane, the center will be vpward, and the ſtyle muſt point downward; if the North face, the center muſt be in the lower part of the meridian line, and the ſtyle-point vpward in all ſuch places as are to the Northward of the equinoctiall line, as it may appeare by conſidering how the lines do fall in the fundamentall Diagram.

Ho.	Ang.Po		Arc.Pla	
	Gr.	M.	Gr.	M.
12	0	0	0	0
	3	45	2	20
	7	30	4	41
	11	15	7	3
1	15	0	9	28
	18	45	11	56
	22	30	14	27
	26	15	17	4
2	30	0	19	46
	33	45	22	35
	37	30	25	32
	41	15	28	38
3	45	0	31	54
	48	45	35	22
	52	30	39	3
	56	15	42	58
4	60	0	47	9
	63	45	51	36
	67	30	56	20
	71	15	61	23
5	75	0	66	42
	78	45	72	17
	82	30	78	3
	86	15	84	0
6	90	0	90	0

CHAP. VI.

To draw the houre-lines in a verticall inclining plane.

ALl thoſe Planes that haue their horizontall line lying Eaſt and Weſt, are in that reſpect ſaid to be verticall; if they be alſo vpright and paſſe through the zenith, they are direct verticals; if they incline to the pole they are direct polars: if to the equinoctiall, they are properly called equinoctiall planes, and are deſcribed before: if to none of theſe three points, they are then called by the generall name of inclining verticals.

Theſe may incline either to the North part of the horizon, or to the South; and each of them hath two faces, one to the zenith, the other to the nadir, in which we are firſt to cõſider the height of the pole aboue the plane, by comparing the inclination of the plane to the horizon, with the latitude of the place.

As in our latitude of 51 *gr.* 30 *m.* if the inclination of the plane *E I W* ſhall be 13 *gr.* Northward, that is, if *I N* the ark of the meridian between the plane and the North part of the horizon ſhall be 13 *gr.* we may take theſe 13 *gr.* out of *P N* 51 *gr.* 30 *m.* the eleuation of the pole aboue the horizon, and there wil remain *P I* 38 *gr.* 30 *m.* for the eleuatiō of the North pole aboue the vpper face of the plane, and therefore 38 *gr.* 30 *m.* for the height of the South pole aboue the lower face of the plane.

Or if the inclination of the plane ſhall be found to be 62 *gr.* to the Southward, we may number them in the meridian from *S* the Southpart of the horizon vnto *L*, and there draw the arke *E L W* repreſenting this plane; ſo the arke of the meridian *P L* ſhall giue the height of the North pole aboue the vpper face of this plane to be 66 *gr.* 30 *m.* and therefore the height of the South pole aboue the lower face of the plane is alſo 66 *gr.* 30 *m.*

In like maner if the inclination of the plane *E Y W* ſhall be 15 *gr.* Southward, that is, if *S Y* the arke of the meridian between the South part of the horizon and the plane, ſhall be 15 *gr.* The height of the North pole aboue the vpper face of the plane, and the height of the South pole aboue the lower face of the plane, will be alſo found to be 66 *gr.* 30 *m.*

But if the plane ſhall fall betweene the zenith and the North pole, then will the North pole be eleuated aboue the lower face, and the South pole aboue the vpward face of the plane, as may appeare by the proiection of the ſphere in the fundamentall Diagram.

Then in the triangles made by the plane, the meridian, and the houre-circles, we haue the ſide which is the height of the pole aboue the plane, together with the angles at the pole, and the right angle at the interſection of the meridian with the plane, by which we may find the arks of the plane betweene the meridian and the houre-circles, after this maner:

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As the ſine of 90 *gr.*
is to the ſine of the pole aboue the plane:
So the tangent of the houre
to the tangent of the houre-line from the meridian.

Thus in the former example, where *PI* the height of the pole aboue the plane was found to be 38 *gr.* 30 *m.* if you ſhall extend the compaſſes from the ſine of 90 *gr.* to the ſine of 38 *gr.* 30 *m.* the ſame extent will reach from the tangent of 15 *gr.* vnto the tangent of 9 *gr.* 28 *m.* for the diſtance of the firſt houre from the meridian, and from 30 *gr.* vnto 19 *gr.* 46 *m.* for the ſecond houre, and ſo forward as in the direct vertical.

And for the two laſt examples, you may extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 66 *gr.* 30 *m*: ſo the ſame extent ſhall reach in the line of tangents from 15 *gr.* vnto 13 *gr.* 48 *m.* for the firſt houre, from 75 *gr.* vnto 73 *gr.* 43 *m.* for the fift houre, from 30 *gr.* vnto 27 *gr.* 54 *m.* for the ſecond houre, from 60 *gr.* vnto 57 *gr.* 48 *m.* for the fourth houre, and from 45 *gr.* vnto 42 *gr.* 31 *m.* for the third houre from the meridian.

Theſe arks being knowne, you may firſt draw the horizontall line, and croſſe it in the middle with a perpendicular that may ſerue both for the meridian and the houre of 12, and the ſubſtylar; then knowing which pole is eleuated aboue the plane, you may accordingly make choice of a fit point in the meridian for the center of your houre-lines, and thence deſcribe an occult arke of a circle, inſcribe the chords of thoſe former arks, and draw the houre lines, and ſet vp the ſtyle, as I ſhewed before in the horizontall plane.

CHAP. VII.

To draw the houre-lines in an verticall declining Plane.

ALl vpright planes whereon a man may draw a verticall line, are in this reſpect ſaid to be verticall; if they ſhall alſo ſtand directly Eaſt and Weſt, they are direct verticals; if directly North and South, they are commonly called meridian planes, and are deſcribed before: if they behold none of theſe foure principall parts of the world, but ſhall ſtand between the prime verticall and the meridian, they are then called by the generall name of declining verticals.

Theſe haue two faces, one to the South, the other to the Northward, which may be diſtinguiſhed in theſe Northerne parts of the world after this manner. If the Sunne coming to the meridian ſhall ſhine vpon the plane, it is the South face; if not, it is the North face of that plane. Againe, if the Sunne ſhall ſhine vpon the plane at high noone, and yet longer in the forenoone then in the afternoone, it is the Southeaſt face; if longer in the afternoone then in the forenoone, it is the Southweſt face of the plane. But how much the declination cometh to, is beſt found as before.

When the declination is found, there be foure things more to be conſidered before we can come to the drawing of the houre-lines, and all foure repreſented in the fundamentall Diagram.

1 *The meridian of the plane and his inclination to the meridian of the place.*

Let the arke *EZW* repreſent the prime verticall, and *BZD* a declining verticall, according to the angle of declination *EZB*, the meridian of the place is repreſented by *PZS*, croſſing the verticall *EZW* at right angles at the zenith in the point *Z*: but the proper meridian of the plane wil be *PR*, which is a perpendicular let downe from the pole vnto the declining verticall, and croſſing it with right angles in the point *R*, ſo the angle *RPZ* ſhall ſhew the inclination of the two meridians and may thus be found.

In the triangle *PRZ* we know the angle at *R* to be a right angle, and the angle at *Z*, for it is the complement of the declination, and the baſe *PZ*, for it is the complement of the latitude. And therefore

As

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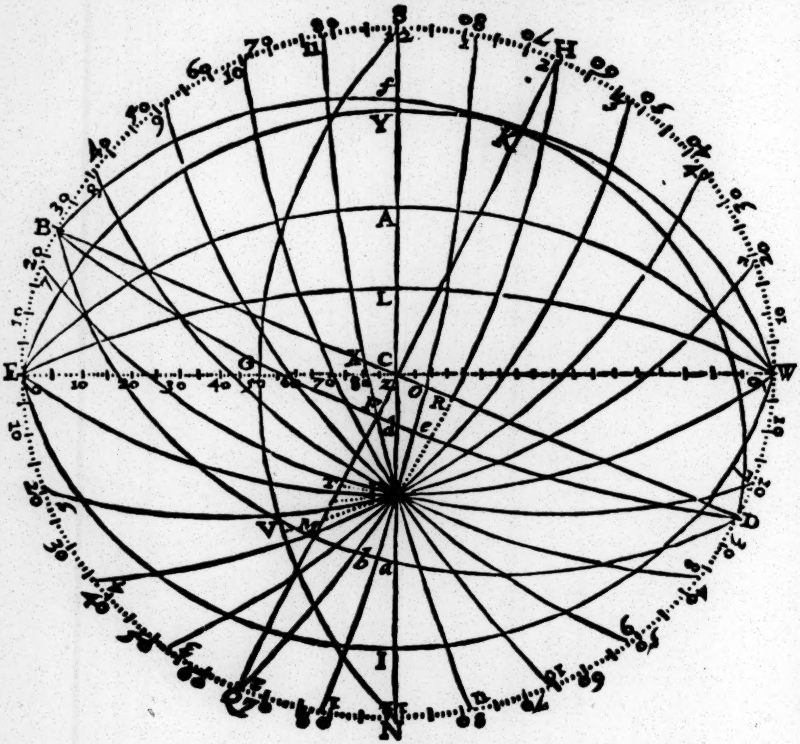

As the ſine of the latitude
is to the ſine of 90*gr.*
So the tangent of the declination
to the tangent of the inclination required.

2 *The height of the ſtyle aboue the Plane.*

This is here repreſented by the perpendicular arke *PR*, and may be found by that which we haue knowne in the former triangle *PRZ*. For

As the ſine of 90 *gr.*
to the coſine of the latitude:
So the coſine of the declination
to the ſine of the height of the ſtyle.

Or if you pleaſe to make vſe, of the angle of the inclination of the two meridians, the proportion will hold.

As the ſine of 90 *gr.*
to the coſine of the inclination of meridians:
So the cotangent of the latitude
to the tangent of the height of the ſtyle.

3 *The diſtance of the ſubſtylar from the meridian.*

This is here repreſented by the arke *Z R*, and may be found by that which we haue knowne in the former triangle *PRZ*.

As the ſine of 90 *gr.*
to the ſine of the declination:
So the cotangent of the latitude
to the tangent of the ſubſtylar from the meridian.

4 *The diſtance of each houre-line from the ſubſtylar.*

The diſtances of the houre-lines from the ſubſtylar, are here repreſented by thoſe arks of the declining verticall belonging to the plane, which are intercepted betweene the proper meridian of the plane and the houre-circles.

To this purpoſe we haue diuers triangles made by the declining plane, together with his proper meridian and the houre-circles. In theſe we haue knowne, firſt the right angle at the interſection of the proper meridian with ẏ plane; then the ſide which is the height of the pole aboue the plane; and thirdly the angles at the pole. For knowing the angle of inclination betweene the meridian of the plane and the meridian of the place, which is alwayes the houre of 12, we may finde the angle betweene the meridian of the plane and the houre of 1, by allowing in 15 *gr.* and the angle betweene

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the meridian of the plane and the houre of **2**, by allowing in 30*gr.* and ſo for the reſt, which being knowne, we may find the arks of the plane from the ſubſtylar to the houre-circles, in this maner.

As the ſine of 90 *gr.*
to the ſine of the height of the pole aboue the plane:
So the tangent of the houre from the proper meridian,
to the tangent of the houre-line from the ſubſtylar.

Thus in our latitude of **51** *gr.* 30 *m.* if the declination of an vpright plane ſhall be found to be **24** *gr.* **20** *m.* from the prime verticall, the one face open to the Southweſt, the other to the Northweſt, I may number theſe **24** *gr.* **20** *m.* in the horizon of the fundamentall Diagram, from *E* vnto *B*, according to the ſituation of the plane, and there draw the verticall *BZD*, which ſhall repreſent the plane propoſed.

Then taking the compaſſes into my hand,

1 I may extend them from the ſine of the latitude **51** *gr.* 30 *m.* vnto the ſine of 90 *gr.* the ſame extent will reach in the line of tangents from **24** *gr.* **20** *m.* the declination giuen, to about 30 *gr.* and ſuch is *ZPR* the angle of inclination between the meridiã of the place & the meridian of the plane; and therefore the meridian of the plane will here fall vpon the circle of the ſecond houre frõ the meridian of the place, (as it may alſo appeare by opening the compaſſes to the neareſt extent, between the pole and the plane) and there I place the letter *R* to make this rectangle triangle *PRZ.*

2 I extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 38 *gr.* 30 *m.* the complement of the latitude, and the ſame extent will reach from the ſine of 65 *gr.* 40 *m.* the complement of the declination, vnto the ſine of 34 *gr.* 33 *m.*

Or I may extend them from the ſine of 90 *gr.* vnto the ſine of 60 *gr.* the complement of the inclination of the meridians, and the ſame extent wil reach from the tangent of 38 *gr.* 30 *m.* the complement of the latitude, vnto the tangent of 34 *gr* 33 *m.* and ſuch is the arke *PR,* the height of the pole aboue the plane.

3 I may extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 24 *gr.* 20 *m.* the declination giuen, and the ſame extent will reach from the tangent of 38 *gr.* 30 *m.* the complement of the latitude, vnto the tangent of 18 *gr.* 8 *m.* and ſuch is the arke *Z R*, the diſtance of the ſubſtylar from the meridian.

4 That I may finde the diſtance of each houre-line from the ſubſtylar, I conſider the angle of inclination of the meridians *R P Z*, and there ſee how that *P Z* the meridian of the place, which is the houre of 12, being 30 *gr.* diſtance from *PR* the meridian of the plane, and that one face of the plane being open to the Southweſt, and the other to the Northeaſt, this meridian of the plane falleth to be the ſame with the houre of 2, (otherwiſe with the houre of 10:) therefore allowing 15 *gr.* for an houre, the houre of 1, *R P O* will be 15 *gr.* and *R P X* the houre of 11, 45 *gr.* diſtant from *P R* the proper meridian of the plane: and ſo I gather the inclination of the reſt of the houre-circles toward this meridian, according to their angles at the pole, as in this Table.

Then taking my compaſſes in my hand, I extend them from the ſine of 90 *gr.* vnto the ſine of 34 *gr.* 33 *m.* the height of the pole aboue the plane, and finde them to reach in the line of tangents from 15 *gr.* the inclination of the houre of 1, to 8 *gr.* 38 *m.* for the arke of 1, from the ſubſtylar, and from 30 *gr.* vnto 18 *gr.* 8 *m.* for the houre of 12, agreeable to the third *Prop.* and from 45 *gr.* vnto 29 *gr.* 33 *m.* for the houre of 11, and ſo the reſt, which I alſo ſet downe in a Table.

Hor.	Ang. Po		Ar. Pla.	
	Gr.	M.	Gr	M.
8	90	0	90	0
9	75	0	64	42
10	60	0	44	30
11	45	0	29	33
12	30	0	18	8
1	15	0	8	38
2	*Merid*		*ſubſtyl*	
3	15	0	8	38
4	30	0	18	8
5	45	0	29	33
6	60	0	44	30
7	75	0	64	42
8	90	0	90	0

Theſe arks being thus found, wil ſerue for the drawing of the houre-lines, both on the Southweſt face, and the Northweſt face of the plane.

For comming to the plane,

1 By the help of a thread and plummet I draw a verticall

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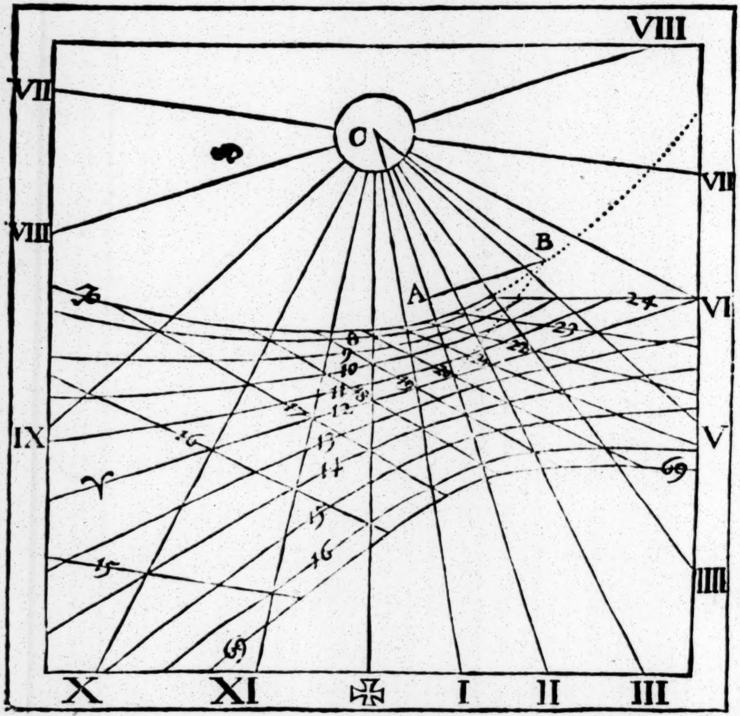
line, ſeruing both for the meridian of the place and the houre of 12.

2 In this meridian line I make choice of a center at *C*, in the vpper part of the line, if it be the South face, as here we ſuppoſe it, that the ſtyle may haue roome to point downward; but in the lower part of the line, if it be the North face of the plane; for there the ſtyle muſt point vpward: and vpon this center I deſcribe an occult circle, repreſenting the declining verticall belonging to the plane.

3 I find a chord of 18 *gr.* 8 *m.* the diſtance of the ſubſtylar from the meridian of the place, and inſcribe it into this cir-



cle,from the meridian vnto *A* toward the right hand, becauſe in this example the meridian of the plane falls among the houres after noone, (for otherwiſe it muſt haue been inſcribed toward the left hand) and there I draw the line *C A* ſeruing for the ſubſtylar.

4 According to the Table of the arkes of the plane from the ſubſtylar,I find a chord of 8 *gr.*38 *m.* and inſcribe it into this circle, from the ſubſtylar toward the meridian, for the houre of 1.In like maner a chord of 29 *gr.*23 *m.*for the houre of 11, and a chord of 44 *gr.*30 *m.* for the houre of 10, and ſo for the reſt of the houres, their halues and quarters.

5 I draw right lines through the center and the termes of theſe chords,and theſe lines ſo drawne are the houre-lines required.

Laſtly, I ſet vp the ſtyle ouer the ſubſtylar,ſo as it may cut the plane in the center, and there make an angle with the ſubſtylar of 34 *gr.*33 *m.* according to the height of the pole aboue the plane ; ſo it ſhall repreſent the axis of the world, and be truly placed for caſting of the ſhadow vpon the houre-lines in this declining plane.

A ſecond example.

After the like maner if in our latitude an vpright plane ſhall decline 85 *gr.*from the prime verticall, the one face of it being open to the Northweſt, and the other to the Southeaſt,we may in ſome ſort repreſent it by the verticall *QZH*, and then working as before.

1 The angle *Z P T*,the inclination of the two meridians will be found to be 86 *gr.*5 *m.* ſo that *P T* the meridian of this plane,will here fall between the houre-circles of 6 and 7 from the meridian.

2 The arke *P T* the height of the pole aboue the plane will be onely 3 *gr.*6 *m.*

3 The arke *Z T* the diſtance of the ſubſtylar from the meridian 38 *gr.* 23 *m.*

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Hor.	Ang. Po		Ar. Pla.		C F		C G	
	Gr.	M.	Gr.	M	In.	Par.	In.	Par.
12	86	5	38	23	91	08	79	21
	78	35	15	3	30	92	26	89
11	71	5	9	6	18	42	16	02
	63	35	6	13	12	52	10	89
10	56	5	4	36	9	25	8	05
9	41	5	2	42	5	43	4	72
8	26	5	1	31	3	05	2	65
7	11	5	0	36	1	20	1	05
	Merid.		*Subſtyl*		0	0	0	0
6	3	55	0	13	0	44	0	38
5	18	55	1	4	2	15	1	86
4	33	55	2	5	4	18	3	64
3	48	55	3	33	7	13	6	20
2	63	55	6	20	12	77	11	10
	71	25	9	10	18	56	16	14
1	78	55	15	28	31	82	27	67
	86	25	40	55	99	67	86	68

4 The Table of the angles at the pole will be alſo gathered, by comparing the meridian of the plane with the reſt of the houre-circles. For the angle *TPZ* betweene *PT* the meridian of the plane, *PZ* the meridian of the place, and the houre of 12, being 86 *gr.* 5 *m.* allowing 15 *gr.* for an houre, the houre of 11½ wil be 78 *gr.* 35 *m.* and the houre of 11 71 *gr.* 5 *m.* diſtant from the meridian of the plane; & ſo the reſt of the houres, as in the ſecond columne of this Table.

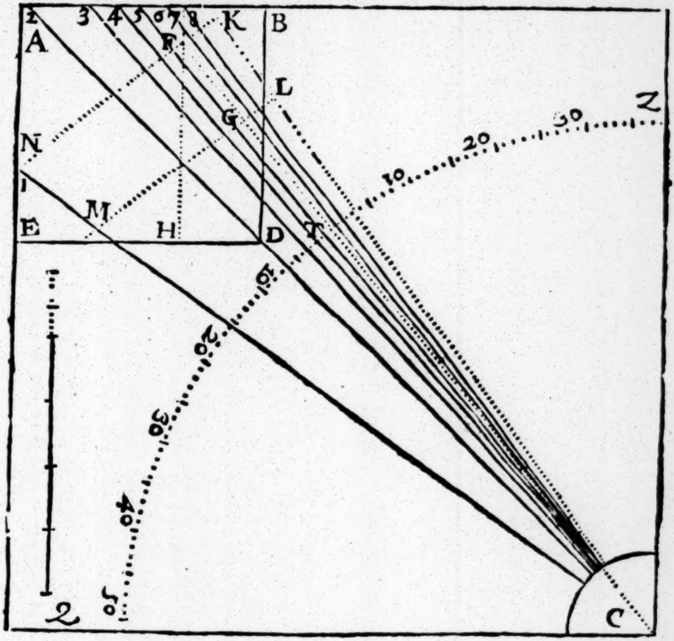
Then hauing the height of the pole aboue the plane, and theſe angles at the pole; the arkes of the plane, betweene the ſubſtylar and the houre-circles, will be found as in the third columne.

Theſe arks being found, will ſerue for the drawing of the houre-lines on either face of this plane.

1 By the help of a thread and plummet I draw *ZC* a verticall plane, ſeruing both for the meridian of the place and the houre of 12.

2 In this meridian line I make choice of a center in the vpper part of the line, if it had been the Southerne face of the plane, but here in *C* the lower part of the line, becauſe we ſuppoſed it to be the Northweſt face of the plane, and the ſtyle muſt point vpward; and vpon this center I deſcribe an occult circle repreſenting the declining verticall belonging to this plane.

3 I finde a chord of 38 *gr.* 23 *m.* the diſtance of the ſubſtylar



ſtylar from the meridian of the place, and inſcribe it into this circle, from *Z* in the meridian, vnto *T* toward the left hand, acording as the proper meridian *PT* falls in the fundamentall Diagram; and here I draw the line *CT* ſeruing for the ſubſtylar.

4 The ſubſtylar being drawne, I may inſcribe the chords of the arks of the plane from the ſubſtylar, and draw the houre-lines, and ſet vp the ſtyle as in the former plane.

Or the arks of the plane from the ſubſtylar being found as before, we may draw the houre-lines vpon the plane otherwiſe then by chords. For hauing drawne the houre-lines as in the laſt figure, vpon paper or paiſt-boord, we ſhall find the

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the moſt part of them, in this and ſuch like planes that haue greater declination, to fall ſo cloſe together, that they can hardly be diſcerned: wherefore to draw them at large to the beſt aduantage of the plane, I leaue out the center, and draw them by tangents, as in the polar plane.

1 I conſider the length and bredth of the plane whereon I am to draw the houre-lines, which I ſuppoſe to be a ſquare, whoſe ſide is 36 inches, and find that the little ſquare *A B D E* wil contain both the ſubſtylar and all thoſe houre-lines which are required in the great ſquare *A Z C Q.*

2 I draw two parallell lines *F N*, *G M*, croſſing the ſubſtylar at right angles in the points *F* & *G*, ſo as they may beſt croſſe all the houre-lines, and yet the one be diſtant from the other as far as the plane will giue me leaue; and I find by the ſight of the figure that if *AB* the ſide of the leſſer ſquare ſhall be 36 inches, the line *C F* will be about 115 inches, and the line *C G* about 100 inches, and therefore *FG* 15 inches. Againe, that the point *F* will fall about 6 inches below the vpper horizontall ſide *A B*, and about 12 inches from the next verticall ſide *B D*; for I need not here ſtand vpon parts.

3 Becauſe theſe two parallell lines are tangent lines in reſpect of circles drawne vpon the ſemidiameters *C F*, *C G*, and ſuch tangents as belong to the arkes of the plane, betweene the ſubſtylar and the houre-lines, the proportion will hold,

As the tangent of 45 *gr.*
to the tangent of the arke of the plane:
So the length of the ſemidiameter
to the length of the tangent line.

As for example, the arke of the plane betweene the ſubſtylar and the houre of 1, is 15 *gr.* 28 *m.* in the former Table, the ſemidiameter *C F* 115 inches, and the ſemidiameter *C G* 100 inches: wherefore I extend the compaſſes from the tangent of 45 *gr.* vnto the tangent of 15 *gr.* 28 *m.* the ſame extent will reach from 115 in the line of numbers vnto 31,82, which ſhewes the length of the tangent line betweene *F* in the ſubſtylar and the houre-line of 1, to be 31 inches, 82 *cent.*

or parts of 100. Againe, the ſame extent will reach from 100 vnto 27, 67; and ſuch is the length of the leſſer tangent from *G* to the houre of 1.

The like reaſon holds for the length of the other tangents from the ſubſtylar to the reſt of the houres, as in the Table; as alſo for the height of the ſtyle aboue theſe tangent lines; and ſo the angle of the ſtyle aboue the plane being 3 *gr.*6 *m.* the height *FK* will be found to be 6 inches 23 *cent.* and the height *GL* 5 inches 42 *cent.*

Where the Reader may obſerue, that if the extent from the tangent of 45 *gr.* to the tangent of 3 *gr.*6 *m.* or to 115 in the line of numbers, be too large for his compaſſes, he may vſe the tangent of 5 *gr.*43 *m.* in ſtead of the tangent of 45 *gr.* as I noted before *Pag.*100.

4 Hauing found theſe lengths and heights, and ſet them downe in a Table, I come to the plane here reſembled by the leſſer ſquare *A B D E*, where I begin with an occult verticall *F H*, about 12 inches from the ſide *B D*, and vpon the center *F*, about 6 inches below the ſide *A B* deſcribe an occult arke of a circle.

5 Into this arke I firſt inſcribe a chord of 38 *gr.*23 *m.* the diſtance of the ſubſtylar from the meridian, to make the angle *H F G* equall to the *Z C T*; ſo the line *F G* ſhall be the ſubſtylar: and then another chord of 51 *gr.*37 *m.* the complement of this diſtance, to make vp the right angle *G F N*; ſo the line *F N* ſhall be the greater of the two tangent lines before mentioned.

6 I ſet off 15 inches from *F* vnto *G*, toward the center, and through *G* draw the leſſer tangent line *G M* parallell to the former.

7 Theſe two occult tangent lines being thus drawne, I looke vnto the former Table for the houre of 1, and there finde the arke of the plane betweene the ſubſtylar and the houre of 1, to be 15 *gr.*28 *m.* and the length belonging to it in the greater tangent line to be 31 inches 82 *cent.* in the leſſer tangent line 27 inches 67 *cent*: wherefore I take out 31 inches 82 parts, and pricke them downe in the greater

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tangent from *F* to *N*, and then 27 inches 67 parts, and prick them downe in the leſſer tangent from *G* to *M*, and draw the *M N* for the houre of 1, which if it were produced would croſſe the ſubſtylar *F G* in the center *C*, and there make the angle *F C N* 15 *gr.* 28 *m.* The like reaſon holdeth for the drawing of all the reſt of the houre-lines.

Laſtly I ſet vp the ſtyle right ouer the ſubſtylar, ſo as the height *F K* may be 6 inches 23 *cent.* and the height *G L* 5 inches 42 *cent.* then ſhall *K L* repreſent the axis of the world, and if it were produced would croſſe the ſubſtylar *F G* in the center *C*, and there make the angle *F C K* to be 3 *gr.* 6 *m.* and ſo be truly placed for caſting of the ſhadow vpon the houre-lines in this declining plane.

CHAP. VIII.

To draw the houre-lines in a meridian inclining plane.

ALl thoſe planes wherein the horizontall line is the ſame with the meridian line, are therefore called meridian planes: if they be right to the horizon, they are called by the generall name of meridian planes without farther addition, and are deſcribed before: if they leane to the horizon, they are then called meridian incliners.

Theſe may incline either to the Eaſt part of the horizon, or to the Weſt, and each of them hath two faces, the vpper toward the zenith, the lower toward the Nadir, wherein knowing the latitude of the place, and the inclination of the plane to the horizon, we are to conſider

1 The inclination of the meridian of the plane to the meridian of the place.
2 The height of the pole aboue the plane.
3 The diſtance of the ſubſtylar from the meridian.
4 The diſtance of each houre-line from the ſubſtylar.

And all theſe foure are repreſented in the fundamentall

Diagram, as in this example.

In our latitude of 51 *gr.*30 *m.* a meridian plane inclineth Eaſtward 50 *gr*; theſe 50 *gr.* I number in the verticall circle from *E* vnto *G*, according to the inclination of the plane, and there draw the arke *S G N* repreſenting the plane propoſed. Then I let downe a perpendicular arke *P V* from the pole to the plane, ſeruing for the meridian of the plane, ſo haue I a rectangle triangle *P V N*, wherein the baſe *P N* is the height of the pole aboue the North part of the horizon, and the angle *P N V* the complement of the inclination to the horizon; and theſe being knowne,

1 I may finde the angle *N P V* of inclination of the two meridians. For

As the coſine of the latitude
is to the ſine of 90 *gr.*
So the tangent of inclination to the horizon,
to the tangent of inclination of meridians.

Extend the compaſſes from the ſine of 38 *gr.*30 *m.* the complement of the latitude, vnto the ſine of 90 *gr.* the ſame extent will reach from the tangent of 50 *gr.*0 *m.* the inclination of the plane to the horizon, vnto the tangent of 62 *gr.* 25 *m.* and ſuch is the inclination of the meridian of the plane to the meridian of the place; which being reſolued into time, doth giue about 4 houres and 10 *m.* from the meridian, for the place of the ſubſtylar among the houre-lines.

2 The height of the pole aboue the plane is here repreſented by the quantitie of the arke of the proper meridian *PV*, betweene the pole and the plane, and may be knowne by that which we haue giuen in the former triangle *P V N*. For

As the ſine of 90 *gr.*
to the ſine of the latitude:
So the coſine of the inclination to the horizon,
to the ſine of the height of the pole aboue the plane.

Extend the compaſſes from the ſine of 90 *gr.* vnto 51 *gr.* 30 *m.* the ſine of the latitude, the ſame extent wil reach from

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the ſine of 40 *gr.* the complement of the inclination of the plane to the horizon, vnto the ſine of 30 *gr.* 12 *m.*

Or as the ſine of 90 *gr.*
to the coſine of inclination of meridians:
So the tangent of the latitude
to the tangent of the height of the pole aboue the (plane.

Extend the compaſſes from the ſine of 90 *gr.* vnto the tangent of 51 *gr.* 30 *m.* the latitude of the place, the ſame extent will reach from the ſine of 27 *gr.* 35 *m.* the complement of the inclination of the two meridians, vnto the tangent of 30 *gr.* 12 *m.* And ſuch is *PV* the height of the pole aboue the plane, and ſuch muſt be the height of the ſtyle aboue the ſubſtylar.

3 The diſtance of the ſubſtylar from the meridian is here repreſented by *NV* the arke of the plane betweene the two meridians, and may be found by that which we haue giuen at the firſt in the former triangle *PVN*. For

As the ſine of 90 *gr.*
to the ſine of the inclination to the horizon:
So the tangent of the latitude
to the tangent of the ſubſtylar from the meridian.

Extend the compaſſes from the ſine of 90 *gr.* vnto the tangent of 51 *gr.* 30 *m.* the latitude of the place, the ſame extent will reach from the ſine of 50 *gr.* the inclination of the plane to the horizon, vnto 43 *gr.* 55 *m.* And ſuch is the arke *NV* the diſtance of the ſubſtylar from the meridian.

4 The diſtances of the houre-lines from the ſubſtylar, are here alſo repreſented by thoſe arkes of the plane, which are here intercepted betweene the proper meridian and the houre-circles, and may be found by that which we haue giuen in the triangles made by the plane, with his proper meridian and the houre-circles. For the angle at *V*, betweene the plane and the proper meridian, is well knowne to be a right angle, and the ſide *PV* is the height of the pole aboue the plane, and the angles at the pole betweene the proper meridian and the houre-circles are eaſily gathered into a Table.

ble. The angle *VPN* betweene *VP* the proper meridian of the plane, and *PN* the generall meridian of the place being *62 gr. 25 m.* the angle between the proper meridian and the circle of the houre of 11, will be *77 gr. 25 m.* and the angle belonging to the houre of 1, *47 gr. 25 m*, and ſo the reſt of the angles at the pole. Then

Hor.	Ang.Po.		Arc.Pla.	
	Gr.	M.	Gr.	M.
11	77	25	66	4
12	62	25	43	55
1	47	25	28	41
2	32	25	17	43
3	17	25	8	58
4	2	25	1	13
	Merid		*Subſtyl*	
5	12	35	6	26
6	27	35	14	44
7	42	35	24	48
8	57	35	38	23
9	72	35	58	3
10	87	35	85	12

As the ſine of *90 gr.*
to the ſine of the pole aboue ẏ plane:
So the tangent of the angle at the pole,
to the tangent of the houre-line
from the ſubſtylar.

Wherefore I extend the compaſſes from the ſine of *90 gr.* vnto the ſine of *30 gr. 12 m.* the height of the pole aboue the plane, and I finde the ſame extent to reach in the line of tangents from *77 gr 25 m.* vnto *66 gr. 4 m.* for the diſtance belonging to the houre of 11; and from the tangent of *62 gr. 25 m.* to *43 gr. 55 m.* for the houre of 12. as when I found the diſtance of the ſubſtylar from the meridian. And ſo for the reſt of the arks of plane betweene the ſubſtylar and the houre-circles, as in the Table.

Theſe arks being thus found, wil ſerue to draw the houre-lines on either ſide of this plane: but ſuppoſing it to be the vpper ſide,

1 I draw the horizontall line *CN*, ſeruing for the meridian and houre of 12.

2 In this line I make choice of a center at *C*, and thence deſcribe an occult arke of a circle repreſenting the plane propoſed.

3 I find a chord of *43 gr. 55 m.* the diſtance of the ſubſtylar from the meridian, and inſcribe it into this circle from *N* vnto *A*, according as I finde the proper meridian *PV* to fall in the fundamental diagram, and there I draw the line *CA* ſeruing for the ſubſtylar.

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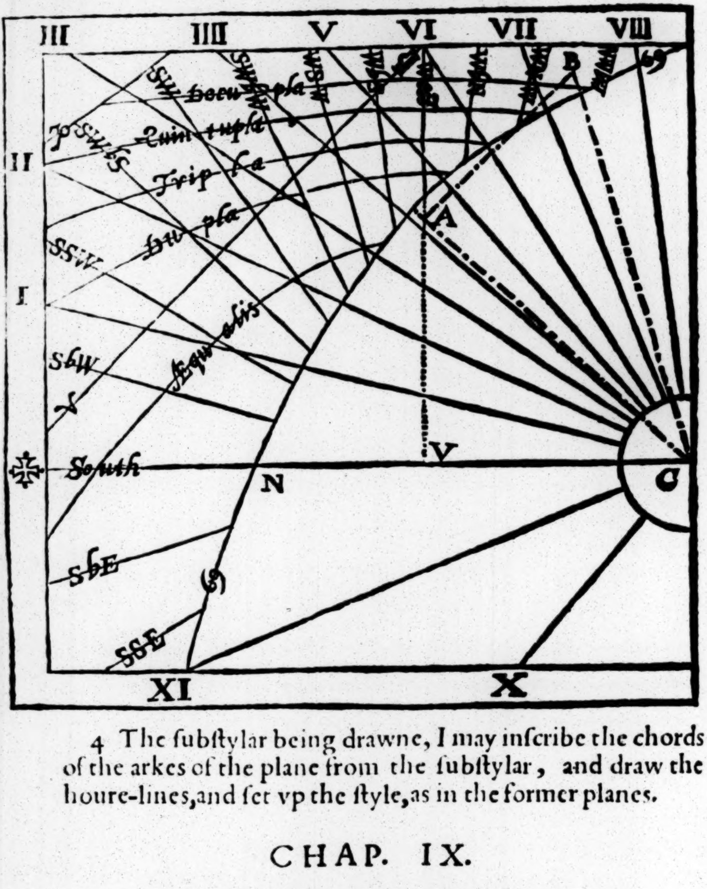

4 The ſubſtylar being drawne, I may inſcribe the chords of the arkes of the plane from the ſubſtylar, and draw the houre-lines, and ſet vp the ſtyle, as in the former planes.

CHAP. IX.

To draw the houre-lines in a polar declining Plane.

Thoſe planes wherein a line may be drawne parallell to the axis of the world, are called polar planes, becauſe that line pointeth vnto the poles, and theſe planes are always parallell to ſome one of the houre-circles. If they be parallell to the houre of 6, they are called direct polar planes; if to the houre of 12, they are called meridian planes; and both theſe are deſcribed before: if to any other of the houre-circles, they are then called by the name of polar declining planes, becauſe of their inclining to the pole, and declining from the verticall.

Theſe kind of planes may be knowne in this ſort: Firſt conſider the inclination of the plane to the horizon, which in theſe parts of the world muſt alwayes be Northward, and more then the latitude of the place. Then find the declination from the verticall. Theſe two being knowne, if the proportion hold,

As the ſine of 90 *gr.*
to the coſine of the declination:
So the tangent of the inclination
to the tangent of the latitude;

it is then a polar declining plane, otherwiſe not.

For example, in our latitude of 51 *gr.* 30 *m.* a plane is propoſed declining from the verticall 65 *gr.* 40 *m.* and inclining Northward 71 *gr.* 51 *m.* the vpper face being open to the Southeaſt, and the lower to the Northweſt. If I number thoſe 65 *gr.* 40 *m.* in the horizon of the fundamentall diagram from *E* vnto *Q*, and draw the line *H C Q*, it ſhall repreſent the horizontall line of the plane; then croſſing it at right angles with the plane *B Z D* drawne through the zenith, I number 71 *gr.* 51 *m.* for the inclination from *D* vnto *R*, and there draw the circle *H R Q*, this circle ſo drawne ſhall repreſent the plane propoſed; and becauſe it alſo paſſeth through the pole, it is therefore a polar plane. But for farther triall I extend the compaſſes from the ſine of 90 *gr.* to the ſine of 24 *gr.* 20 *m.* the complement of the declination, and I find the ſame extent to reach from the tangent of 71 *gr.* 51 *m.* the inclination propoſed, vnto the tangent of 51 *gr.* 30 *m.* which is the true latitude of the place, and therefore it is a polar plane.

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Here then the ſtyle will be parallell to the plane, and the houre-lines parallell one to the other, as in the meridian and direct polar planes. But that we may know how to draw the houre-lines, and where to place the ſtyle, we are to conſider

1 *The arke of the plane betweene the horizon and the houre lines.*

In a meridian plane the arke betweene the horizon and the houre lines, is alwayes equall to the latitude of the place; in a direct polar it is an arke of *90gr*; in theſe declining polars it is greater then the latitude, and yet leſſe then *90 gr.* and may be knowne in this maner.

As the ſine of *90 gr.*
  to the coſine of the latitude:
So the ſine of the declination
  to the coſine of the arke betweene the horizon and the houre-lines.

Extend the compaſſes from the ſine of *90 gr.* vnto the ſine of 38 *gr.* 30 *m.* the complement of the latitude, the ſame extent will reach from the ſine of 65 *gr.* 40 *m.* the declination propoſed, vnto the ſine of 34 *gr.* 34 *m.* whoſe complement 55 *gr.* 26 *m.* the arke of the plane betweene the horizon and the ſubſtylar, to which all the houre-lines muſt be parallell.

2 *The inclination of the meridian of the plane to the meridian of the place.*

The ſubſtylar in a direct polar plane is alwayes the ſame with the houre of 12: in a meridian plane it is the ſame with the houre-line of 6: in theſe declining polars it muſt be placed betweene 12 and 6, according to the inclination of the meridian of the plane to the meridian of the place, which is thus knowne.

As the ſine of *90 gr.*
  to the ſine of the latitude:

So the tangent of the declination of the plane,
to the tangent of the inclination of meridians.

Extend the compaſſes from the ſine of *90 gr.* to the ſine of *51 gr.30 m.* the latitude of the place, the ſame extent will reach from the tangent of *65 gr.40 m.* the declination propoſed, vnto the tangent of *60 gr.* and ſuch is the angle of inclination betweene the meridian of the place and the proper meridian of the plane, which reſolued into time doth make foure houres; and ſo the ſubſtylar muſt here be placed vpon the houre of 8 in the morning.

This angle being knowne, the reſt of the angles at the pole are eaſily gathered. For if the houre of 12 be *60 gr.* diſtant from the meridian of the plane, the houre of 1 will be *75 gr.* and the houre of 11, will be *45 gr.* diſtant, and the reſt of the houres, as in the Table following. Then comming to the plane,

1 I draw an occult horizontall line *HQ*, wherein I make choice of a center *H*, and deſcribe an occult circle for the horizon of the plane.

2 I find a chord of *55 gr.26 m.* and inſcribe it into this circle, from *Q* vnto *B*, according to the ſituation of the plane; ſo the line *HB* ſhall be the meridian of the plane: and therefore the ſubſtylar and the line *AC* croſſing it at right angles, ſhall be the equator.

3 I conſider the length of the plane, and how many houres I am to draw vpon it, that ſo I may proportion the height of the ſtyle; and I finde by the fundamentall diagram and the former table, that it will containe all the houres from Sun riſing vntill 1 afternoone: and therefore the meridian of the plane falling on the houre of 8 in the morning, there will be foure houres on the one ſide, and fiue on the other ſide of the ſubſtylar. But in all polar planes the height of the ſtyle aboue the ſubſtylar muſt be equall to the diſtance of the third houre from the ſubſtylar, or about $\frac{4}{7}$ of the fourth houre, or little more then $\frac{1}{4}$ of the fift houre, and thereupon I allow the height of this ſtyle to be equall to *CB*, which you may ſuppoſe to be ten inches.

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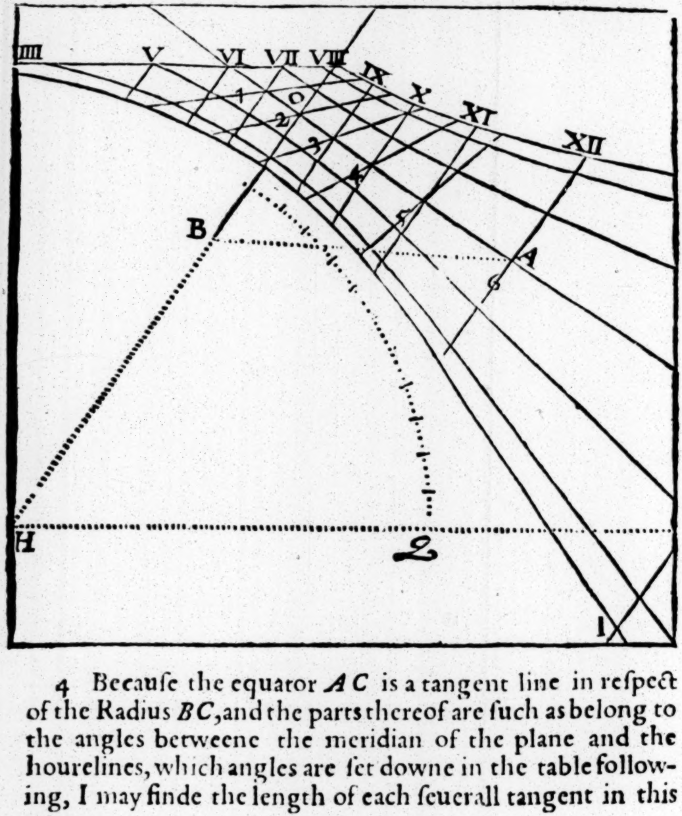

4 Becauſe the equator *A C* is a tangent line in reſpect of the Radius *BC*, and the parts thereof are ſuch as belong to the angles betweene the meridian of the plane and the hourelines, which angles are ſet downe in the table following, I may finde the length of each ſeuerall tangent in this maner.

As the tangent of 45 *gr.*
  is to the tangent of the houre:
So the parts of the Radius,
  to the parts of the tangent line.

The angle *A B C* betweene the meridian of the plane and the houre of 12, the meridian of the place is 60 *gr.* in the

Hor.	An. Po.		Tangent	
	Gr.	M.	In.	Par.
4	60	0	17	32
5	45	0	10	00
6	30	0	5	77
7	15	0	2	68
8	*Meria*		*Subſtyl*	
	3	45		65
	7	30	1	32
	11	15	1	99
9	15	0	2	68
	18	45	3	39
	22	30	4	14
	26	15	4	93
10	30	0	5	77
	33	45	6	68
	37	30	7	67
	41	15	8	77
11	45	0	10	00
	48	45	11	40
	52	30	13	03
	56	15	14	97
12	60	0	17	32
	63	45	20	28
	67	30	24	14
	71	15	29	46
1	75	0	37	32
	78	45	50	27
	82	30	75	96
	86	15	152	57
2	90	0	Infinit	

the former table, and the Radius *B C* is ſuppoſed to be ten inches; whereupon I extend the compaſſes from the tangent of 45 *gr.* vnto the tangent of 60 *gr.* the ſame extent will reach from 10 in the line of numbers, vnto 17. 32, which ſhewes the length of the tangent *A C* betweene the ſubſtylar and the houre of 12, to be 17.32 *cent.* The like reaſon holds for the reſt of the houres.

5 Theſe lengths being thus found and ſet downe in the table, I take out 17 inches 32 *cent.* and prick them in the equator from *C* vnto *A* for the houre of 12, and 37 inches 32 *cent.* and prick them downe for the houre of 1. And ſo the reſt of the houre-points.

6 This done, if I draw right lines through each of theſe points, croſſing the equator at right angles, they ſhall be the houre-lines required: and if I ſet the ſtyle ouer the ſubſtylar, ſo as the edge of it may be parallell to the plane, and the height of it be ten inches equall to the former Radius *B C*, it ſhall repreſent the axis of the world, and be truly placed for caſting of the ſhadow vpon the houre-lines in this declining polar plane.

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CHAP. X.

To draw the houre-lines in a declining inclining plane.

IF a plane ſhall decline from the prime verticall, and incline to the horizon, and yet not lie euen with the poles of the world, it is then called a declining inclining plane.

Of theſe there are ſeuerall ſorts; for the inclination being Northward, the plane may fall between the horizon and the pole, as the circle *B M D* in the fundamentall Diagram; or betweene the zenith and the pole, as *B F D*: or the inclination may be Southward, and ſo be repreſented by *B K D*, it may alſo fall either below the interſection of the meridian and the equator, or aboue it; and each of theſe haue two faces, the vpper toward the zenith, and the lower toward the nadir; wherein hauing the latitude of the place with the declination and inclination of the plane, we are farther to conſider,

1 The arke of the meridian betweene the pole and the plane.
2 The inclination of the plane to the meridian.
3 The arke of the plane betweene the horizon and the meridian.
4 The angle of inclination betweene both meridians.
5 The height of the pole aboue the plane.
6 The diſtance of the ſubſtylar from the meridian.
7 The diſtances of each houre-line from the ſubſtylar.

And all theſe ſeuen may be repreſented in the fundamentall diagram, as in this example.

In our latitude of 51 *gr.* 30 *m.* a plane is propoſed, declining from the verticall 24 *gr.* 20 *m.* and inclining Northward 36 *gr.* the vpper face lying open to the Southweſt, the lower to the Northeaſt. If I number theſe 24 *gr.* 20 *m.* in the horizon from *E* to *B*, and there draw the line BCD, it ſhall repreſent the horizontall line of the plane: then croſſing it at right angles with the plane *HZQ* drawne through the zenith, I number 36 *gr.* for the inclination from *Q* vnto *M*, and there draw the circle B M D, croſſing the meridian in the point *a*; this circle ſo drawne ſhall repreſent the plane propoſed; and becauſe it doth not paſſe through the pole, is therfore no polar, but an ordinary declining inclining plane.

1 The arke of the meridian of the place between the pole and the plane, is here repreſented by *P a*, and may be found by reſoluing the triangle D N *a*, wherein the angle at *N* is knowne to be a right angle, the angle at D is the angle of inclination, the ſide D N the complement of the declination, which being knowne,

As the ſine of 90 *gr.*
to the coſine of declination:
So the tangent of inclination to the horizon,
to the tangent of the meridian betweene the horizon and the plane.

Extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 65 *gr.* 40 *m.* the complement of the declination, the ſame extent will reach from the tangent of 36 *gr.* the inclination propoſed, vnto the tangent of 33 *gr.* 30 *m.* and ſuch is the ark of the meridian *N a* between the horizon and the plane. This arke *N a* being compared with the arke *N P*, which is the eleuation of the pole aboue the horizon, and is here ſuppoſed to be 51 *gr.* 30 *m.* the difference *N a* cometh to 18 *gr.* and ſuch is the arke of the meridian required betweene the pole and the plane.

2 The inclination of the plane to the meridian is here repreſented by the angle N *a* D, and may be found by that which we haue giuen in the former triangle D N *a*. For

As the ſine of 90 *gr.*
to the ſine of the declination from the verticall:
So the ſine of inclination to the horizon,
to the coſine of the inclination to the meridian.

Extend the compaſſes from the ſine of 90 *gr.* vnto the ſine

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of 24 *gr.* 20 *m.* the declination of the plane, the ſame extent will reach from the ſine of 36 *gr.* the inclination giuen, vnto the coſine of 76 *gr.* And ſuch is N *a* D the angle of inclination betweene the plane D *a*, and N *a* the meridian of the place. Or

As the ſine of the arke of the meridian betweene the
horizon and the plane,
is to the ſine of 90 *gr.*
So the cotangent of the declination
to the tangent of the inclination to the meridian.

Extend the compaſſes from the ſine of 33 *gr.* 30 *m.* the ark of the meridian betweene the horizon and the plane, vnto the ſine of 90 *gr.* the ſame extent will reach from the tangent of 65 *gr.* 40 *m.* the complement of the declination vnto the tangent of 76 *gr.* And ſuch is the inclination of the plane to the meridian, the ſame as before.

3 The arke of the plane betweene the horizon and the meridian, is here repreſented by D *a*, and may alſo be found by that which we haue giuen in the former triangle D N *a*.

As the ſine of 90 *gr.*
to the coſine of inclination to the horizon:
So the cotangent of the declination
to the tangent of the plane required.

Extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 54 *gr.* the complement of the inclination of the plane to the horizon, the ſame extent will reach from the tangent of 65 *gr.* 40 *m.* the complement of the declination, vnto the tangent of 69 *gr.* 54 *m.* And ſuch is D *a* the arke of the plane, betweene the horizon and the meridian of the place.

4 The inclination of meridians is here repreſented by the angle *a P b*. For if I let downe a perpendicular *P b* from the pole vnto the plane, this perpendicular ſhall be the meridian of the plane; and we ſhall haue another triangle *a b P*, wherein the angle at *b* is a right angle, becauſe of the perpendicular, the angle at *a* is the inclination of the plane to the meridian of the place, and the ſide *P a* is the arke of the

meridian betweene the pole and the plane, which being knowne,

As the coſine of the arke of the meridian betweene the
pole and the plane
is to the ſine of *90 gr.*
So the cotangent of the inclination to the meridian,
to the tangent of inclination of the meridian of the
plane to the meridian of the place.

Extend the compaſſes from the ſine of *72 gr.* the complement of the arke ***Pa***, betweene the pole and the plane, vnto the ſine of *90 gr.* the ſame extent wil reach from the tangent of **14** ***gr.*** the complement of the inclination of the plane to the meridian, vnto the tangent of **14** ***gr.*** **41** *m.* And ſuch is the angle ***a P b*** of inclination betweene the meridian of the place and the proper meridian of the plane, which reſolued into time, doth make about *59 minutes*, and ſo the ſubſtylar muſt here be placed neare the houre of 1 after noone.

5 The height of the pole aboue the plane is here repreſented by ***Pb***, the arke of the proper meridian between the pole and the plane, and may be found by that which wee haue giuen in the triangle ***a b P***. For

As the ſine of *90 gr.*
to the ſine of the meridian of the place betweene the
pole and the plane:
So the ſine of the inclination to the meridian,
to the ſine of the height of the pole aboue the plane.

Extend the compaſſes from the ſine of *90 gr.* vnto the ſine of 18 *gr.* the arke *P a* of the meridian of the place from the pole to the plane, the ſame extent wil reach from the ſine of ***b a P*** the inclination of the plane to the meridian of the place, vnto the ſine of 17 *gr.* 26 ***m.*** Or

As the ſine of **90** ***gr.***
to the coſine of inclination of meridians:

So

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So the tangent of the meridian of the place betweene
the pole and the plane,
to the tangent of the height of the pole aboue the plane.

Extend the compaſſes from the ſine of *90 gr.* vnto the ſine of *75 gr. 19 m.* the complement of ***a P b*** the inclination of the two meridians, the ſame extent will reach from the tangent of 18 *gr.* the arke ***P a*** of the generall meridian between the pole and the plane, vnto the tangent of 17 *gr. 26 m.* And ſuch is ***Pb*** the height of the pole aboue the plane; and ſuch muſt be the height of the ſtyle aboue the ſubſtylar.

6 This diſtance of the ſubſtylar from the meridian of the place, is here repreſented by ***a b*** the arke of the plane between the two meridians, and may be found by that which we had giuen at the firſt in the former triangle ***a b P***. For

As the ſine of *90 gr.*
to the ſine of the inclination to the meridian:
So the tangent of the meridian of the place betweene
the pole and the plane,
vnto the tangent of the ſubſtylar from the meridian of the place.

Extend the compaſſes from the ſine of *90 gr.* vnto the ſine of 14 *gr.* the complement of *b a* P, the inclination of the plane to the meridian, the ſame extent will reach from the tangent of 18 *gr.* the arke of the generall meridian betweene the pole and the plane, vnto the tangent of 4 *gr.* 30 *m.* And ſuch is the arke of the plane betweene the two meridians; and ſuch muſt be the diſtance from the houre of 12 to the ſubſtylar.

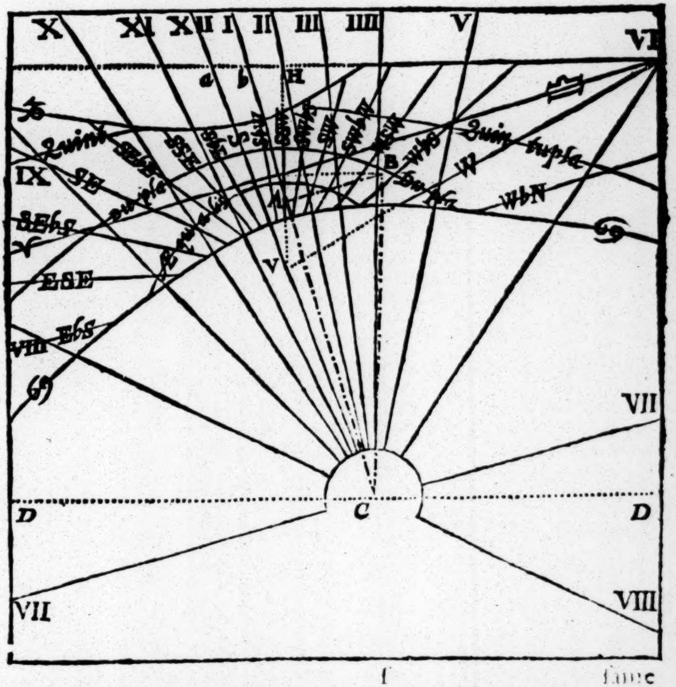
7 The diſtances of the houre-lines from the ſubſtylar, are here alſo repreſented by thoſe arks of the plane, which are

Hor.	Ang. Po.		Arc. Pla.	
	Gr.	M.	Gr.	M.
7	89	41	88	57
8	74	41	47	35
9	59	41	27	9
10	44	41	16	31
11	29	41	9	41
12	14	41	4	30
	Merid.		*Subſtyl*	
1	0	19	0	6
2	15	19	4	42
3	30	19	9	56
4	45	19	16	52
5	60	19	27	45
6	75	19	48	51

are intercepted between the proper meridian and the houre-circles. For in theſe triangles the angle at *b* betweene the plane and the proper meridian is a right angle, the ſide P*b* is the height of the pole aboue the plane, and then the angles at the pole between the proper meridian and the houre-circles being gathered into a table.

As the ſine of *90 gr.*
to the ſine of the pole aboue the plane:
So the tangent of the angle at the pole,
to the tangent of the houre-line from the ſubſtylar.

Extend the compaſſes from the ſine of *90 gr.* vnto the ſine of 17 *gr.* 26 *m.* the height of the pole aboue the plane, the



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ſame extent will reach from the tangent of 14*gr*.41 *m*. the angle at the pole belonging to the houre of 12, vnto the tangent of 4*gr*.30 *m*.for the arke of the plane betweene the ſubſtylar and the houre of 12; and frõ the tangent of 29*gr*. 41 *m*.vnto the tangent of 9*gr*.41 *m*.for the houre of 11, and ſo for the reſt of the arks of the plane between the ſubſtylar and the houre-lines, as in the former table.

Theſe arkes being thus found, will ſerue for the drawing of the houre-lines on either ſide of the plane: but ſuppoſing it to be the vpper ſide, I conſider how the lines doe fall in the fundamentall diagram, and accordingly

1 I draw an occult horizontall line DD, wherein I make choice of the center *C*, and thence draw an occult circle for the horizon of the plane.

2 I finde a chord of 69*gr*.54*m*. the arke of the plane betweene the horizon and the meridian, and inſcribe it into this circle from D vnto *a*, and there draw the line *C a* for the houre of 12.

3 I finde a chord of 4*gr*.30 *m*. the arke of the plane betweene the two meridians, and inſcribe it into this circle from *a* vnto *b*, and there draw the line *C b* for the ſubſtylar.

4 The ſubſtylar being drawne, I may inſcribe the chords of the arkes of the plane from the ſubſtylar, and draw the houre-lines, and ſet vp the ſtyle as in the former planes.

A ſecond example of a Plane falling betweene the pole and the zenith.

In like maner if in our latitude a plane be propoſed declining from the verticall 24*gr*.20 *m*.as before,but inclining to the horizon 75 *gr*.40 *m*.Northward,the vpper face being opẽ to the Southweſt,the lower to the Northeaſt,this plane ſhall be here repreſented by the circle BFD,croſſing the meridian in the point *d*,betweene the pole and the zenith,and the proper meridian of this plane,by the perpendicular arke P *e*.

Then in this triangle DN*d* knowing the ſide D N the complement of the declination, with the angle of inclination to the horizon at D, and the right angle at N, theſe former Canons will giue *N d* the arke of the meridian between the horizon and the plane to be 74 *gr.* 20 *m*; and therfore *Pd* the arke of the meridian betweene the pole and the plane will be 22 *gr.* 50 *m.* the angle *D d N* of the inclination of the plane to the meridian, will be found to be 66 *gr.* 29 *m.* and *D d* the arke of the plane between the horizon and the meridian 83 *gr.* 36 *m.*

Againe, in the triangle *P e d* knowing the ſide *Pd* the arke of the meridian betweene the pole and the plane, with the angle of inclination to the meridian at *d*, and the right at *e*, the angle *d P e* of the inclination of the two meridians will be found to be 25 *gr.* 17 *m.* and *P e* the height of the pole aboue the plane to be 20 *gr.* 50 *m.* and *d e* the diſtance of the ſubſtylar from the meridian about 9 *gr.* 32 *m.*

Declinatiõ	24	20
Inclinatiõ	75	40
Diff. meri.	83	36
diſt. ſubſty	9	32
Alti. Styl.	20	50

Laſtly, hauing found the height of the pole aboue the plane, and gathered the angles at the pole, the arks of the plane from the ſubſtylar to the houre-lines will be as in this table.

Hor.	Ang. Po.		Arc. Pla.	
	Gr.	M.	Gr.	M.
8	85	17	76	56
9	70	17	44	47
10	55	17	27	11
11	40	17	16	43
12	25	17	9	32
1	10	17	3	41
	Merid		*Subſtyl*	
2	4	43	1	40
3	19	43	7	16
4	34	43	13	50
5	49	43	22	46
6	64	43	37	0
7	79	43	62	58

This done, if we conſider how the lines doe fall in the fundamentall diagram, we may there ſee how the North pole is eleuated aboue the lower face, & the South pole aboue the vpper face of the plane, and accordingly make choice of a center, draw the horizontall, ẏ meridian, the ſubſtylar, & the houre-lines, and ſet vp the ſtyle as in the other planes.

A third example of a Plane inclining to the Southward.

If in our latitude a plane were pro-

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poſed declining from the verticall 24 *gr.* 20 *m.* as before, but inclining to the horizon 14 *gr.* 20 *m.* Southward, the vpper face being open to the Northeaſt, the lower to the Southweſt, this plane ſhall be here repreſented by the circle *BKD* croſſing the meridian in the point *f* betweene the equator and the horizon, and the proper meridian of this plane by the perpendicular arke *Pg* let downe from the pole to the plane, neare the houre of 11, at the North part of the horizon, as may partly appeare by the neareſt extent of the compaſſes.

Then in the triangle BS*f*, knowing the ſide *BS* the cōplement of the declination, with the angle of inclination to the horizon at B, and the right angle at *S*, we may find *Sf* the ark of the meridian betweene the horizon and the plane to be 13 *gr.* 6 *m.* And therefore *Pf* the arke of the meridian betweene the pole and the plane to the Southward 115 *gr.* 24 *m.* but 64 *gr.* 36 *m.* to the Northward, the angle B*f*S or D*f*N of the inclination of the plane to the meridian, will be found 84 *gr.* 9 *m*; & B*f* or D*f* the arke of the plane between the horizon & the meridian 66 *gr.* 20 *m.*

Againe, in the triangle *Pgf* knowing the ſide *Pf* the arke of the meridian betweene the pole and the plane, with the angle of inclinatiō to the meridian at *f*, and the right angle at *g*, the angle *fPg* of the inclination of the two meridians wil be found to be 13 *gr.* 27 *m.* and *Pg* the height of the pole aboue the plane, about 64 *gr.* and *fg* the diſtance of the ſubſtylar from the meridian 12 *gr.* 8 *m.*

Hauing found the height of the pole aboue the plane, and gathered the angles at the pole, the arkes of the plane from the ſubſtylar to the houre-lines will be found as in this table.

Declinatiō	24	20
Inclinatiō	14	20
diff. merid	13	27
diſt. ſubſty	12	8
Alt. Styl.	64	0

Hor.	Ang. Po		Arc. Pla.	
	Gr.	M.	Gr.	M.
6	76	33	75	6
7	61	33	58	56
8	46	33	43	30
9	31	33	28	55
10	16	33	14	58
11	1	33	1	25
	Merid		*Subſtyl*	
12	13	27	12	8
1	28	27	25	57
2	43	27	40	23
3	58	27	55	38
4	73	27	71	41
5	88	27	88	15

This done, if we conſider how the lines do fall in the fundamentall diagram, we may there ſee how the North pole is eleuated aboue the vpper face, and the South pole aboue the lower face of this plane, and accordingly make choice of the center, draw the horizontall, the meridian, the ſubſtylar, and the houre-lines, and ſet vp the ſtyle as in the former planes.

CHAP. XI.

To deſcribe the Tropiques and other circles of declination in an equinoctiall Plane.

SVch circles as are parallell to the equinoctiall, and yet fall within the tropiques, may be deſcribed on any plane by help of theſe lines of proportion, but after a different maner, according as the ſtyle ſhall be either perpendicular, or parallel to the plane, or cut the plane with oblique angles.

In an equinoctiall plane where the ſtyle is perpendicular to the plane, the tropiques and other circles of declination will be perfect circles: wherefore conſider the length of the ſtyle in inches and parts, and the declination of the circle which you intend to deſcribe in degrees and minutes, the proportion will hold,

As the tangent of 45 *gr.*
to the length of the ſtyle:
So the cotangent of the parallell,
to the ſemidiameter of his circle.

Suppoſe the length of the ſtyle aboue the plane to be 10 inches, and that it were required to finde the ſemidiameter of the tropique, whoſe declination is knowne to be 23 *gr.* 30 *m*: extend the compaſſes from the tangent of 45 *gr.* vnto the tangent of 66 *gr.* 30 *m.* the ſame extent will reach in the line of numbers from 10 vnto 23, which ſhewes the ſemidiameter of the tropique to be 23 inches. So if the declination be 20 *gr.* the ſemidiameter will be 27 inches 47. *cent*; if

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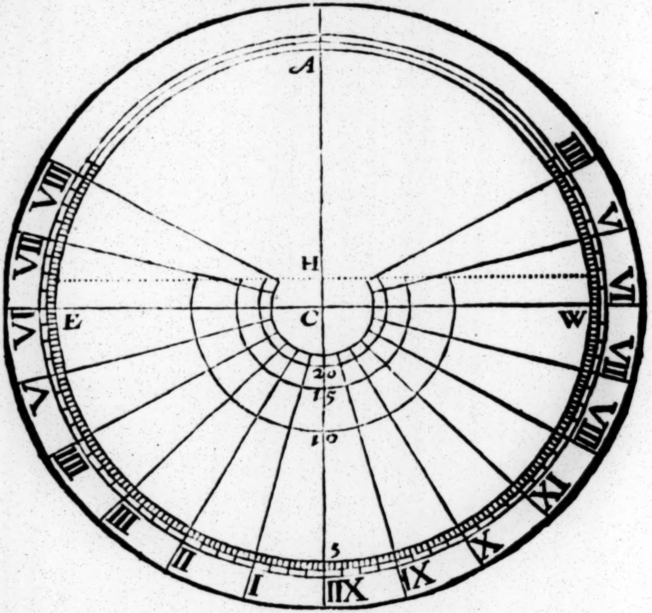

15 *gr.* then 37.32; if 10 *gr.* then 56.71; if 5 *gr.* then 114,30; and ſo in the reſt.

Or if it were required to proportion the ſtyle to the plane,

As the tangent of 45 *gr.*
 to the tangent of the declination:
So the ſemidiameter of the plane,
 to the length of the ſtyle.

As if the ſemidiameter of the greateſt parallell vpon the plane were but ſix inches, and that parallell ſhould be the fift degree of declination: extend the compaſſes from the tangent of 45 *gr.* vnto the tangent of 5 *gr.* the ſame extent wil reach in the line of numbers from 6.00 vnto about 0.53, which ſhewes that the length of the ſtyle muſt be 53 parts of an inch diuided into 100; then the length of the ſtyle being knowne, the ſemidiameter of the other circles will be found as before.

I begin here with the fift parallell, and thence proceed vnto the tropique, becauſe the ſhadow of the reſt neare the equinoctial, would be ouerlong, and the equinoctiall it ſelfe cannot be deſcribed. The parallels of North declination are to be ſet on the North face, and the parallels of South declination on the South face of the plane. Neither need theſe parallels to be drawne in full circles, but onely to the horizontall line, which ſhall be deſcribed in *Cap.* xviij.

Hauing by theſe meanes ſet vp the ſtyle to his true height, and drawne the circles of declination, if we ſhall place the plane ſo as it ſhall make an angle with the horizon equall to the cōplement of the latitude, and then turne it vntil the top of the ſtyle caſt the ſhadow vpon the parallell of declination belonging to ŷ time, the meridian of the plane will ſhew the meridian of the place, and the ſhadow of the ſtyle the houre of the day, without the help of a magneticall needle.

CHAP. XII.

To deſcribe the Tropiques and other circles of declination in a polar plane.

IN all polar planes, whether they be parallel to the meridian or to the circle of the houre of 6, or otherwiſe declining, the equinoctiall will be a right line, but the tropiques and other circles of declination wil be ſections hyperbolicall, and be thus deſcribed.

Conſider the length of the ſtyle, the declination of the parallell, and the angle at the pole betweene the ſubſtylar and the houre-line, whereon you meane to deſcribe the parallell.

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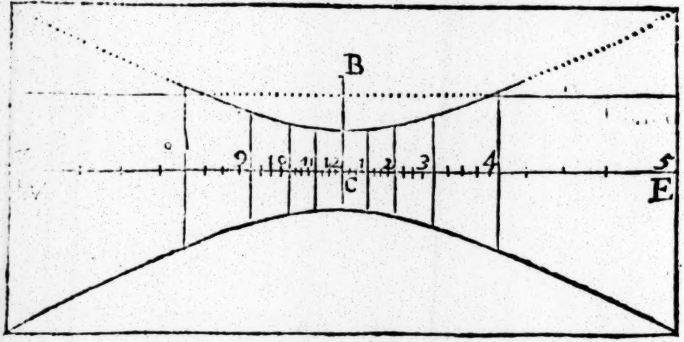
If you would find where the parallels doe croſſe the ſubſtylar;

As the tangent of 45 *gr.*
to the tangent of declination:
So is the length of the ſtyle,
to the diſtance of the parallell from the equinoctiall.

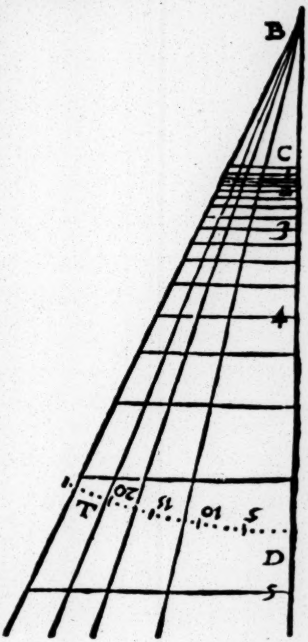


As in the example of the polar plane, where the length of the ſtyle *B C* was found to be 1 inch 61 *cent.* if you deſire to know the diſtance betweene the equinoctiall and the tropique vpon the ſubſtylar line:extend the compaſſes from the tangent of 45 *gr.* vnto the tangent of 23 *gr.* 30 *m.* the ſame extent will reach in the line of numbers from 1.61 vnto 0.70; and therefore the diſtance required is 70 parts of an inch diuided into 100. The like reaſon holdeth for all other parallels of declination croſſing the ſubſtylar.

But if you would finde where the parallels doe croſſe any other of the houre-lines, firſt find the diſtance betweene the axis of the ſtyle and the houre-line, then the diſtance betweene the equinoctiall and the parallell, both theſe may be repreſented in this maner.

On the center *B* and any ſemidiameter *B D* deſcribe an occult arke of a circle, and therein deſcribe a chord of 23 *gr.* 30 *m.*

30 *m.* from *D* vnto *T*, with ſuch other intermediate declinations as you intend to deſcribe on the plane, ſo the line B D ſhall be the equator, and B T the tropique, and the other intermediate lines the lines of declination.



That done, conſider your plane, which for example may be either the meridian or yͤ declining polar plane, wherein hauing drawne both the equator, and the houre-lines as before, firſt take out the height of the ſtylo, and prick that downe in this equator from *B* vnto *C*; then take out all the diſtances, betweene *B* the top of the ſtyle and the ſeuerall points wherein the houre-lines doe croſſe the equator, transfer them into this equator B D from the center *B*, and at the termes of theſe diſtances erect lines perpendicular to the equator, croſſing the lines of declination, and note them with the number of the houre from whence they were taken: ſo theſe perpendiculars ſhal repreſent thoſe houre-lines, and the ſeuerall diſtances between the equator and the lines of declination, ſhall giue the like diſtances between the equator and the parallels of declination vpon your plane. Vpon this ground it followeth,

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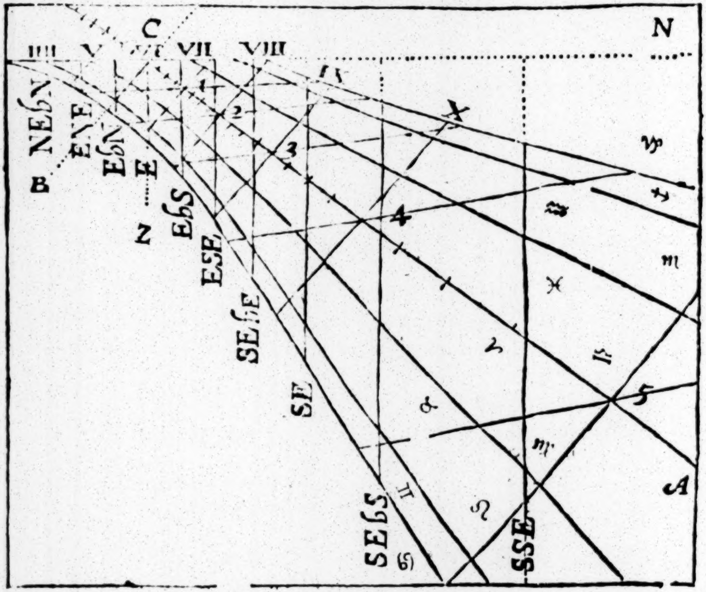
To find the diſtance betweene the axis and the houre-lines.

As the coſine of the houre from the ſubſtylar,
is to the ſine of 90 *gr.*
So the length of the ſtyle,
to the diſtance between the axis and the houre-line.



As if in the former example of the meridian plane, where ***B C*** the height of the ſtyle is ſuppoſed to be 10 inches, it were required to find the diſtance between ***B*** the top of the ſtyle and the point wherein the houre of 11 in the morning doth croſſe the equator, which is here repreſented by ***B*5**, becauſe it is the fift houre from the ſubſtylar, whoſe angle at the pole is 75 *gr*. Extend the compaſſes from the ſine of 15 *gr*. the complement of the fift houre from the ſubſtylar, vnto the ſine of 90 *gr*. the ſame extent wil reach from 10.00 in the line of numbers vnto 38.64; and therefore the diſtance *B* 5 between the axis and the houre-line, is 38 inches & 64 *cent*. and may be called the ſecant of the houre. Then in the rectangle B 5 T, hauing the ſide B 5, and the angle of declination at B.

To finde the diſtance betweene the equinoctiall and the parallell.

As the tangent of 45 *gr*.
to the tangent of the declination:
So the diſtance betweene the axis and the houre-line,
to the diſtance between the equinoctiall and the parallell.

Extend the compaſſes from the tangent of 45 *gr*. vnto the tangent of 23 *gr*. 30 *m*. the declinatiõ of the tropique, ſo the ſame extent will reach in the line of numbers from 38 64, the diſtance between the axis and the fift houre-line vnto 16.80; and therefore the diſtance is 16 inches and 80 *cent*. The like reaſon holdeth for all the reſt, which may be gathered and ſet downe in ſuch a Table as this which followeth.

Wherein I haue ſet downe theſe diſtances for ſeuerall declinations, for 11 *gr*. 30 *m*. for 16 *gr*. 55 *m*. for 20 *gr* 12 *m*. for 21 *gr* 41 *m*. and for the declination of the Tropique 23 *gr*. 30 *m*. which may be applied to the like declinations in all meridian and direct polar planes.

As in the former example of the polar plane, where *B C* the height of the ſtyle is found to be 1 inch 61 *cent*. if it were required to find the diſtance betweene B the top of the ſtyle and the points wherein the houre-lines of 7 in the morning or 5 after noon, do croſſe the equator (which diſtances, I called the ſecants of thoſe houres.) either you may extend the compaſſes from the ſine of 15 *gr*. the complement of the houre from the ſubſtylar vnto the ſine of 90 *gr*. ſo the ſame

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Hor.	Ang.Po		Tangent		Secant.		11	30	16	55	20	12	21	41	23	30
	Gr.	M.	In.	Par.	In.	Pa.	In.	Pa.	In.	Pa.	In.	Pa	In.	Pa.	In.	Pa
0	0	0	0	0	10	0	2	3	3	4	3	08	3	98	4	35
	3	45	0	65	10	02	2	04	3	05	3	69	3	99	4	36
	7	30	1	32	10	09	2	05	3	07	3	71	4	01	4	39
	11	15	1	99	10	20	2	07	3	10	3	75	4	05	4	43
1	15	0	2	68	10	35	2	10	3	15	3	81	4	12	4	50
	18	45	3	39	10	56	2	15	3	21	3	89	4	20	4	59
	22	30	4	14	10	82	2	20	3	29	3	99	4	30	4	70
	26	15	4	93	11	15	2	26	3	39	4	10	4	43	4	85
2	30	0	5	77	11	55	2	34	3	51	4	24	4	60	5	02
	33	45	6	68	12	03	2	44	3	66	4	42	4	78	5	23
	37	30	7	67	12	60	2	56	3	83	4	64	5	02	5	48
	41	15	8	77	13	30	2	70	4	05	4	89	5	29	5	78
3	45	0	10	00	14	14	2	87	4	30	5	20	5	63	6	15
	48	45	11	40	15	17	3	08	4	62	5	58	6	03	6	00
	52	30	13	03	16	43	3	34	5	00	6	04	6	54	7	14
	56	15	14	97	18	00	3	66	5	48	6	62	7	00	7	83
4	60	0	17	32	20	00	4	07	6	08	7	36	7	95	8	70
	63	45	20	28	22	61	4	60	6	88	8	32	9	00	9	83
	67	30	24	14	26	13	5	31	7	95	9	61	10	39	11	36
	71	15	29	45	31	11	6	33	9	47	11	45	12	37	13	53
5	75	0	37	32	38	64	7	86	11	74	14	20	15	36	16	80
	78	45	50	27	51	26	10	43	15	60	18	89	20	38	22	28
	82	30	75	96	76	61	15	58	23	32	28	19	30	47	33	31
	86	15	152	57	152	90	31	10	46	54	56	20	60	81	66	48
6	90	0	Infin.		Infin.		Infin.		Infin.		Infin.		Infin.		Infin.	

extent wil reach in the line of numbers from 1.61 the length of the ſtyle, vnto 6 21, according to the former Canon. Or elſe you may make vſe of the former Table, extending the compaſſes in the line of numbers from 10.00 the length of the ſtyle in the Table, vnto 1.61 the length of the ſtyle belonging to your plane, ſo the ſame extent ſhall reach from from 38.64 the ſecant in the Table, vnto 6.21, and ſuch is your ſecant required, the diſtance betweene the top of the ſtyle and the point of interſection, wherein the fift houre-line from the ſubſtylar doth croſſe the equator.

Again, the ſame extent will reach from 16.80 the diſtance in the Table belonging to the fift houre-line betweene the equator and the parallell of 23 *gr.* 30 *m.* declination, vnto 2.70 for the like diſtance vpon your plane; and ſo for the reſt, which may be gathered and ſet downe in a Table.

Hor.		An. Po		Tang		Secant		Trop.	
		Gr.	M	In.	P.	In.	P.	In	P
12		0	0	0	0	1	61	0	70
11	1	15	0	0	43	1	63	0	72
10	2	30	0	0	93	1	85	0	80
9	3	45	0	1	61	2	27	0	99
8	4	60	0	2	79	3	22	1	40
7	5	75	0	6	00	6	21	2	70

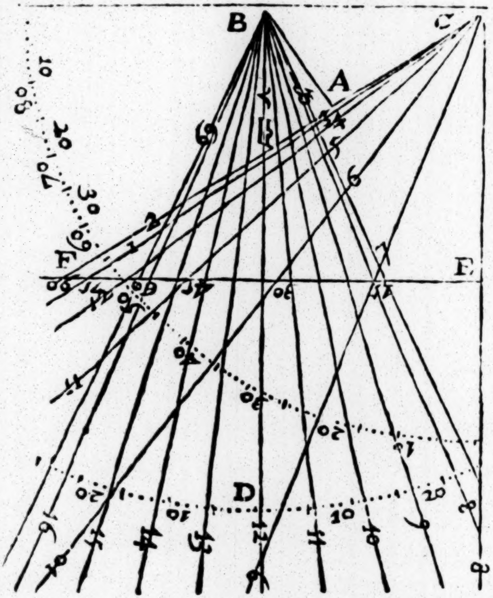
That done, and the equator drawne as before, if you would draw the tropiques in the polar plane, looke into the Table, and take 70 *cent.* out of the line of inches, and pricke them downe in the ſubſtylar on either ſide of the equator, and ſo 72 *cent.* on the firſt houre, and 80 on the ſecond houre, and 2 inches 70 *cent.* on the fift houre from the ſubſtylar, and the reſt of theſe diſtances on their ſeuerall houre-lines, and then draw a crooked line through all theſe points, ſo as it makes no angles, the line ſo drawne ſhall be the Tropique required. In like maner you may draw any other parallell of declination.

CHAP. XIII.

To deſcribe the Tropiques and other circles of declination in ſuch a Plane as is neither equinoctiall nor polar.

IN Planes neither equinoctiall nor polar, the equator will be a right line, the tropiques and other parallels of declination will be conicall ſections, ſome of them parabolicall, ſome ellipticall, but the moſt of them hyperbolicall.

To finde the points of interſection of theſe parallels with the houre-lines, wee are to conſider, firſt the length of the axis of the ſtyle in inches and parts of inches; ſecondly the height of the ſtyle aboue the plane; thirdly the angles at the pole betweene the proper meridian and the houre-circles. Theſe being knowne, will help vs to find, firſt the angle betweene the axis and the houre-lines on the plane; and then the diſtance between the center and the parallels: both theſe may be repreſented in this maner.



Let the triangle *A B C* be made equal to the ſtyle belonging to your plane, *A C* the ſubſtylar, *B C* the axis of the ſtyle, AB the length of the ſtyle perpendicular to the plane. Then hauing drawn the line BD perpēdicular to the axis on the cēter B, & any ſemidiameter B D deſcribe an occult ark of a circle, and therein inſcribe a chord of 23 *gr.* 30 *m.* from *D* vnto *T*, on either ſide of the line, with ſuch other intermediate declinations as you intend to deſcribe on the plane, ſo the perpendicular B D ſhall be the equator, and B T the tropiques, and the other intermediate lines the parallels of declination. Wherefore you may take out the diſtance *C* ♈ from the center to the equator, and pricke it downe on the ſubſtylar of your plane from the center at *C* vnto ♈, ſo the line drawne through ♈ perpendicular to your ſubſtylar, ſhal be the equator of your plane.

That done, take the diſtance of each houre-line betweene the center and the equator of your plane, and pricke them downe in the equator of this figure, from the center at *C*, noting the place, where they croſſe the equator, with the number belonging to the houre, and drawing the houre-lines from *C* through the lines of declination.

Or hauing the *Sector* you may draw an occult line *C E* perpendicular to the axis B C, and therein pricke downe the tangent of the height of the ſtyle aboue the plane, from *C* vnto *E*. Then draw the line *E F* parallell to the axis, croſſing the ſubſtylar produced in the point *F*, this line *E F* will be the line of ſines vpon the *Sector*, and therein you may pricke downe the ſines of the complement of the angles at the pole from *E* toward *F*, and draw the houre-lines by thoſe points through the lines of declination, ſo the angles at *C* betweene the axis B C and thoſe houre-lines, ſhall be the angles betweene the axis of your ſtyle and the houre-lines on your plane, and the ſeuerall diſtances betweene the point *C* and the lines of declination, ſhall giue you the like diſtances betweene the center, and the parallels of declination vpon the houre-lines in your plane. Vpon this ground it followeth,

1 *To proportion the ſtyle vnto the plane.*

Conſider the height of the ſtyle aboue the plane, and the length of the ſubſtylar betweene the center and the place which you intend for the tropique. If it be the tropique which is fartheſt from the center, adde 113 *gr.* 30 *m*: if the nearer tropique, adde 66 *gr.* 30 *m.* vnto the height of the ſtyle, the remainder vnto 180 *gr.* ſhall giue you the altitude of the Sun

aboue

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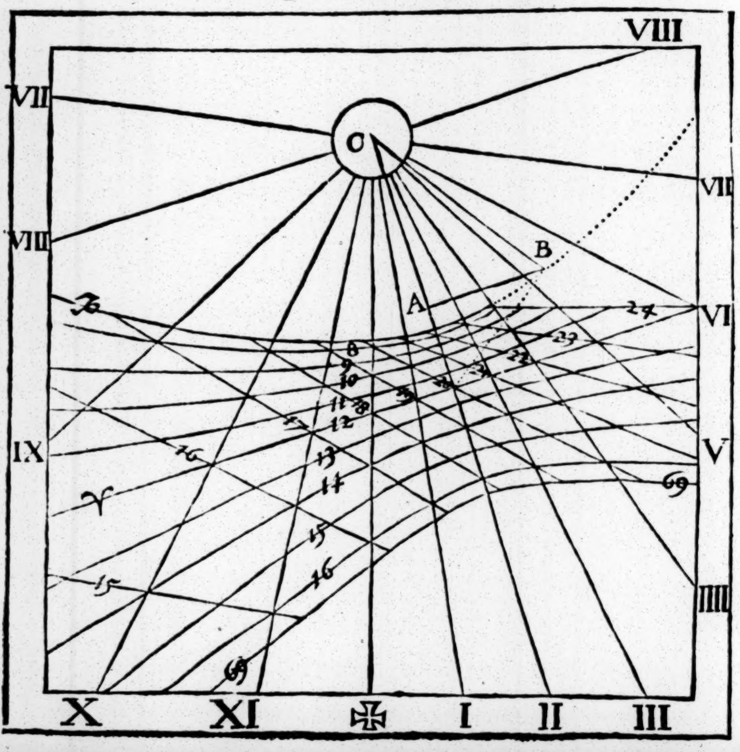
aboue the plane when he commeth to that tropique. As in our latitude the height of the ſtyle aboue an horizontall plane is 51 *gr.* 30 *m.* adde vnto this 113 *gr.* 30 *m.* the ſumme is 165 *gr.* which being taken out of 180 *gr.* the remainder wil be 15 *gr.* and ſuch is the altitude of the Sun aboue this plane when he commeth to be in the Winter tropique: but if you adde 66 *gr.* 30 *m.* vnto 51 *gr.* 30 *m.* the remainder to 180 *gr.* will be 62 *gr.* And ſuch is the altitude of the Sunne in the Summer Tropique. Then

As the ſine of 66 *gr.* 30 *m.*
to the ſine of the Sunnes altitude:
So the length of the ſubſtylar line,
to the length of the axis of the ſtyle.



As in the firſt example of the declining verticall, where the height of the ſtyle was found to be 34 *gr.* 33 *m.* and is here repreſented before *pag* 150. by the angle *BC* ♋; adde to this height 113 *gr.* 30 *m.* for the angle *CB* ♋, the ſum wil be 148 *gr.* 3 *m.* and the remainder to 180 *gr.* wil be 31 *gr.* 57 *m.* and ſuch is the angle *B* ♋ *C* of the altitude of the Sun aboue the plane, when he cometh to be in the tropique of ♋, which is here the fartheſt tropique from the center.

Then ſuppoſing the length of the ſubſtylar line between the center and the place which is fit for the fartheſt tropique to be about 21 inches, extend the compaſſes from the ſine of 66 *gr.* 30 *m.* vnto the ſine of 31 *gr.* 57 *m.* the ſame extent will reach in the line of numbers from 21 vnto 12.11, and ſo the length of the axis of the ſtyle ſhould be 12 inch. 11 *cent.* Or it may ſuffice to make it iuſt 12 inches, as a more eaſie ground for the reſt of the worke.

But if it were required to proportion the ſtyle vnto the plane, ſo as it may caſt the ſhadow to the ful lẽgth of the ſubſtylar line at all times of the yeare, you may then conſider the Sunne in the tropique, which is to be ſet neareſt vnto the center, and adde 66 *gr.* 30 *m.* vnto 34 *gr.* 33 *m.* ſo the remainder vnto 180 *gr.* will be 78 *gr.* 57 *m.* And if you extend the compaſſes from the ſine of 66 *gr.* 30 *m.* vnto the ſine of 78 *gr.* 57 *m.* the ſame extent will reach in the line of numbers from 21 vnto 22.47 for the length of the axis of the ſtyle.

2 *Hauing the length of the axis, and the height of the ſtyle aboue the plane, to find the length of the ſides of the ſtyle.*

The ſtyle of a plane neither equinoctiall nor polar, may be either a ſmall rod of iron ſet parallell to the axis of the world, or perpendicular to the plane, or elſe a thin plate of iron or braſſe made in forme of a rectangle triangle *B A C*, with the baſe *BC* parallell to the axis of the world, the ſide *AB* perpendicular to the plane, & the ſide *AC* the ſame with the ſubſtylar line, wherein knowing *BC*, and the angle BAC,

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As the ſine of 90 *gr.*
to the length of the axis:
So the ſine of the height of the ſtyle,
to the length of the perpendicular ſide:
And ſo the coſine of the height of the ſtyle,
to the length of the ſubſtylar ſide.

Thus in the former example, the length of the axis being ſuppoſed to be 12 inches, and the height of the ſtyle 34 *gr.* 33 *m.* Extend the compaſſes from the ſine of 90 *gr.*(or elſe from the ſine of 5 *gr.*45 *m.*) vnto 12 in the line of numbers, the ſame extent will reach from the ſine of 34 *gr.*33 *m.* vnto 6.80 in the line of numbers for the length of the perpendicular ſide, and from the ſine of 55 *gr.*27 *m.* vnto 9.88 for the length of the ſubſtylar ſide.

3 *To find the diſtance between the center and the equator vpon the ſubſtylar line.*

This is here repreſented by C♈, and may be found by reſoluing the rectangle triangle C B♈.

As the coſine of the height of the ſtyle,
is to the ſine of 90 *gr.*
So the length of the axis,
to the diſtance of the equator from the center.

Extend the compaſſes from the ſine of 55 *gr.* 27 *m.* vnto the ſine of 90 *gr.* the ſame extent will reach in the line of numbers from 12 vnto 14.57. Wherefore if you take 14 *inch.* 57 *cent.* and pricking them down on your ſubſtylar line frō C vnto ♈, draw a line through ♈, croſſing the ſubſtylar at right angles, the line ſo drawne ſhall be the equator.

4 *To find the angles contained between the equator and the houre-lines vpon your plane.*

Theſe angles made by B♈ and the houre-lines, are complements of thoſe which are at *C*, betweene B C the axis and thoſe ſeuerall houre-lines, and depend vpon the angles at the pole, between the proper meridian and the houre-circles.

As the ſine of *90 gr.*
to the coſine of the angle at the pole:
So the cotangent of the height of the ſtyle,
to the tangent of the angle between the equator and the houreline.

In our example the height of the ſtyle is 34 *gr.* 33 *m.* and the proper meridian falleth to be the ſame with the circle of the ſecond houre after noone, whereupon the angle at the pole, betweene this proper meridian, and the circles of the houre of 1 on the one ſide, and 3 on the other ſide, wil be 15 *gr*; ſo between this meridian and the houre-circles of 12 and 4, the angle will be 30 *gr.* &c. as in the Table.

Ho.		An.Po		Arc.Pla		Ar.Equ		C	♈	C	♋	C	♑
		Gr.	M.	Gr.	M	Gr	M	In.	P.	In.	P.	In.	P.
ſubſty		0	0	0	0	55	27	14	57	20	80	11	21
1	3	15	0	8	38	54	30	14	74	21	36	11	25
12	4	30	0	18	8	51	30	15	33	23	44	11	40
11	5	45	0	29	33	45	45	16	75	29	06	11	76
10	6	60	0	44	30	36	0	20	00	50	84	12	77
9	7	75	0	64	42	20	36	34	10	Infin.		15	82
8	8	90	0	90	0	0	0	Infinit				27	60

If then it be required to find the angle, which the houreline of 4 after noone doth make with the plane of the equator, that is the angle C 4 B contained betweene the houreline C 4 and the line B 4, drawne from the top of the ſtyle vnto the interſectiõ of the houre-line of 4 with the equator.

Extend the compaſſes from the ſine of *90 gr.* vnto the ſine of *60 gr.* the complement of the angle at the pole, the ſame extent wil reach from the tangent of 55 *gr.* 27 *m.* the complement of the height of the pole, vnto the tangẽt of 51 *gr.* 30 *m.*

and ſuch is the angle C 4 B in the diagram *Pag.*150.

Or in croſſe-worke, if it were required to finde the angle C 9 B, looke into the Table for the houre of 9, and there you ſhall find the angle at the pole to be 75 *gr*, and if you extend the compaſſes from the ſine of 90 *gr.* vnto the tangent of 55 *gr.* 27 *m.* the ſame extent will reach from the ſine of 15 *gr.* the complement of 75 *gr.* vnto the tangent of 20 *gr.* 36 *m.* and ſuch is the angle C 9 B, made at the equator betweene the line B 9 drawne from the top of the ſtyle, and the houre-line C 9 drawne from the center. The like reaſon holdeth for the reſt, which may be found and ſet downe in a table: then may you either draw theſe angles at *C* in the former figure more perfectly, and thence finiſh your worke, or elſe proceed

5 *To find the diſtance betweene the center and the parallels of declination.*

The diſtances betweene the center and the parallels of declination, may be found by reſoluing the triangles made by the axis *BC*, the lines of declination, and the houre-lines. For hauing the angles at the equator, and knowing the declination of the parallell, if the parallell ſhall fall betweene the equator and the center, adde the declination vnto the angle at the equator; or if it ſhall fall without the equator, take the declination out of the angle at the equator, ſo ſhall you haue the angle at the parallell. Then

As the ſine of the angle at the parallell,
to the coſine of the declination:
So the length of the axis of the ſtyle,
to the diſtance between the center and the parallell.

Thus in our example, the angle at the equator belonging to the houre of 4 after noone, was found before to be 51 *gr.* 30 *m:* if you would find the diſtance between the center and the equator, extend the compaſſes frō the ſine of 51 *gr.* 30 *m.*

vnto the ſine of *90 gr.* the complement of the declination, the ſame extent will reach in the line of numbers, from 12 vnto 15.33, and ſuch is the diſtance vpon the houre-line of 4 between the center and the equator.

If you would finde the diſtance vpon this houre-line, between the center and the inner tropique, whoſe declination is knowne to be 23 *gr.* 30 *m.* adde the declination to the angle at the equator, ſo the angle at the parallell wil be 75 *gr.* wherefore extend the compaſſes from the ſine of 75 *gr.* vnto the ſine of 66 *gr.* 30 *m.* the complement of the declination, the ſame extent will reach in the line of numbers, from 12 vnto 11.40, and ſuch is the length of the houre-line of 4 betweene the center and the tropique of ♑.

If you would finde the diſtance vpon this houre-line between this center and the tropique of ♋, which is here the fartheſt from the center, take the declination out of the angle at the equator, ſo the angle at the parallell will be 28 *gr.* wherefore extend the compaſſes from the ſine of 28 *gr.* vnto the ſine of 66 *gr.* 30 *m.* the ſame extent will reach in the line of numbers, from 12 vnto 23.44, and ſuch is the diſtance betweene the center and the tropique of ♋ vpon this houre-line of 4. The like reaſon holdeth for all the reſt, which may be gathered and ſet downe in a table.

That done and the equator drawne as before, if you would draw the tropique of ♋, looke into the table, and there finding vnder the title C ♋ the diſtance of the ſubſtylar between the center and the parallel of ♋ to be 20 inch. 80 *cent.* take 20 inch. 80 *cent.* out of the line of inches, and prick them downe in the ſubſtylar of your plane from C vnto ♋.

Or if either the center fall without your plane, or the extent be too large for your compaſſes, you may pricke downe the difference betweene C ♈ and C ♋. As here the diſtance C ♈ between the center & the equator is 14.57, the diſtance C ♋ 20.80, the differẽce 6.23, therfore taking 6 inch. 23 *cent.* pricke them downe on the ſubſtylar from ♈ vnto ♋, and you ſhall haue the ſame interſection of the tropique and the ſubſtylar, as before; & the like reaſon holdeth for pricking down

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of the reſt of theſe diſtances on their ſeuerall houre-lines.

Then hauing the points of interſection betweene the houre-lines and the parallell, you may ioyne them all in a crooked line without making of any angles, the line ſo drawne ſhall be the tropique required. And after this maner may you draw any other parallell of declination, whereof you haue examples in the moſt of the former Diagrams.

CHAP. XIIII.

To deſcribe the parallels of the Signes in any of the former Planes.

THe equator and the tropiques before deſcribed, do ſhew the Suns entrance into 4 of the Signes, the equator into ♈ and ♎, the one tropique into ♋, and the other into ♑, the reſt of the intermediate Signes will be deſcribed in the ſame maner as the tropiques, if firſt we know their declination.

The maner of finding the declination not onely of the beginning of the Signes, but of all other points of the ecliptique, is before ſet downe in 2. *Prop. Aſtronomicall, pag.* 52. by which you may finde the declination of the beginning of ♉, ♍, ♏, and ♓ to be 11 *gr.* 30 *m.* and of ♊, ♌, ♐ and ♒ to be 20 *gr.* 12 *m.* If then you inſcribe the chords of 11 *gr.* 30 *m.* and of 20 *gr.* 12 *m.* into the former figure *B D T Pag.* 145. from *D* toward *T*, the lines drawne from *B* through the termes of thoſe chords ſhall be the Signes required.

And with theſe declinations, the height of the ſtyle, and the length of the axis, you may finde the angles at the parallell, and then the diſtances betweene the center and the parallell, which being pricked downe vpon their ſeuerall houre-lines ſhall giue you the points of interſection, by which you may draw the parallels of the Signes, as in the figures belonging to the polar planes.

CHAP. XV.

To deſcribe the parallels of the length of the day in any of the former Planes.

THe length of the day will alwayes be 12 houres long when the Sunne cometh to be in the equator, and this holdeth in all latitudes; but at other times of the yeare the ſame place of the Sunne, wil not giue the ſame length of the day in another latitude; wherefore the latitude being known, we are firſt

To find the declination of the Sunne agreeing to the length of the day.

Conſider the difference betweene the length of an equinoctiall day and the day propoſed, and turne the time into degrees and minutes.

As the ſine of *90 gr.*
is to the ſine of halfe the difference:
So the cotangent of the latitude,
to the tangent of the declination.

As if the length of the day propoſed were 15 houres, the difference betweene this and an equinoctiall day (whoſe length is alwayes 12 houres) would be three houres, which make 45 *gr.* and the halfe difference is 22 *gr.* 30 *m*: wherefore extend the compaſſes from the ſine of *90 gr.* vnto the tangent of 38 *gr.* 30 *m.* the complement of the latitude, the ſame extent wil reach from the ſine of 22 *gr.* 30 *m.* vnto the tangent of 16 *gr.* 55 *m.* for the declination of the Sunne at ſuch time as the length of the day is either *9* or 15 houres; and from the ſine of 30 *gr.* vnto the tangent of 21 *gr.* 40 *m.* for the declination belonging to 8 or 16 houres, and from the ſine of

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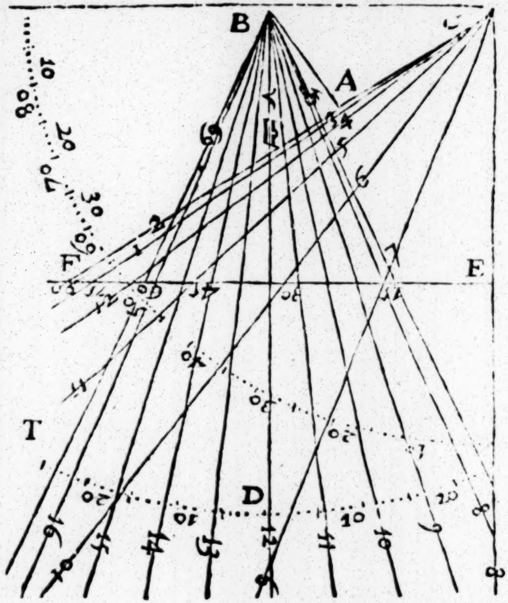
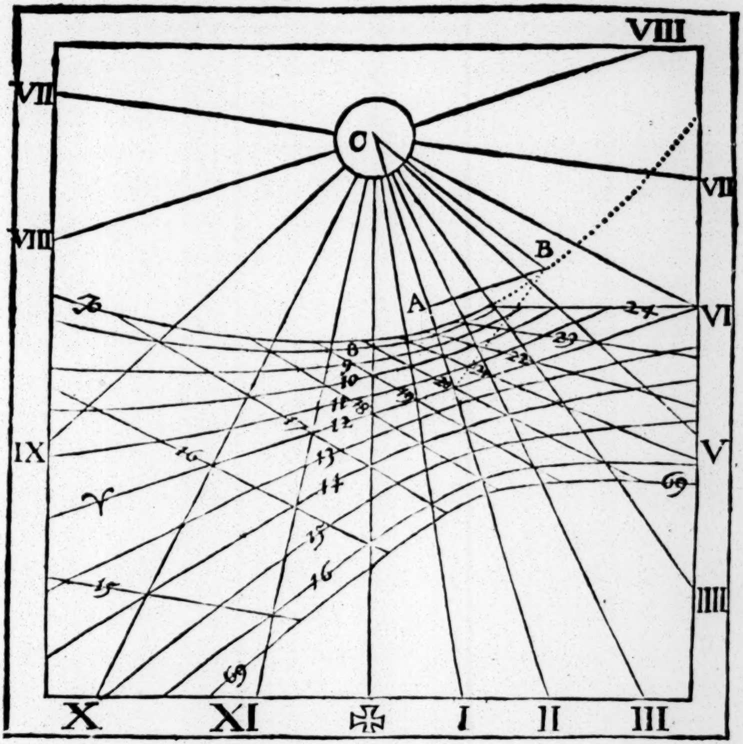


15 *gr.* vnto the tangent of 11 *gr.* 38 *m.* for the declination belonging to 10 or 14 houres, and from the ſine of 7 *gr.* 30 *m.* vnto the tangent of 5 *gr.* 56 *m.* for the declination of the Sun when the length of the day is either 11 or 13 houres.

If then you inſcribe the chords of theſe arks into the former figure *B D T*, the lines drawne from *B* through the termes of theſe arks, ſhall be the lines belonging to the diurnall arkes, and the ſeuerall diſtances betweene them and the point *C* giue the like diſtances betweene the center and the parallels of the length of the day vpon the houre-lines in your plane.

Or comparing theſe angles of declination with the angles at the equator, you may haue the angles at the parallell, and then find the diſtances between the center and the parallell, which being pricked downe vpon the ſeuerall houre-lines,



ſhall giue you the points of interſection, by which you may draw the parallels of the length of the day, whereof you haue another example in the diagram belonging to an horizontal plane, *pag.* 105. And by the ſame reaſon you may draw the parallels of thoſe circles to which the Sunne is verticall, the parallels of the principall feaſts, or what elſe depends on the declination of the Sunne.

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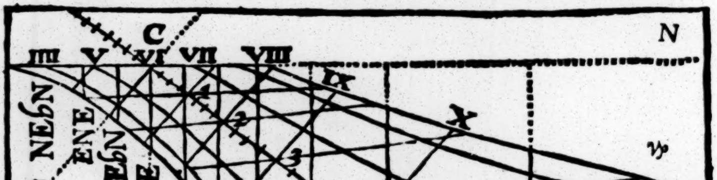
CHAP. XVI.

To draw the old vnequall houres in the former Planes.

IT was the maner of the Ancients to divide the day into twelue equall houres, and the night into twelue other equal houres, and ſo the whole day and night into 24 houres. Of theſe 24, thoſe which belonged vnto the day, were either longer or ſhorter (excepting the two equinoctial dayes) then thoſe which belonged vnto the night; and the Summer houres alwayes longer then the houres in the Winter, according to the lengthening of the dayes, whereupon they are called the old vnequall (and by ſome the Planetary) houres.



To expreſſe theſe in the former Planes: firſt draw the common houre-lines, the equator, and the tropiques, as before: then deſcribe two occult parallels of the length of the day, one for 9 houres, the other for 15 houres; for ſo you may draw a ſtreight line for the firſt vnequall houre through 5 *ho.* 45 *m.* in the parallell of 15, and through 8 *ho.* 15 *m.* in the parallel of 9. This ſtreight line ſhal paſſe directly through 7 *ho.* 0 *m.* in the equator, and ſo cut off a twelfth part of the arks aboue the horizon, both from theſe two parallels and the equator: and being continued vnto the tropiques, it ſhall alſo cut off about a twelfth part from them, and all the reſt of the parallels of declination, without any ſenſible error.

In like maner may you draw the ſecond vnequall houre through 7 *ho.* in the parallell of 15, through 8 *ho.* in the equator, and through 9 *ho.* in the parallell of 9, and ſo in the reſt, as in this Table.

Hora	15		Æq	9	
	Ho.	M.	Ho	Ho.	M
0	4	30	6	7	30
1	5	45	7	8	15
2	7	0	8	9	0
3	8	15	9	9	45
4	9	30	10	10	30
5	10	45	11	11	15
6	12	0	12	12	0
7	1	15	1	0	45
8	2	30	2	1	30
9	3	45	3	2	15
10	5	0	4	3	0
11	6	15	5	3	45
12	7	30	6	4	30

And of theſe vnequall houres you haue a farther example in the diagram belonging to the polar declining plane, *Pag.* 130.

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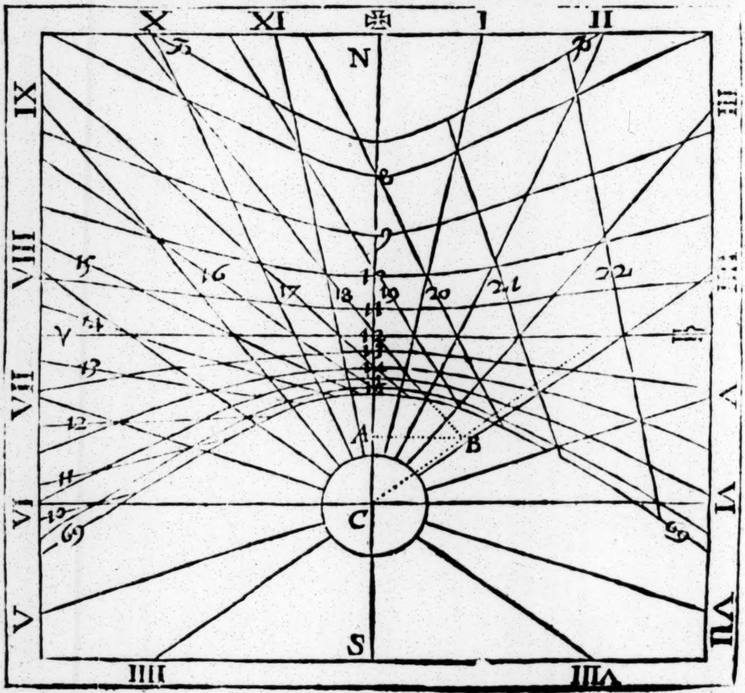
CHAP. XVII.

To draw the houres from Sun riſing and ſun ſetting in the former Planes.

TO know how many houres are paſt ſince the Sun riſing, or how many remaine to the Sun ſetting; firſt draw the common houre-lines, the equator, and the tropiques, as before: then deſcribe two occult parallels of the length of the day, one for 8 houres, and the other for 16 houres. For ſo you



may draw the firſt houre from the Sun riſing through the common houres of 5 in the parallell of 16, of 7 in the equator, and of 9 in the parallell of 8. In like maner the ſecond houre from Sun riſing through the common houres of 6 in the parallell of 16, of 8 in the equator, and of 10 in the parallell of 8. And ſo the reſt in their order.

The firſt houre before Sun ſetting, or the 23 houre from the laſt Sun ſetting, may be drawne in like ſort through the common houres of 3 after noone in the parallell of 8, of 5 in the equator, and of 7 in the parallell of 15. The ſecond houre before Sun ſetting, or the 22 houre after the laſt Sun ſetting through the common houres of 2 in the parallell of 8, of 4 in the equator, and of 6 in the parallell of 16. And ſo the reſt in the like order, whereof you haue another example in the Diagram belonging to the declining verticall, *Pag.* 116.

CHAP. XVIII.

To draw the horizontall line in the former planes.

THe common houre-lines do commonly depend on the ſhadow of the axis, but the parallels of the Signes, and of the length of the day, the houre-lines from Sun riſing and Sun ſetting, with many others, depend on the ſhadow of the top of the ſtyle, or ſome one point in the axis, which here ſignifieth the center of the world, and is repreſented by the point B. And theſe lines ſo depending, are then only vſeful when they fall betweene the two tropiques, and within the horizon.

There may be ſeuerall horizontall lines drawne vpon euery plane, as I ſhewed before in finding the inclination of a plane; but the proper horizontall line which is here meant, muſt always be in the ſame plane with *B* the top of the ſtyle; ſo that in an horizontall plane there can be no ſuch hori-

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zontall line, but in all other planes it may be found by applying the horizontall leg of the *Sector* vnto the top of the ſtyle, and then working as before; and the interſection of this line with the meridian or ſubſtylar line, may be found by proportion.

1 *To find the interſection of the horizon with the meridian, in an equinoctiall plane.*

As the tangent of 45 *gr.*
  to the tangent of the latitude:
So is the height of the ſtyle,
  to the diſtance between the ſtyle and the horizontal (line.

As in the example of the former equinoctiall plane, *Pag.* 142. extend the compaſſes from the tangent of 45 *gr.* vnto 51 *gr.* 30 *m.* the tangent of the latitude, the ſame extent will reach in the line of numbers, from 52 the length of the ſtyle vnto *66*, and ſuch is the diſtance betweene the ſtyle and the horizontall line; wherefore I take *66* parts out of a line of inches, and pricke them downe in the meridian line from *C* vnto *H* aboue the ſtyle in the vpper face, but below the ſtyle in the lower face of the plane, ſo a right line drawne through *H*, parallell to the houre of *6*, ſhall be the horizontall line.

2 *To find the interſection of the horizon with the meridian, in a direct polar plane.*

As the tangent of 45 *gr.*
  to the cotangent of the latitude:
So the length of the ſtyle,
  to the diſtance between the ſtyle & the horizontal line.

As in the example of the former polar plane, *Pag.* 144. extend the compaſſes from the tangent of 45 *gr.* vnto tangent of 38 *gr.* 30 *m.* the complement of the latitude, the ſame extent will reach in the line of numbers, from 1.*61* the length of the ſtyle, vnto 1.28, and ſuch is the diſtance vpon the meridian betweene the ſtyle and the horizontall line.

In all vpright planes, whether they be direct verticall, or declining, or meridian planes, the horizontall line muſt alwayes be drawne through *A* the foot of the ſtyle, as may appeare in the examples before, *Pag.* 102. 107. 116.

And generally in all planes whatſoeuer, the horizontal line muſt be drawne through the interſection of the equator with the houre of 6. Or if that interſection fall without the plane, yet if any arks of the length of the day be drawne on the plane, the horizontall line may be drawne through their interſections, with the houres of the Suns riſing or ſetting.

CHAP. XIX.

To deſcribe the verticall circles in the former Planes.

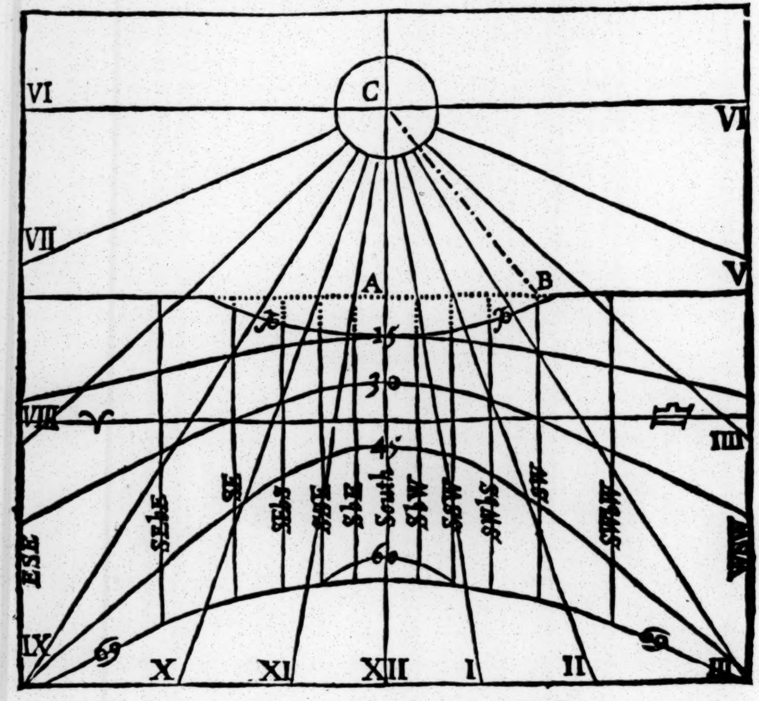
THe vertical circles commonly called Azimuths, are great circles drawne through the zenith, by which we may know in what part of the heauen the Sunne is, how far from the Eaſt or Weſt, and how neare vnto the meridian.

In all vpright planes, whether they be direct verticals, or declining, or meridian planes, the ſemidiameter of the horizon wil be the ſame with *AB* the perpendicular ſide of the ſtyle, and theſe Azimuths will be parallels one to the other, and the diſtance of each Azimuth, from the foote of the ſtyle vpon the horizontall line, may be found in this manner.

Conſider the length of the ſtyle in inches and parts of inches, and the diſtance of each Azimuth from the ſtyle, according to the angle at the zenith in degrees and minutes.

As the tangent of 49 *gr.*
to the tangent of the azimuth:

So the length of the ſtyle,
to the length of the horizontall line between the ſtyle
and the azimuth.



As if it were required to draw the common azimuths on the South face of the verticall plane before deſcribed, where *AB* the length of the ſtyle may be ſuppoſed to be **10** inches. Here the plane hauing no declination, the ſtyle is in the plane of the meridian, and ſo pointeth directly into the South. The point of *S b E* is **11** *gr.* **15** *m.* diſtant from the ſtyle, and *S S E* **22** *gr.* 30 *m.* and the reſt in their order: wherefore extend the compaſſes from the tangent of 45 *gr.* vnto **10** in the line of numbers, the ſame extent will reach from the tangent of 11 *gr.*15 *m.*vnto 1.99 in the line of numbers for the length of the tangent line, betweene the ſtyle and the point *S b E*, and from the tangent of 22 *gr.*30 *m.*vnto 4.14 for *S S E*, and ſo for the reſt, as in this Table.

Azimuths.	An Zen. Gr.	M.	Tangen In.	Pa.
South	0	0	0	0
S b E	11	15	1	99
S S E	22	30	4	14
SE b S	33	45	6	68
S E	45	0	10	00
SE b E	56	15	14	97
E S E	67	30	24	14
E b S	78	45	50	27
East	90	0	Infin.	

In like maner in the firſt example of the declining plane, where the ſtyle ſtandeth according to the declination 24 *gr.* 20 *m.* diſtant from the South toward the Weſt. The next point of *S b W* is but 13 *gr.*5 *m.* diſtant from the ſtyle; and the ſecond of *S S W* onely 1 *gr.* 50 *m.* and the third of *S W b S* is againe 9 *gr.*25 *m.* and the reſt in their order. Wherefore hauing before found the length of the ſtyle to be 6 inches 80 parts, extend the compaſſes from the tangent of 45 *gr.* vnto 6.80 parts in the line of numbers, the ſame extent wil reach from the tangent of 24 *gr.* 20 *m.* vnto 3.07 in the line of numbers for the length of the tangent line between the ſtyle & the South, and from the tangent of 13 *gr.* 5 *m.* vnto 1.58 for the point of *SbW*; and ſo for the reſt, as in this Table.

Azimuths.	An. Zen. Gr.	M	Tangen In.	Pa.
SE b E	80	35	41	00
S E	69	20	18	03
SE b S	58	5	10	91
S S E	46	50	7	25
S b E	35	35	4	86
South	24	20	3	07
S b W	13	5	1	58
S S W	1	50	0	22
	The foote of		*the ſtyl*	
SWbS	9	25	1	13
S W	20	40	2	57
SWbW	31	55	4	24
W S W	43	10	6	37
W b S	54	25	9	50
West	65	40	15	02
W b N	76	55	29	26
W N W	88	10	212	45

That done, if you take theſe parts out of a line of inches, and pricke them downe in the horizontall line on either ſide of the ſtyle, drawing right lines perpendicular to the horizon through theſe interſections, but ſo as they may be contained betweene the horizontall and the tropiques, the lines ſo drawne ſhall be the azimuths required.

In an horizontall plane theſe azimuths are drawne more eaſily. For here the perpendicular ſide of the ſtyle is the ſame with the axis of the horizon, and the foote of the ſtyle is the verticall point, in which all the azimuth lines doe meete as their circles do in the zenith: wherefore let any circle deſcribed on the center *A*, at the foote of the ſtyle, be diuided firſt into foure parts, beginning at the meridian, and then each quarter ſubdiuided either into eight equall parts, according to the points of the Mariners compaſſe, or into 90 *gr.* according to the Aſtronomical diuiſion; if you draw right lines through the center and theſe diuiſions, the lines ſo drawne ſhall be the azimuths required.

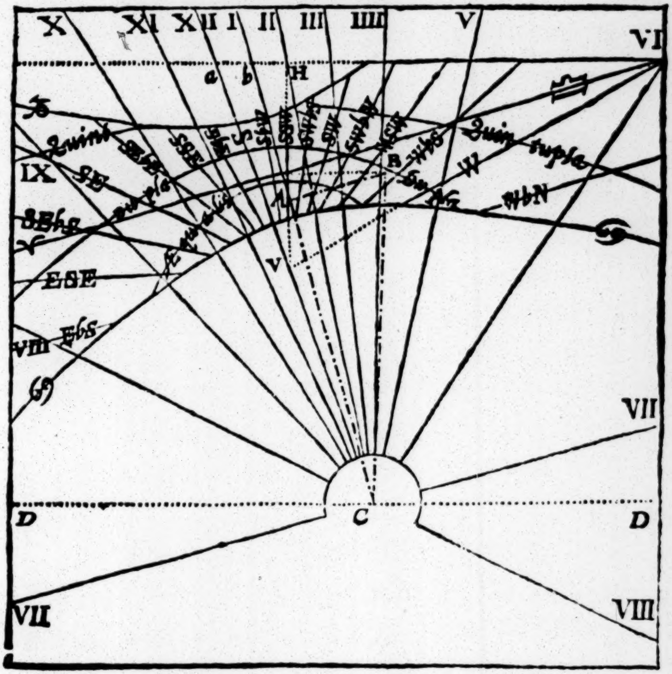
In all other planes inclining to the horizon, theſe vertical circles will meete in a point, but that verticall point being more or leſſe diſtant from the foote of the ſtyle, the angles at this point will be vnequall.

1 *To find the diſtance between the foote of the ſtyle and the verticall point.*

The verticall point wherein all the verticall lines do meet, will be alwayes in the meridian, directly vnder or ouer the top of the ſtyle; and the angle betweene the perpendicular ſide of the ſtyle and the verticall line, will be equall to the inclination of the plane to the horizon. Wherefore

As the tangent of 45 *gr.*
to the tangent of the inclination of the plane:
So is the length of the ſtyle,
to the diſtance betweene the foote of the ſtyle and the verticall point.

Thus in the firſt exãple of the declining inclining planes, where the vpper face of the plane looking Southweſt, the declination was 24 *gr.* 20 *m.* the inclination 36 *gr*; and you may ſuppoſe *AB* the length of the ſtyle to be 6 inches: if you extend the compaſſes from the tangent of 45 *gr.* vnto the



the tangent of 36 *gr.* the ſame extent will reach in the line of numbers from 6.00 vnto 4.36, for the diſtance *AV* betweene A the foote of the ſtyle and V the verticall point.

2 *To find the diſtance betweene the foote of the ſtyle and the horizontall line.*

As the tangent of the inclination of the plane,
 is to the tangent of 45 *gr.*
So the length of the ſtyle,
 to the diſtance betweene the foote of the ſtyle and
 the horizontall line.

So the ſame extent of the compaſſes as before, wil reach in the line of numbers from 6.00 vnto 8.26 for the diſtance *AH*

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betweene the foote of the ſtyle and the horizontall line.

Then may you take 4 inches *36 cent.* and pricking them downe from *A* the foot of the ſtyle vnto *V* the verticall point in the meridian, draw the line *VA*, which being produced ſhall cut the horizon in the point *H* with right angles, and be that particular azimuth which is perpendicular to the plane.

Or you may take 8 inches *26 cent.* and pricke them downe in the former line *VA* produced from *A* vnto *H*, and ſo draw the horizontall line through *H* perpendicular vnto *VH*, which horizontall line being produced wil croſſe the equator in the ſame point wherein the equator croſſeth the houre-line of *6*, vnleſſe there be ſome former error.

3 *To find the angles made by the azimuth lines at the verticall point.*

The angles at the zenith depend on the declination of the plane, as in our example, where the ſtyle ſtandeth according to the declination 24 *gr.* 20 *m.* diſtant from the South toward the Weſt, the azimuth of 10 *gr.* from the meridian Eaſtward will be 34 *gr.* 20 *m.* the azimuth of 10 *gr.* Weſtward will be onely 14 *gr.* 20 *m.* diſtant from the ſtyle, and ſo the reſt in their order.

Or if you would rather deſcribe the common azimuths, the point of *S b E* wil be 35 *gr.* 35 *m.* the point of *SbW* 13 *gr.* 5 *m.* diſtant from the ſtyle, and ſo the reſt in their order. Then

As the ſine of 90 *gr.*
  to the coſine of the inclination of the plane:
So the tangent of the angle at the zenith,
  to the tangent of the angle at the verticall point between the line drawne through the foot of the ſtyle and the azimuth required.

Wherefore the inclination of the plane in our example being 36 *gr.* extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 54 *gr.* the ſame extent ſhall reach in the line of tangents, from 24 *gr.* 20 *m.* vnto 20 *gr.* 5 *m.* for the angle *HVa* at ỹ vertical point, between the line *VH* drawne through *A* the foote of the ſtyle & the South. Againe, the ſame extent will reach from the tangent of 13 *gr.* 5 *m.* vnto 10 *gr.* 38 *m.* for the angle belonging to *SbW*; and ſo for the reſt, as in this table.

Azimuths.	Ang. Ze. Gr.	Ang. Ze. M.	Ang. Ve. Gr.	Ang. Ve. M.
SEbE	80	35	78	25
SE	69	20	65	0
SEbS	58	5	52	25
SSE	45	50	40	46
SbE	35	35	30	3
South	24	20	20	5
SbW	13	5	10	39
SSW	1	50	1	29
	Style.		0	0
SWbS	9	25	7	38
SW	20	40	16	58
SWbW	31	55	26	45
WSW	43	10	37	11
WbS	54	25	48	30
West	65	40	60	48
WbN	76	55	73	58
WNW	88	10	87	44

Theſe angles being knowne, if on the center *V*, at the verticall point, you deſcribe an occult circle, and therein inſcribe the chords of theſe angles from the line *VH*, and then draw right lines through the verticall point, and the termes of thoſe chords, the lines ſo drawne ſhall be the azimuths required.

The like reaſon holdeth for the drawing of the azimuths vpon all other inclining planes, wheof you haue another example in the Diagram belonging to the meridian incliner, *Pag.* 126.

Or for further ſatisfaction you may finde where each azimuth line ſhall croſſe the equator.

As the ſine of 90 *gr.*
to the ſine of the latitude:
So the tangent of the azimuth from the meridian,
to the tangent of the equator from the meridian.

Extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of our latitude 51 *gr.* 30 *m.* the ſame extent will reach in the line of tangents from 10 *gr.* vnto 7 *gr.* 50 *m.* for the interſection of the equator with the azimuth of 10 *gr.* from the meridian. Againe, the ſame extent will reach from 20 *gr.*

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vnto 15 *gr.* 54 *m.* for the azimuth of 20 *gr.* And ſo the reſt, as in theſe tables.

Azim.		*Equat.*	
Gr.	*M.*	*Gr.*	*M.*
10	0	7	50
20	0	15	54
30	0	24	20
40	0	43	18
50	0	13	0
60	0	53	35
70	0	65	3
80	0	77	18
90	0	90	0

Azim.		*Equat.*	
Gr.	*M.*	*Gr.*	*M.*
11	15	8	51
22	30	17	58
33	45	27	36
45	0	38	2
56	15	49	30
67	30	62	6
78	45	75	44
90	0	90	0

By which you may ſee that the azimuth 90 *gr.* diſtant from the meridian, which is the line of Eaſt and Weſt, will croſſe the equator at 90 *gr.* from the meridian in the ſame point, with the horizontall line and the houre of 6. And that the azimuth of 45 *gr.* will croſſe the equator at 38 *gr.* 2 *m.* from the meridian, that is, the line of *S E* will croſſe the equator at the houre of 9 and 28 *m.* in the morning, and the line of *S W* at 2 *ho.* 32 *min.* in the afternoone; and ſo for the reſt, whereby you may examine your former worke.

CHAP. XX.

To deſcribe the parallels of the horizon in the former planes.

THe parallels of the horizon, commonly called Almicanters, or parallels of altitude (whereby we may know the altitude of the Sunne aboue the horizon) haue ſuch reſpect vnto the horizon, as the parallels of declination vnto the equator, and ſo may be deſcribed in like maner.

In an horizontall plane, theſe parallels will be perfect circles; wherefore knowing the length of the ſtyle in inches and parts, and the diſtance of the parallell from the horizon in degrees and minutes.

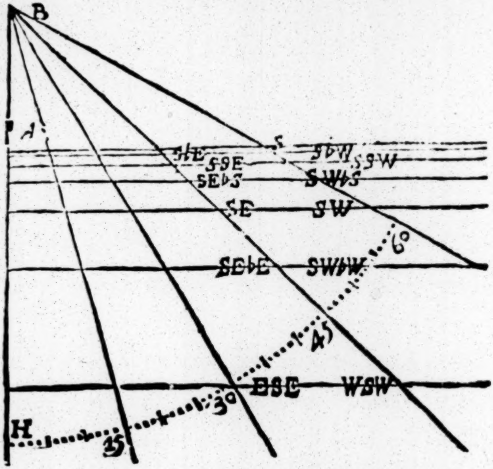
As the tangent of 45 *gr.*
  is to the length of the ſtyle:
So the cotangent of the parallell,
  to the ſemidiameter of his circle.

Thus in the example of the horizontall plane, *Pag.* 164. if *A B* the length of the ſtyle ſhall be 5 inches, and that it were required to finde the ſemidiameter of the parallell of 62 *gr.* extend the compaſſes from the tangent of 45 *gr.* vnto 5.00 in the line of numbers, the ſame extent will reach from the tangent of 28 *gr.* the complement of the parallell vnto 2.65, and if you deſcribe a circle on the center *A* to the ſemidiameter of 2 inches 65 *cent.* it ſhall be the parallell required.

In all vpright planes, whether they be direct verticals, or declining, or meridian planes, theſe parallels will be conicall ſections, and may be drawne through their points of interſection, with the azimuth lines, in the ſame maner as the parallels of declination, through their points of interſection with the houre-lines. To this end you may firſt finde the diſtance betweene the top of the ſtyle and the azimuth; and then the diſtance betweene the horizon and the parallell, both which may be repreſented in this maner.

On the center *B* and any ſemidiameter *B H*, deſcribe an occult arke of a circle, and therein inſcribe the chords of ſuch parallels of altitude as you intend to draw on the plane, (I haue here put them for 15.30.45 and 60 *gr.*) then draw right lines through the center and the termes of thoſe chords, ſo the line *B H* ſhall be the horizon, and the reſt the lines of altitude, according to their diſtance from the horizon.

That



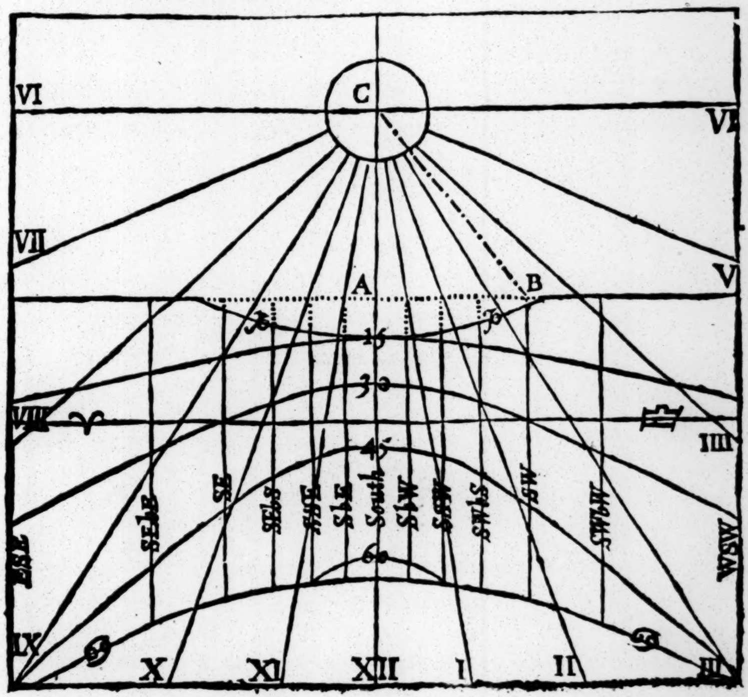
That done, conſider your plane (which here for example is the South face of our vertical plane, *p.*168) wherein hauing drawne both the horizontall & verticall lines, as I ſhewed before, firſt take out *A B* the length of the ſtyle, & pricke that downe in this horizontall line from *B* vnto *A*; then take out all the diſtances between *B* the top of the ſtyle and the ſeuerall points wherein the verticall lines do croſſe the horizontal, transfer them into this horizontal line BH, from the center *B*, and at the terms of theſe diſtances erect lines perpendicular to the horizon, noting them with the number or letter of the azimuth from whence they were taken, ſo theſe perpendiculars ſhall repreſent thoſe azimuths, and the ſeuerall diſtances betweene the horizon and the lines of altitude ſhall giue the like diſtances, betweene the horizontall and the parallels of altitude vpon the azimuths in your plane. Vpon this ground it followeth,

To

1 *To find the diſtance betweene the top of the ſtyle, and the ſeuerall points wherein the azimuths do croſſe the horizontall line.*

Hauing drawne the horizontall and azimuth lines as before, looke into the table by which you drew them, and there you ſhall haue the angles at the zenith. Then

As the coſine of the angle at the zenith,
is to the ſine of 90 *gr.*
So the length of the ſtyle,
to the diſtance required.



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Azi-	Ang Ze		Tangent		Secant.		Par. 15.		Par. 30	
muths.	Gr.	M.	Inch	P.	Inch.	P.	Inch.	P.	Inch.	P.
South.	0	0	0	0	10	00	2	68	5	77
S b E	11	15	1	99	10	20	2	73	5	90
S S E	22	30	4	14	10	82	2	90	6	24
S E b S	33	45	6	68	12	03	3	23	6	94
S E	45	0	10	00	14	14	3	80	8	16
S E b E	56	15	14	97	18	00	4	82	10	40
E S E	67	30	24	14	26	13	7	02	15	08
E b S	78	45	50	27	51	26	13	73	29	60
Eaſt.	90	0	Infinit		Infinit		Infinit		Infinit	

As in our example of the verticall plane, where *A B* the length of the ſtyle was ſuppoſed to be 10 inches, extend the compaſſes from the ſine of 78 *gr.* 45 *m.* (the complement of 11 *gr.* 15 *m.* the angle at the zenith, belonging to *S b E* and *S b W*) vnto the ſine of 90 *gr.* the ſame extent wil reach from 10.00 the length of the ſtyle, vnto 10.20 for the diſtance betweene the top of the ſtyle and the interſection of the azimuth *S b E* with the horizontall line, which diſtance may be called the *ſecant* of the azimuth, and may ſerue for the drawing of the parallell of 45 *gr.* from the horizon. The like reaſon holdeth for the reſt of theſe diſtances here repreſented in the line B H.

2 *To finde the diſtance betweene the horizon and the parallels.*

As the tangent of 45 *gr.*
   to the tangent of the parallell:
So the ſecant of the azimuth,
   to the diſtance required.

As if it were required to draw the parallell of 15 *gr.* from the horizon, vpon this verticall plane; extend the compaſſes from the tangent of 45 *gr.* vnto the tangent of 15 *gr.* the ſame extent will reach in the line of numbers from 10.00 the ſecant of the South azimuth vnto 2.68, and therefore the diſtance betweene the horizon and the parallell of 15 *gr.* is 2 inches 68 *cent.* vpon the South azimuth. Againe, the ſame extent will reach from 10.20 the ſecant of *S b E* vnto 2.73 for the like diſtance belonging to *S b E* and *S b W*; and ſo for the reſt, which may be gathered and ſet downe in the table.

That done, and the horizon and azimuths being drawne, pricke downe 10 inches from the horizontall line vpon the South azimuth, & 10 inches 20 *cent.* on the azimuths of *SbE* and *SbW*, and 10 inches 82 *cent.* on the azimuths of *S S E* and *S S W*, and 12 inches 3 *cent.* on the azimuth of *SEbS* and *S W b S*, and ſo the reſt of theſe diſtances on their ſeuerall azimuths: then if you draw a crooked line through all theſe points, that may make no angles, the line ſo drawne ſhall be the parallell of 45 *gr.* from the horizon. In like maner may you draw the parallell of 15 *gr.* or any other parallell of altitude vpon any verticall plane.

If the plane incline to the horizon, after we haue found the verticall point, and drawne the horizontall line, we are farther to finde the length of the axis of the horizon, then the angles betwixt this axis and the azimuth lines, and ſo the ſeuerall diſtances betweene the parallels and the verticall point, all which may be repreſented in this manner.

On the center *B*, and any ſemidiameter, deſcribe an occult quadrant of a circle, and therein inſcribe the chords of ſuch parallels of altitude as you intend to draw on the plane, drawing right lines through the center and the termes of theſe chords, ſo the line B H ſhall be the horizon, and his perpendicular *B V* the axis of the horizon, and the reſt the lines of altitude, according to their diſtance from the horizon.

That done, conſider your plane, which here for example is the firſt of our three declining inclining planes, wherein hauing drawne both the horizontall and verticall lines as I ſhewed before, firſt take out the axis of the horizon, which

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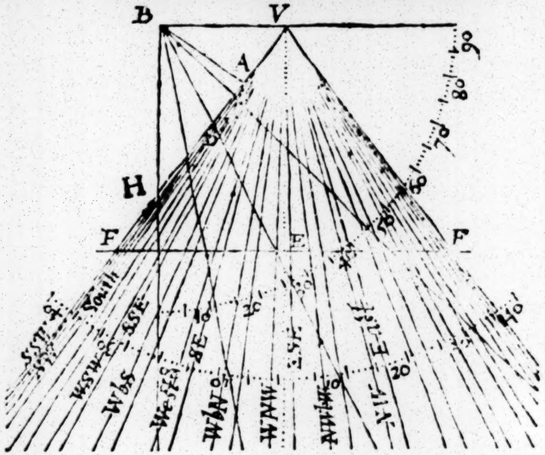


is the line between B the top of the ſtyle and V the verticall point, and pricke that downe in this figure from *B* vnto *I*; then take out both the line *VH* and all the reſt of the diſtances between V the verticall point and the ſeuerall points wherein the verticall lines doe croſſe the horizontall line of this figure, from the point *V*, noting the place where they croſſe the horizontall line with the number or letter of the azimuth from whence they were taken, and drawing the azimuth lines from *V* through the lines of altitude.

Or hauing the *Sector* you may draw an occult line *VE* perpendicular to the axis *VB*, and therein prick downe the tangent of the complement of the inclination of the plane from *V* vnto *E*: then draw the line *E F* parallell to the axis, croſſing the line *VH* produced in the point *F*, ſo this line *E F* will be as the line of ſines vpon the *Sector*, and therein you may pricke downe the ſines of the complement of the angles at the zenith from *E* towards *F*, and draw the verticall lines by thoſe points through the lines of altitude, ſo the angles at V, betweene the axis V B and thoſe azimuth lines, ſhall be the angles betweene the axis of the horizon and the azimuth lines on your plane, and the ſeuerall diſtances between the point V and the lines of altitude, ſhall giue the like diſtances betweene the verticall point and the parallels of altitude vpon the azimuths in your plane. Vpon this ground it followeth,

1 *To find the length of the axis of the horizon.*

The verticall point is alwayes either directly ouer or vnder the top of the ſtyle, and the diſtance betweene them is that which I call the axis of the horizon, which may thus be found,

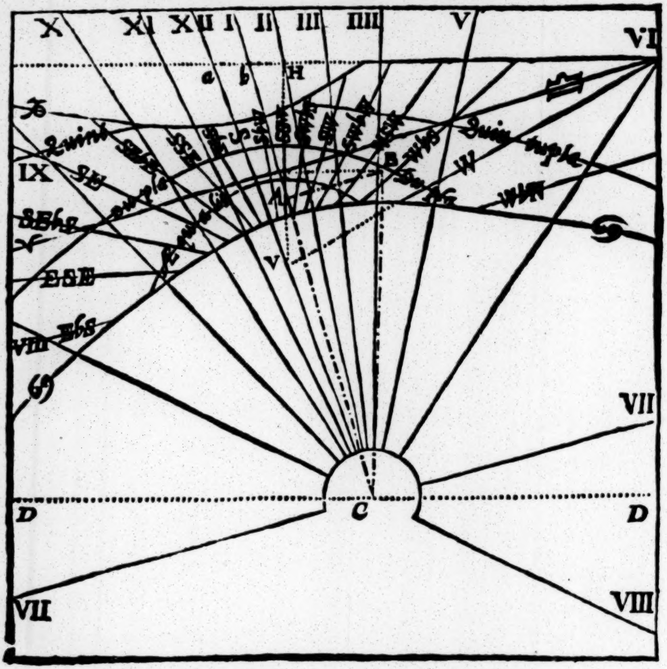
As the coſine of the inclination,
to the ſine of *90 gr.*
So the length of the ſtyle,
to the length of the axis of the horizon.

For example in the firſt of the three declining inclining planes, the inclination to the horizon is 36 *gr.* the length of the ſtyle *AB* ſix inches, extend the compaſſes from the ſine of 54 *gr.* the complement of the inclination vnto the ſine of 90 *gr.* the ſame extent wil reach in the line of numbers from 6.00 vnto 7.42, & ſuch is VB the length of the axis required.

2 *To find the angles contained between the horizon and the verticall lines vpon your plane.*

The angles at the vertical point betweene the axis of the horizon and the azimuth lines vpon your plane are repreſented in this figure by thoſe at V, betweene V B and the azimuths. The angles betweene the horizon and the azimuth lines being complements to the former, are repreſented either by thoſe which are made by V E or by B H, and the azimuth lines which are drawne from V.

That you may find them, looke into the Table, by which you drew the azimuth lines, there ſhall you finde the angles at the zenith. Then



As the ſine of *90gr.*
to the coſine of the angle at the zenith:
So the tangent of the inclination to the horizon,
to the tangent of the angle betweene the horizon
and the verticall line.

In our example where the inclination to the horizon is *36gr.* and the angle at the zenith betweene the azimuth at the ſtyle and the meridian, is according to the declination *24gr.20 m.* extend the compaſſes from the ſine of *90gr.* vnto the tangent of *36gr.* the ſame extent wil reach from the ſine of *65gr.40m.* the complement of the angle at the zenith, vnto the tangent of *33gr. 30 m.* for the angle contained between the horizon and the South part of the meridian line. Again, the ſame extent wil reach frō the coſine of *35 gr. 35 m.* the angle at the zenith belonging to *S b E* vnto the tangent of *30 gr. 3 m.* for the angle betweene the horizon and the azimuth line of *S b E*. The like reaſon holdeth for the reſt, which may be found and ſet downe in the Table.

Azimuths.	Ang. Ze.		Ang V		Ang. Ho		Horizon		11	18	26	34	45	0
	Gr	M.	Gr.	M.	Gr.	M	Inch.	P.	Inch.	P	Inch	P.	Inch.	P.
Eaſt.	114	25	119	12	16	40	In-				38	60	11	05
E*b*S	103	5	106	2	9	20	fi-		210	24	22	40	9	00
E*S*E	91	50	92	16	1	20	nite.		41	98	15	57	7	60
*S*E*b*E	80	35	78	25	6	47	62	82	23	44	12	07	6	68
*S*E	69	20	65	0	14	23	29	87	16	79	10	12	6	00
SE*b*S	58	5	52	25	21	0	20	70	13	61	8	99	5	79
SSE	46	50	40	46	26	25	16	68	11	50	8	31	5	53
S b E	35	35	30	3	30	35	14	58	10	90	7	50	5	42
South	24	20	20	5	33	30	13	44	10	32	7	66	5	35
S b W	13	5	10	39	35	17	12	84	10	02	7	55	5	33
S S W	1	50	1	29	35	59	12	62	9	90	7	47	5	31
	Style.		0	0	36	0	12	62	9	90	7	47	5	31
SWbS	9	25	7	38	35	37	12	74	9	96	7	50	5	32
SW	20	40	16	58	34	12	13	20	10	20	7	59	5	34
SWbW	31	55	26	45	31	40	14	13	10	67	7	81	5	39
WSW	43	10	37	11	27	55	15	85	11	50	8	15	5	49
WbS	54	25	48	30	22	55	19	05	12	94	8	73	5	66
Weſt	65	40	60	48	16	40	25	87	15	51	9	60	5	96
W*bN*	76	55	73	58	9	20	45	75	20	64	11	32	6	46
W*N*w	88	10	87	44	1	20	318	88	33	27	14	18	7	25
NwbW	99	25	101	35	6	47	Infi-		92	40	19	60	8	48
*N*W	110	40	115	0	14	23	nit.				31	44	10	30

Then may you either draw theſe angles at V in the former figure more perfectly, and thence finiſh your worke, or elſe proceed.

3 *To find the diſtance between the verticall point and the parallels of the horizon.*

Theſe diſtances may be found by reſoluing the triangles in the laſt figure made by the axis, the lines of altitude, and the azimuth lines. For hauing the length of the axis and the angles at the horizon, if you adde the diſtance of the parallel from the horizon vnto the angle at the horizon, you ſhall haue the angle at the parallell. Then

As the ſine of the angle at the parallell,
  to the coſine of the altitude:
So the length of the axis,
  to the diſtance betweene the verticall point and the parallell.

Thus in our example if it were required to finde the diſtance vpon the ſtylar azimuth V H, betweene the verticall point and the horizon, you haue the rectangle triangle VBH, wherein the angle at the horizon here repreſented by BHV is (equall to the inclination of the plane) 36 *gr.* and B V the axis of the horizon betweene the plane and the top of the ſtyle, is 7 inches 42 *cent.* Wherefore extend the compaſſes from the ſine of 36 *gr.* vnto the ſine of 90 *gr.* the complement of the altitude, the ſame extent will reach in the line of numbers from 7.42 vnto 12.62, and ſuch is the diſtance of the perpendicular azimuth line V H betweene the verticall point and the horizon.

In like maner if you would find the diſtance vpon the meridian between the verticall point and the horizon, extend the compaſſes from the ſine of 33 *gr.* 30 *m.* the angle at the horizon, to the ſine of 90 *gr.* the ſame extent will reach in the line of numbers from 7.42 vnto 13.44, and ſuch is *Va* the diſtance betweene the verticall point and the horizon vpon the line of the South azimuth, that is, vpon the meridian line.

But if you would find the diſtance vpon the meridian betweene the verticall point and any other parallell of the horizon, as vpon the parallell of 26 *gr.* 34 *m.* then adde theſe 26 *gr.* 34 *m.* vnto 33 *gr.* 30 *m.* the angle at the horizon, ſo ſhall you haue 60 *gr.* 4 *m.* for BDV the angle at the parallell. And if you extend the compaſſes from the ſine of 60 *gr.* 4 *m.* vnto the ſine of 63 *gr.* 26 *m.* the complement of the parallell from the horizon, the ſame extent will reach in the line of numbers from 7.42 the length of the axis, vnto 7.66, and ſuch is the diſtance VD between the verticall point and the parallel of 26 *gr.* 34 *m.* vpō the meridian line. The like reaſon holdeth for all the reſt, which may be gathered & ſet down in ẏ table.

That done, and the horizon drawne as before, if you would draw the parallel of 26 *gr.* 34 *m.* from the horizon, looke into the table, and there finding vnder the title of the parallel of 26.34, the diſtance on ẏ South azimuth line to be 7.66, take 7 inches 66 *cent.* out of a line of inches, and prick them down on the meridian of your plane, from the verticall point at *V.*

Or if either the verticall point fall without your plane, or the extent at any time be too large for your compaſſes, you may pricke downe the diſtance betweene the horizon and the parallel. As here the diſtance betweene the vertical point and the parallell is 7.66, betweene the verticall point and the horizon 13.44, the difference between them 5.78 is the diſtance frō the horizon to the parallel, which being pricked downe vpon the meridian, ſhall giue the ſame interſection as before. And the like reaſon, holdeth for the pricking down the reſt of theſe diſtances on their ſeuerall azimuths.

Hauing the points of interſection betweene the azimuths and the parallell, you may ioyne them all in a crooked line without making of angles, the line ſo drawne ſhall be the parallell required. And vpon this ground it followeth,

To deſcribe ſuch parallels on the former planes, as may ſhew the proportion of the ſhadow vnto the gnomon.

The proportion of a mans ſhadow vnto his height, or o-

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ther ſhadow to his gnomon ſet perpendicular to the horizon, may be ſhewed by parallels to the horizon, if they be drawne to a due altitude, which may thus be found:

As the length of the ſhadow, to the length of the gnomon:
So the tangent of 45 *gr.* to the tangent of the altitude.

As if it were required to finde the altitude of the Sunne when the ſhadow of a man ſhall be decuple to his height, extend the compaſſes from 10 vnto 1 in the line of numbers, the ſame extent will reach in the tangent of 45 *gr.* vnto the tangent of 5 *gr.* 42 *m*; which ſhewes that when the Sun commeth to the altitude of 5 *gr.* 42 *m.* your ſhadow, vpon a leuell ground, will be ten times as much as your height. In the ſame maner you may finde that at 7 *gr.* 7 *m.* of altitude your ſhadow wil be octuple, at 9 *gr.* 27 *m.* ſextuple, at 11 *gr.* 18 *m.* quintuple, at 14 *gr.* 2 *m.* quadruple, at 18 *gr.* 26 *m.* triple, at 26 *gr.* 34 *m.* double to your height, at 33 *gr.* 41 *m.* as 3 vnto 2, at 36 *gr.* 52 *m.* as 4 vnto 3, at 38 *gr.* 40 *m.* as 5 vnto 4, at 45 *gr.* equall, at 51 *gr.* 20 *m.* as 4 vnto 5, at 53 *gr.* 7 *m.* as 3 vnto 4, at 56 *gr.* 19 *m.* as 2 vnto 3, at 59 *gr.* 2 *m.* as 3 vnto 5, at 63 *gr.* 26 *m.* as 1 vnto 2, &c.

If then you draw a parallell to the horizon at 5 *gr.* 42 *m.* another at 7 *gr.* 7 *m.* and ſo the reſt, when the ſhadow of the ſtyle falleth on the parallell, you haue the proportion, and thereby may you know the ſhadow by the height, and the height by the ſhadow, whereof you haue examples *Pag.* 126. and 137.

I might here proceed to ſhew the deſcription of the circles of poſition, the Signes of the Zodiacke in the meridian, the Signes aſcending and deſcending, with ſuch other gnomonicall concluſions; but theſe would proue ſuperfluous to ſuch as vnderſtand the doctrine of the Sphere; and for others, that which is deliuered may ſuffiſe for ordinary vſe, it being my intention not ſo much to explane the full vſe of ſhadowes (whereof I haue lately giuen a large example in an other place) as the vſe of theſe lines of proportion, that were not extant heretofore.

An Appendix concerning

The deſcription and vſe of a ſmall portable Quadrant, for the more eaſie finding of the houre and Azimuth.

CHAP. I.

Of the deſcription of the Quadrant.

HAuing deſcribed theſe ſtanding planes, I will now ſhew the moſt of theſe concluſions by a ſmall Quadrant. This might be done generally for all latitudes, by a quarter of the generall Aſtrolabe, deſcribed before in the vſe of the *Sector, pag.* 58; and particularly for any one latitude, by a quarter of the particular Aſtrolabe, there alſo deſcribed, *pag.* 63. which if it be a foote ſemidiameter, may ſhew the azimuth vnto a degree, and the time of the day vnto a minute; but for ordinary vſe this ſmaller Quadrant may ſuffice, which may be made portable in this maner.

1 Vpon the center *A*, and ſemidiameter *AB*, deſcribe the arke *BC*: the ſame ſemidiameter will ſet of 60 *gr.* and the halfe of that will be 30 *gr.* which being added to the former 60 *gr.* will make the arke *BC* to be 90 *gr.* the fourth part of the whole circle, and thence comes the name of a Quadrant.

2 Leauing ſome little ſpace for the inſcription of the moneths and dayes, on the ſame center *A*, and ſemidiameter *AT*, deſcribe the arke *TD*, which ſhall ſerue for either tropique.

3 Diuide the line *AT* in the point *E*, in ſuch proportion, as that *AT* being 10000, *AE* may be 6556, and there draw another arke *EF*, which ſhall ſerue for the Equator.

4 Diuide *AF* the ſemidiameter of the equator in the point *G*, ſo as *AF* being 10000, the line *AG* may be 4343,

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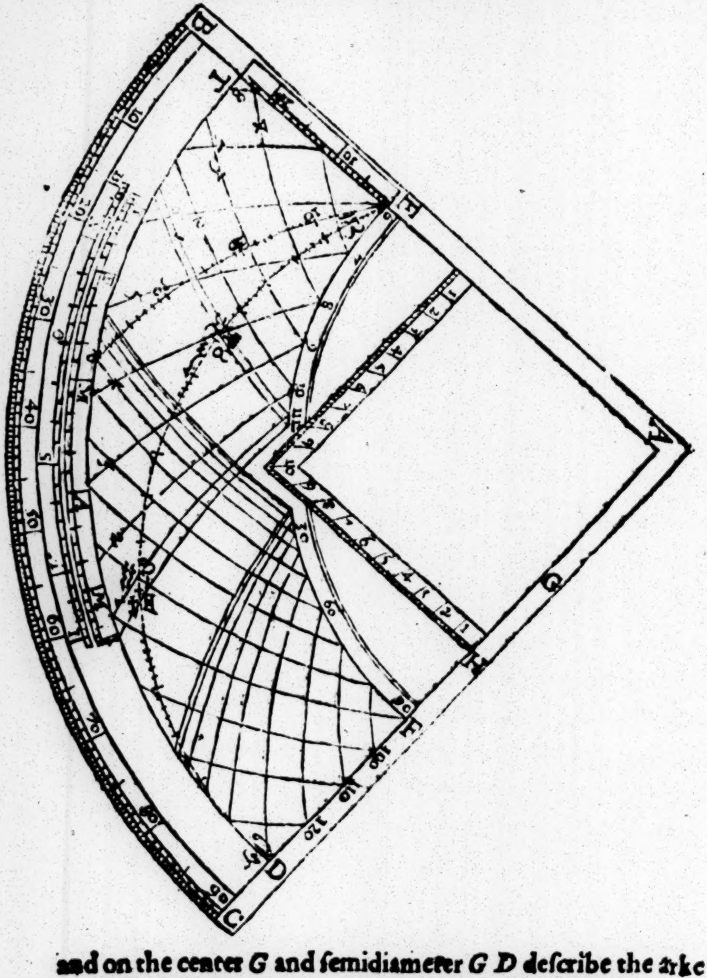

and on the center *G* and ſemidiameter *G D* deſcribe the arke *ED*, which ſhall ſerue for a fourth part of the ecliptique.

5 This part of the ecliptique may be diuided into three

Signes,

Gr.	♈		♉		♊	
	Gr.	M.	Gr.	M.	Gr.	M.
0	0	0	27	54	57	48
5	4	35	32	42	63	3
10	9	11	37	35	68	21
15	13	48	42	31	73	43
20	18	27	47	33	79	7
25	23	9	52	38	84	32
30	27	54	57	48	90	0

A Table of right Aſcenſions.

Signes, and each Signe into 30 *gr.* by a table of right aſcenſions, made as before *pag.*60. As the right aſcenſion of the firſt point of ♉ being 27 *gr.* 54 *m.* you may lay a ruler to the center *A* & 27 *gr.* 54 *m.* in the Quadrant *BC*, the point where the ruler croſſeth the Ecliptique, ſhall be the firſt point of ♉. In like maner the right aſcenſion of the firſt point of ♊ being 57 *gr.* 48 *m.* if you lay a ruler to the center *A*, and 57 *gr.* 48 *m.* in the quadrant, the point where the ruler croſſeth the ecliptique, ſhalbe the firſt point of ♊. And ſo for the reſt: but the lines of diſtinction between Signe & Signe, may be beſt drawne from the center *G*.

6 The line *E T* betweene the equator and the tropique, which I call the line of declination, may be diuided into 23 *gr.* ½ out of this Table. For let *A E* the ſemidiameter of the equator be 10000, the diſtance betweene the equator and 10 *gr.* of declination may be 1917 more; between the equator and 20 *gr.* 4281; the diſtance of the tropique from the equator 5252.

7 You may put in the moſt of the principall ſtarres betweene the equator and the tropique of ♋, by their declination from the equator, and right aſcention from the next equinoctial point. As the declination of the *wing of Pegaſus*, being 13 *gr.* 7 *m.* the right aſcenſion 358 *gr.* 34 *m.* from the firſt point of ♈, or 1 *gr.* 26 *m.* ſhort of it. If you draw an occult parallell through 13 *gr.* 7 *m.* of declination, and then lay the ruler to the center *A*, and 1 *gr.* 26 *m.* in the quadrant *B C*, the point where the ruler croſſeth the parallell ſhall be the place for the *wing of Pegaſus*, to which you may

Gr.	Parts.
1	176
2	355
3	537
4	723
5	913
6	1106
7	1302
8	1503
9	1708
10	1917
11	2130
12	2348
13	2571
14	2799
15	3032
16	3270
17	3514
18	3763
19	4019
20	4281
21	4550
22	4825
23	5108
Tro	5252

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ſet the name and the time when he cometh to the South, in this maner, ***W.Peg.*** * 23 ***Ho.*** 54 ***M.*** and ſo for the reſt of theſe fiue, or any other ſtarres.

		Ho.	M.	R. Aſcen		Decl.	*M*
Pegaſus wing	*	23	54	1	26	13	7
Arcturus	*	13	58	29	37	21	10
Lions heart	*	9	48	32	58	13	45
Buls eye	*	4	15	63	33	15	42
Vultures heart	*	19	33	66	56	7	58

8 There being ſpace ſufficient between the equator and the center, you may there deſcribe the quadrat, and diuide each of the two ſides fartheſt frō the center *A* into 100 parts, ſo ſhall the Quadrant be prepared generally for any latitude.

But before you draw the particular lines, you are to fit foure tables vnto your latitude.

Firſt a table of meridian altitudes for diuiſion of the circle of dayes and moneths, which may be thus made: Conſider the latitude of the place and the declination of the Sun for each day of the yeare. If the latitude and declination be alike both North or both South, ad the declinatiō to the complement of the latitude; if they be vnlike, one North, and the other South, ſubſtract the declination from the complement of the latitude, the remainder will be the meridian altitude belonging vnto the day.

Thus in our latitude of 51 *gr.* 30 *m.* Northward, whoſe complement is 38 *gr.* 30 *m.* the declination vpon the tenth day of Iune will be 23 *gr.* 30 *m.* Northward, wherefore I adde 23 *gr.* 30 *m.* vnto 38 *gr.* 30 *m.* the ſumme of both is 62 *gr.* for the meridian altitude at the tenth of Iune. The declination vpon the tenth of December will be 23 *gr.* 30 *m.* Southward, wherefore I take theſe 23 *gr.* 30 *m.* out of 38 *gr.* 30 *m.* there wil remain 15 *gr.* for the meridian altitude at the tenth of December; and in this maner you may find the meridian altitude for each day of the yeare, and ſet them downe in a table.

Dies.	0		5		10		15		20		25		30	
Mo.	Gr.	M	Gr.	*M.*	Gr.	M	Gr.	*M.*	Gr.	M	Gr.	*M*	Gr.	*M.*
Ianuary	16	31	17	24	18	26	19	37	20	57	22	24	23	58
February	24	17	25	59	27	45	29	35	31	29	33	25		
March	34	35	36	33	38	32	40	30	42	27	44	22	46	15
April	46	37	48	26	50	11	51	50	53	25	54	53	56	15
May	56	15	57	29	58	35	59	33	60	22	61	2	61	31
Iune	61	36	61	54	62	0	61	58	61	45	61	22	60	49
Iuly	60	49	60	6	59	14	58	13	57	4	55	48	54	24
Auguſt	54	7	52	36	50	59	49	17	47	31	45	41	43	49
September	43	26	41	30	39	33	37	35	35	38	33	41	31	46
October	31	46	29	53	28	3	26	16	24	35	22	59	21	29
Nouember	21	12	19	51	18	39	17	36	16	43	16	0	15	28
December	15	28	15	7	15	0	15	2	15	17.	15	44	16	22

The Table being made, you may inſcribe the moneths, and dayes of each moneth into your quadrant, in the ſpace left below the tropique. For lay the ruler vnto the center *A*, and **16** *gr.* 3**1** *m.* in the quadrant *BC*, there may you draw a line for the end of December and beginning of Ianuary; then laying your ruler to the center *A*, and **24** *gr.* **17** *m.* in the quadrant, there draw the end of Ianuary and beginning of February, and ſo the reſt, which may be noted with *I, F, M, A, M, I*, &c. the firſt letters of each moneth, and wil here fall betweene 15 *gr.* and 62 *gr.*

The ſecond Table which you are to fit, may ſerue for the drawing and diuiding of the horizon. For drawing of the horizon.

As the cotangent of the latitude,
 to the tangent of the greateſt declination:
So the ſine of 90 *gr.*
 to the ſine of interſection, where the horizon ſhall
 croſſe the tropiques.

So in our latitude of 51 *gr.* 30 *m.* we ſhall find the horizon to cut the tropique in 33 *gr.* 9 *m*: wherefore if you lay the ru-

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ler to the center *A*, and 33 *gr.* 9 *m.* in the quadrant, the point where the ruler croſſeth the tropique ſhall be the point where the horizon croſſeth the tropique. And if you finde a point at *H*, in the line *AC*, whereon ſetting the compaſſes, you may bring the point at *E*, and this point in the tropique both into a circle, the point *H* ſhall be the center, and the ark ſo drawne ſhall be the horizon. Then for the diuiſion of this horizon.

As the ſine of 90 *gr.*
  to the ſine of the latitude:
So the tangent of the horizon,
  to the tangent of the arke in the quadrant, which ſhall diuide the horizon.

So in our latitude of 51 *gr.* 30 *m.* we ſhall finde 7 *gr.* 52 *m.* belonging to 10 *gr.* in the horizon, and 15 *gr.* 54 *m.* belonging to 20 *gr.* And ſo the reſt, as in this Table.

Ho	Gr.	M	Ho	Gr.	M.	Ho	Gr.	M.	Ho	Gr.	M.	Ho	Gr.	M.	Ho	Gr.	M.
0	0	0	15	11	51	30	24	19	45	38	2	60	53	35	75	71	5
	0	47		12	39		25	11		39	1		54	41		72	19
	1	34		13	27		26	4		40	0		55	48		73	33
	2	21		14	16		26	57		41	0		56	56		74	48
	3	8		15	4		27	50		42	0		58	4		76	3
5	3	55	20	15	54	35	28	43	50	43	0	65	59	13	80	77	18
	4	42		16	43		29	37		44	1		60	22		78	33
	5	29		17	33		30	32		45	3		61	31		79	49
	6	17		18	22		31	27		46	5		62	41		81	6
	7	4		19	12		32	22		47	8		62	52		82	21
10	7	52	25	20	2	40	33	18	55	48	11	70	65	3	85	83	37
	8	39		20	53		34	14		49	14		66	15		84	53
	9	27		21	44		35	10		50	19		67	27		86	10
	10	14		22	36		36	7		51	24		68	39		87	26
	11	2		23	27		37	4		52	29		69	52		88	43
15	11	51	30	24	19	45	38	2	60	53	35	75	71	5	90	90	0

Wherefore you may lay the ruler to the center *A*, and 7 *gr*.52 *m*. in the quadrant *B C*, the point where the ruler croſſeth the horizon ſhall be 10 *gr*. in the horizon; and ſo for the reſt: but the lines of diſtinction between each fift degree, will be beſt drawne from the center *H*.

The third Table for drawing of the houre-lines, muſt be a Table of the altitude of the Sunne aboue the horizon at euery houre, eſpecially when he cometh to the equator, the tropiques, and ſome other intermediate declinations.

If the Sunne be in the equator, and ſo haue no declination.

As the ſine of 90 *gr*.
  to the coſine of the latitude:
So the coſine of the houre from the meridian,
  to the ſine of the altitude.

Thus in our latitude of 51 *gr*.30 *m*. at ſix houres from the meridian the Sunne wil haue no altitude, at fiue the altitude will be 9 *gr*.17 *m*; at foure 18 *gr*.8 *m*; at three 26 *gr*.7 *m*; at two 32 *gr*.37 *m*. at one 36 *gr*.58 *m*; at noone it will be 38 *gr*. 30 *m*. equall to the complement of the latitude.

If the Sunne haue declination, the meridian altitude wil be found as before, for the Table of dayes and moneths.

If the houre propoſed be ſix in the morning or ſix at night.

As the ſine of 90 *gr*.
  to the ſine of the latitude:
So the ſine of the declination,
  to the ſine of the altitude.

Thus in our latitude the declination of the Sunne being 23 *gr*.30 *m*. the altitude will be found to be 18 *gr*.11 *m*: the declination being 11 *gr*.30 *m*. the altitude will be 9 *gr*.

If the houre propoſed be neither twelue nor ſix.

As the coſine of the houre from the meridian,
  to the ſine of 90 *gr*.

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So the tangent of the latitude,
to the tangent of a fourth arke.

So in our latitude and one houre from the meridian, this fourth ark will be found to be 52 *gr.*28 *m.* at two 55 *gr.*26 *m.* at three 60 *gr.*39 *m.* at foure 68 *gr.*22 *m.* and at fiue houres from the meridian 78 *gr.*22 *m.*

Then conſider the declination of the Sunne and the houre propoſed; if the latitude and declination be both alike, as with vs in North latitude, North declination, and the houre fall between noone and ſix, take the declination out of the fourth arke, the remainer ſhall be your fift arke.

But if either the houre fall between ſix and midnight, or the latitude and declination ſhall be vnlike, adde the declination vnto the fourth arke, and the ſumme of both ſhall be your fifth arke: or if the ſumme ſhall exceed 90 *gr.* you may take the complement vnto 180 *gr.* This fifth arke being knowne:

As the ſine of the fourth arke,
to the ſine of the latitude:
So the coſine of the fift arke,
to the ſine of the altitude.

Thus in our latitude of 51 *gr.*30 *m.* Northward, the Sunne hauing 23 *gr.*30 *m.* of North declination, if it ſhall be required to finde the altitude of the Sunne for ſeuen in the morning; here becauſe the latitude and declination are both alike to the Northward, and the houre propoſed falleth betweene noone and ſix, you may take 23 *gr.*30 *m.* the arke of the declination out of 78 *gr.*22 *m.* the fourth arke belonging to the fift houre from the meridian, ſo there will remaine 54 *gr.*52 *m.* for your fift arke. Then working according to the Canon, you ſhall find,

As the ſine of 78 *gr.*22 *m.* your fourth arke,
to the ſine of 51 *gr.*30 *m.* for the latitude:

So the ſine of 35 *gr.* 8 *m.* the cōplement of your fift ark,
to the ſine of 27 *gr.* 17 *m.* the altitude required.

If in the ſame latitude and declination, it were required to finde the altitude for fiue in the morning, here the houre falling betweene ſix and midnight; if you adde 23 *gr.* 30 *m.* vnto 78 *gr.* 22 *m.* the ſumme will be 101 *gr.* 52 *m.* and the complement to 180 *gr.* will be 78 *gr.* 8 *m.* for your fifth arke. Wherefore

As the ſine of 78 *gr* 22 *m.*
to the ſine of 51 *gr.* 30 *m.*
So the coſine of 78 *gr* 8 *m.*
to the ſine of 9 *gr.* 32 *m.* for the altitude required.

If in the ſame latitude of 51 *gr.* 30 *m.* Northward, the Sunne hauing 23 *gr.* 30 *m.* of South declination, it were required the altitude for nine in the morning; here becauſe the latitude and declination are vnlike, the one North, and the other South, you may adde 23 *gr.* 30 *m.* the arke of declination, vnto 60 *gr.* 39 *m.* the fourth arke belonging to the third houre from the meridian, ſo ſhall you haue 84 *gr.* 9 *m* for your fift arke. Wherefore

As the ſine of 60 *gr.* 39 *m.*
to the ſine of 51 *gr.* 30 *m.*
So the coſine of 84 *gr.* 9 *m.*
to the ſine of 5 *gr.* 15 *m.* for the altitude required.

And ſo by one or other of theſe meanes you may finde the altitude of the Sunne for any point of the ecliptique at all houres of the day, and ſet them downe in ſuch a Table as this.

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A Table for the altitude of the Sunne in the beginning of each Signe at all houres of the day, calculated for 51 gr.30 m. of North latitude.

Ho.		♋		♊ ♌		♉ ♍		♈ ♎		♓ ♏		♒ ♐		♑	
		Gr.	M.	Gr.	*M.*	Gr.	*M.*	Gr.	M.	Gr	*M.*	Gr.	M.	Gr.	M.
12		62	0	58	42	50	0	38	30	27	0	18	18	15	0
11	1	59	43	56	34	48	12	36	58	25	40	17	6	13	52
10	2	53	45	50	55	43	12	32	37	21	51	13	38	10	30
9	3	45	42	43	6	36	0	26	7	15	58	8	12	5	15
8	4	36	41	34	13	27	31	18	8	8	33	1	15		
7	5	27	17	24	56	18	18	9	17	0	6				
6	6	18	11	15	40	9	0	0	0						
5	7	9	32	6	50									11	37
4	8	1	32											21	40

Laſtly, you may finde what declination the Sunne hath when he riſeth or ſetteth at any houre.

As the ſine of 90 *gr.*
to the ſine of the houre from ſix:
So the cotangent of the latitude,
to the tangent of the declination.

And ſo in the latitude of 51 *gr.* 30 *m.* you ſhall finde that when the Sun riſeth, either at fiue in the Summer, or ſeuen in the Winter, his declination is 11 *gr.* 37 *m*: when he riſeth at foure in the Summer, or eight in the Winter, his declination is 21 *gr.* 40 *m.* which may be alſo ſet downe in the Table.

That done, you may there ſee that in this latitude the meridian altitude of the Sunne in the beginning of ♋ is 62 *gr.* in ♊ 58 *gr.* 42 *m.* in ♉ 50 *gr.* in ♈ 38 *gr.* 30 *m.* &c. But the beginning of ♋ and ♑ is repreſented by the tropiques *TD*, drawne at 23 *gr.* 30 *m.* of declination, and the beginning of ♈ and ♎, by the equator *EF*. If you draw an occult parallell betweene the equator and the tropique, at 11 *gr.* 30 *m.* of declination,

clination, it ſhall repreſent the beginning of ♉, ♍, ♏, and ♓; if you draw another occult parallell though 20 *gr.* 12 *m* of declination, it ſhall repreſent the beginning of ♊, ♌, ♐, and ♒.

Then you may lay a ruler to the center *A*, and 62 *gr.* in the quadrant *BC*, and note the point where it croſſeth the tropique of ♋; then moue the ruler to 58 *gr.* 52 *m.* and note where it croſſeth the parallell of ♊; then to 50 *gr.* and note where it croſſeth the parallell of ♉; and againe to 38 *gr.* 30 *m.* noting where it croſſeth the equator; ſo the line drawne through theſe points ſhall ſhew the houre of 12 in the Summer, while the Sunne is in ♈, ♉, ♊, ♋, ♌, or ♍. In like maner if you lay the ruler to the center *A*, and 27 *gr.* in the quadrant, and note the point where it croſſeth the parallel of ♓, then moue it to 18 *gr.* 18 *m.* and note where it croſſeth the parallell of ♒; and againe to 15 *gr.* noting where it croſſeth the tropique of ♑; the line drawne through theſe points ſhall ſhew the houre of 12 in the Winter, while the Sunne is in ♎, ♏, ♐, ♑, ♒ & ♓, and ſo may you draw the reſt of theſe houre-lines: onely that of 7 from the meridian in the Summer, and 5 in the Winter, will croſſe the line of declination at 11 *gr.* 37 *m.* and that of 8 in the Summer, and 4 in the Winter at 21 *gr.* 40 *m.*

The fourth table for drawing of the azimuth lines, muſt likewiſe be fitted for the altitude of the Sunne aboue the horizon at euery azimuth, eſpecially when he cometh to the equator, the tropiques, and ſome other intermediat declinations.

If the Sunne be in the equator, and ſo haue no declination:

As the ſine of 90 *gr.*
to the coſine of the azimuth from the meridian:
So the cotangent of the latitude,
to the tangent of the altitude at the equator.

Thus in our latitude of 51 *gr.* 30 *m.* at 90 *gr.* from the meridian, the Sunne will haue no altitude; at 80 *gr.* the altitude

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will be *7 gr.52 m*; at *70 gr.* it will be *15 gr.30 m*; at *60 gr.* it will be *21 gr.41 m*.

If the Sunne haue declination, the meridian altitude will be eaſily found as before, for the table for dayes and moneths. And for all other azimuths.

As the ſine of the latitude,
 to the ſine of the declination:
So the coſine of the altitude at the equator,
 to the ſine of a fourth arke.

When the latitude and declination are both alike in all azimuths from the prime verticall vnto the meridian, adde this fourth arke vnto the arke of altitude at the equator.

When the latitude and declination are both alike, and the azimuth more then *90 gr.* diſtant from the meridian, take the altitude at the equator out of this fourth arke.

When the latitude and declination are vnlike, take this fourth arke out of the arke of altitude at the equator, ſo ſhall you haue the altitude of the Sunne belonging to the azimuth.

Thus in our latitude of *51 gr.*30 *m*. Northward, if it were required to finde the altitude of the Sunne in the azimuth of *60 gr.* from the meridian, when the declination is *23 gr.* 30 *m*. Norhward, you may finde the altitude at the equator belonging to this azimuth to be *21 gr.*41 *m*. by the former Canon, and by this laſt Canon you may finde the fourth arke to be *28 gr.*15 *m*. Then becauſe the latitude and declination are both alike to the Northward, if you adde them both together, you ſhall haue *49 gr.*56 *m*. for the altitude required.

If the declination had been *23 gr.*30 *m*. to the Southward, you ſhould then haue taken this fourth ark out of the ark at the equator, which becauſe it cannot here be done, it is a ſigne that the Sunne is not then aboue the horizon. But if you take the arke at the equator out of this fourth arke, you ſhall haue *6 gr.34 m*. for the altitude of the Sunne when he is in the azimuth of 60 *gr.* from the North, and 120 *gr.* from the South part of the meridian. The like reaſon holdeth for the reſt of theſe altitudes, which may be gathered and ſet down in a table.

Laſtly when the Sun riſeth or ſetteth vpon any azimuth, to find his declination.

> As the ſine of 90 *gr.*
> to the coſine of the latitude:
> So the coſine of the azimuth from the meridian,
> to the ſine of the declination.

And thus in our latitude of 51 *gr.* 30 *m.* when the azimuth is 80 *gr.* from the meridian, the declination will be found to be 6 *gr.* 12 *m*; if the azimuth be 70 *gr.* the declination will be found 12 *gr.* 18 *m*; if 60 *gr.* then 18 *gr.* 8 *m.* And ſo for the reſt, which may be alſo ſet downe in the Table.

A Table for the altitude of the Sunne in the beginning of each Signe for euery tenth azimuth, in 51 gr. 30 m. of North latitude.

Az.	♋		♊	♌	♉	♍	♈	♎	♓	♏	♒	♐	♑	
	Gr.	M	Gr.	M	Gr.	M.	Gr.	M	Gr.	M	Gr.	M.	Gr.	M
0	62	0	58	42	50	0	38	30	27	0	18	18	15	0
10	61	43	58	24	49	38	38	4	26	30	17	45	14	25
20	60	51	57	28	48	33	36	46	25	0	16	5	12	41
30	59	52	55	52	45	40	34	34	22	27	13	15	9	45
40	57	10	53	29	43	55	31	21	18	48	9	14	6	34
50	54	3	50	12	40	11	27	5	13	50	3	57	0	6
60	49	56	45	53	35	23	21	41	8	0				
70	44	40	40	25	29	27	15	13	1	0				
80	38	11	33	46	21	29	7	52						
90	30	38	26	10	14	45	0	0						
100	22	27	18	2	6	45							6	12
110	14	14	9	58									12	18
120	6	34	2	30									18	8

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That done, if you would draw the line of Eaſt or Weſt, which is 90 *gr.* from the meridian, lay the ruler to the center *A*, and 30 *gr.* 38 *m.* numbred in the quadrant from *C* toward *B*, and note the point where it croſſeth the tropique of ♋; then moue the ruler to 26 *gr.* 10 *m.* and note where it croſſeth the parallell of ♊; then to 14 *gr.* 45 *m.* and note where it croſſeth the parallell of ♉; then to 0 *gr.* 0 *m.* and you ſhall finde it to croſſe the equator in the point *F*; ſo a line drawne through theſe points, ſhall ſhew the azimuth belonging to Eaſt and Weſt. The like reaſon holdeth for all the reſt.

Theſe lines being thus drawne, if you ſet two ſights vpon the line *A C*, and hang a thread and plummet on the center *A*, with a bead vpō the thread, the foreſide of the quadrant ſhall be fully finiſhed.

Or in ſtead of the fiue ſtarres before mentioned, you may place the *Nocturnall* (deſcribed before in the vſe of the *Sector, pag.* 60.) on the backſide of the Quadrant, and ſo alſo it will be fitted both for day and night.

CHAP. II.

Of the vſe of the Quadrant in taking the altitude of the Sunne, Moone, and Starres.

THe Quadrant is the fourth part of a circle, diuided equally into 90 *gr.* and here numbred by 10.20 30.&c. vnto 90 *gr.* each degree being ſubdiuided into 4.

Lift vp the center of the Quadrant, ſo as the thread with the plummet may play eaſily by the ſide of it, and the Sunne beames may paſſe through both the ſights; ſo ſhall the degrees cut by the thread, ſhew what is the altitude at the time of obſeruation, as may appeare by this example.

Vpon the 14 day of April, about noone, the Sun-beames paſſing through both the ſights, the thread fell vpon 51 *gr.* 20 *m.* and this was the true meridian altitude of the Sun for that day in this our latitude of 51 *gr.* 30 *m.* for which this Quadrant was made.

Againe, towards three of the clocke in the afternoone, the thread fell vpon 38 *gr.* 40 *m.* and ſuch was the Sunnes altitude at that time.

CHAP. III.

Of the Ecliptique.

1 *The place of the Sunne being giuen to finde his right aſcenſion.*

THe Ecliptique is here repreſented by the arke, figured with the characters of the twelue Signes, ♈, ♉, ♊, &c. each Signe being diuided vnequally into 30 *gr.* and they are to be reckoned from the character of the Signe.

Let the thread be laid on the place of the Sunne in the Ecliptique, and the degrees which it cutteth in the Quadrant ſhall be the right aſcenſion required.

As if the place of the Sunne giuen be the fourth degree of ♊, the thread laid on this degree ſhall cut 62 degrees in the Quadrant, which is the right aſcenſion required.

But if the place of the Sunne giuen be more then 90 *gr.* from the beginning of ♈, there muſt be more then 90 *gr.* allowed to the right aſcenſion; for this inſtrument is but a quadrant: and ſo if the Sunne be in 26 *gr.* of ♋, you ſhall finde the thread to fall in the ſame place, and yet the right aſcenſion to be 118 *gr.*

2 *The right aſcenſion of the Sunne being giuen, to find his place in the Ecliptique.*

Let the thread be laid on the right aſcenſion in the Quadrant, and it ſhall croſſe the place of the Sunne in the Ecliptique, as may appeare in the former example.

CHAP. IIII.

Of the line of declination.

1 *The place of the Sunne being giuen to find his declination.*

THe line of declination is here drawne from the center to the beginning of the Quadrant, and diuided from the beginning of ♈ downward into 23 *gr.* 30 *m.*

Let the thread be laid, and the beade ſet on the place of Sunne in the ecliptique; then moue the thread to the line of declination, and there the bead ſhal fall vpon the degrees of the declination required.

As if the place of the Sunne giuen be the fourth degree of ♊, the beade firſt ſet to this place, and then moued to the line of declination, ſhall there ſhew the declination of the Sunne at that time to be 21 *gr.* from the equator.

2 *The*

2 *The declination of the Sunne being giuen, to find his place in the Ecliptique.*

Let the thread and beade be firſt laid to the declination, and then moued to the Ecliptique.

As if the declination be 21 *gr.* the beade firſt ſet to this declination, and then moued to the ecliptique, ſhall there ſhew the fourth of ♊, the fourth of ♐, the 26 of ♋, and the 26 of ♑; and which of theſe foure is the place of the Sunne, may appeare by the quarter of the yeare.

CHAP. V.

Of the circle of Moneths and Dayes.

THis circle is here repreſented by the arke, figured with theſe letters, *I, F, M, A, M,* &c. ſignifying the moneths January, February, March, April, &c. each moneth being diuided vnequally, according to the number of the dayes that are therein.

1 *The day of the moneth being giuen, to find the altitude of the Sunne at noone.*

Let the thread be laid to the day of the moneth, and the degrees which it cutteth in the Quadrant ſhall be the meridian altitude required.

As if the day giuen be the 15 of May, the thread laid on this day ſhall cut 59 *gr.* 30 *m.* in the quadrant, which is the meridian altitude required.

2 *The meridian altitude being giuen, to finde the day of the moneth.*

The thread being ſet to the meridian altitude, doth alſo fall on the day of the moneth.

As if the altitude at noone be 59 *gr.* 30 *m.* the thread being ſet to this altitude, doth fal on the 15 day of May and the 9 of Iuly; and which of theſe two is the true day, may be knowne by the quarter of the yeare, or by another dayes obſeruation. For if the altitude proue greater, the thread wil fall on the 16 day of May and the 8 of Iuly: or if it proue leſſer, the thread will fall on the 14 of May and the 10 of Iuly; whereby the queſtion is fully anſwered.

CHAP. VI.

Of the Houre-lines.

THat arke which is drawne vpon the center of the quadrant by the beginning of declination, doth here repreſent the equator: that arke which is drawne by 23 *gr.* 30 *m.* of declination, and is next aboue the circle of moneths and dayes, repreſenteth the tropiques: thoſe lines which are betweene the equator and the tropiques, being vndiuided and numbred at the equator by 6.7 8.9.10.11.12. at the tropique by 1.2.3.4.&c. do repreſent the houre-circles: that which is drawne from 12 in the equator to the middle of Iune, repreſenteth the houre of 12 at noone in the Summer; and thoſe which are drawn with it to the right hand, are for the houres of the day in the Summer, and the houres of the night in the Winter. That which is drawne from 12 in the equator to the middle of December, repreſenteth the houre of 12 in the Winter; and thoſe which are drawne with it to the left hand, are for the houres of the day in the Winter, and the houres of the night in the Summer; and of both theſe, that which is drawne from 11 to 1, ſerues for 11 in the forenoone, and 1 in the afternoon. That which is drawne frō 10 to 2, ſerues for 10 in the forenoon, & 2 in the afternoon: for the Sun on the ſame day is about the ſame height two houres before noon, as two houres after noone. The like reaſon holdeth for the reſt of the houres.

1 *The day of the moneth, or the height at noone being knowne, to finde the place of the Sunne in the Ecliptique.*

The thread being laid to the day of the moneth, or the height at noone, (for one giues the other by the former propoſition) marke where it croſſeth the houre of 12, and ſet the bead to that interſection; then moue the thread till the bead fall on the ecliptique, and it ſhall fall on the place of the Sun.

As if the day giuen be the 15 of May, or the meridian altitude 59 *gr.* 30 *m.* lay the thread accordingly, and put the bead to the interſection of the thread with the houre of 12; then moue the thread till the bead fall on the ecliptique, and it ſhall there ſhew the fourth of ♊, the fourth of ♐, the 26 of ♋, and the 26 of ♑; and which of theſe is the place of the Sunne, may appeare by the quarter of the yeare, or another dayes obſeruation.

2 *The place of the Sun in the Ecliptique being knowne, to finde the day of the moneth, &c.*

Let the thread and beade be firſt laid on the place of the Sunne in the Ecliptique, and then moued to the line of 12.

As if the place of the Sunne giuen, be the fourth of ♊, the bead being laid to this degree, and then moued to the houre of 12, in the Summer, the thread will fall on the 15 day of May, and the 9 of Iuly; or if it be moued to the houre of 12 in the Winter, the thread will fall on the 6 of Ianuary and the 16 of Nouember; which of theſe is the day of the moneth required, may appeare by the quarter of the yeare.

In this and the former propoſitions, you haue two wayes to rectifie the bead, by the place of the Sunne, and by the day of the moneth; the better way is by the place of the Sunne, for in the other the Leap-yeare may breed ſome ſmall difference.

There is yet a third way. For the Sea-men hauing a table for the declination on each day of the yeare, may ſet the bead thereto in the line of declination.

3 *The houre of the day being giuen to find the altitude of the Sunne aboue the horizon.*

The bead being ſet for the time by either of the three ways, let the thread be moued from the houre of 12 toward the line of declination, till the bead fall on the houre giuen; and the degrees which it cuts in the Quadrant, ſhall ſhew the altitude of the Sunne at that time.

As if the time giuen be the tenth of April, the Sunne being then in the beginning of ♉, the bead being rectified, you ſhall finde the height at noone 50 *gr.* 0 *m.* at 11 in the morning 48 *gr.* 12 *m.* at 10 but 43 *gr.* 12 *m.* at 9 but 36 *gr.* at 8 but 27 *gr.* 30 *m.* at 7 but 18 *gr.* 18 *m.* at 6 but 9 *gr.* at 5 it meeteth with the line of declination, and hath no altitude at all, and therefore you may think it did riſe much about that houre.

Then if you moue the thread again from the line of declination toward the houre of 12, you ſhal find that the Sun is 8 *gr.* 33 *m.* below the horizon at 4 in the morning, & neare 16 *gr.* at 3, and 21 *gr.* 51 *m.* at 2, and 25 *gr.* 40 *m.* at 1, and 27 *gr.* at midnight.

4 *The altitude of the Sunne being giuen, to finde the houre of the day.*

The altitude being obſerued as before, let the bead be ſet for the time, then bring the thread to the altitude, ſo the bead ſhall ſhew the houre of the day.

As if the 10 of April hauing ſet the bead for the time, you ſhall finde by the quadrant, the altitude to be 36 *gr.* the bead at the ſame time will fall vpon the houre-line of 9 and 3: wherefore the houre is 9 in the forenoone, or 3 in the afternoone. If the altitude be neare 40 *gr.* you ſhall find the bead at the ſame time to fall halfe way betweene the houre-line of 9 and 3, and the houre-line of 10 and 2: wherefore it muſt be either halfe an houre paſt 9 in the morning, or halfe an houre paſt 2 in the afternoone;and which of theſe is the true time of the day, may be ſoone knowne by a ſecond obſeruation: for if the Sunne riſe higher, it is the forenoone; if it become lower, it is the afternoone.

5 *The houre of the night being giuen, to find how much the Sunne is below the horizon.*

The Sunne is alwayes ſo much below the horizon at any houre of the night, as his oppoſite point is aboue the horizon at the like houre of the day; and therefore the beade being ſet, if the queſtion be made of any houre of the night in the Summer, then moue it to the like houre of the day in the Winter; if of any houre of the night in Winter, then moue it to the like houre of the day in Summer; ſo the degrees which the thread cutteth in the Quadrant ſhall ſhew how much the Sunne is below the horizon at that time.

As if it be required to know how much the Sunne is below the horizon the 10 of April at 4 of the clock in the morning; the bead being ſet to his place according to the time in the Summer houres, bring it to 4 of the clocke in the afternoone in the Winter houres, and ſo ſhall you finde the thread to cut 8 *gr.* and about 30 *m.* in the quadrant; and ſo much is the Sunne below the horizon at that time.

6 *The depreſſion of the Sunne ſuppoſed, to giue the houre of the night with vs, or the houre of the day to our Antipodes.*

Here alſo becauſe the Sun is ſo much aboue the horizon at all houres of the day, as his oppoſite point is below the horizon at the like houre of the night; therefore firſt ſet the bead according to the time, then bring the thread to the degree of the Suns depreſſion below the horizon, ſo ſhall the bead fall on the contrary houre-lines, and there ſhew the houre of the night in regard of vs, which is the like houre of the day to our Antipodes.

As if the 10 of April the Sunne being then in the beginning of ♉, and by ſuppoſition 8 *gr.* 30 *m.* below the horizon in the Eaſt, it be required to know what time of the night it is; firſt ſet the bead according to the day in the Summer houres, then bring the thread to 8 *gr.* 30 *m.* in the quadrant, ſo ſhall the bead fall among the Winter houres, on the line of 4 of the clocke in the afternoone: wherefore to our Antipodes it is 4 of the clocke in their afternoone, and to vs it is then 4 of the clocke in the morning.

7 *The time of the yeare or the place of the Sunne being giuen, to find the beginning of day-breake, and end of twi-light.*

This propoſition differeth little from the former: for the day is ſaid to begin to breake, when yͤ Sun cometh to be but 18 *gr.* below our horizon in the Eaſt, and twi-light to end when it is gotten 18 *gr.* below the horizon in the Weſt: wherefore let the bead be ſet for the time, and then bring the thread to 18 *gr.* in the quadrant, ſo ſhall the bead fall on the contrary houre-lines, and there ſhew the houre of twi-light as before.

So if it be required to know at what time the day begins to breake on the tenth of April, the Sun being then in the beginning of ♉; firſt ſet the bead according to the time in the Summer houres, and then bring the thread to 18 *gr.* in the quadrant, ſo ſhall the bead fall among the Winter houres a little more then a quarter before 3 in the morning; and that is the time when the day begins to break vpon the tenth of April.

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CHAP. VII.

Of the Horizon.

THe Horizon is here repreſented by the arke drawne, from the beginning of declination towards the end of February, diuided vnequally, and numbred by 10.20.30.40.

1 *The day of the moneth, or the place of the Sunne being knowne, to finde the amplitude of the Sunnes riſing and ſetting.*

Let the bead rectified for the time, be brought to the horizon, and there it ſhall ſhew the amplitude required.

As if the day giuen be the 15 of May, the Sunne being in the fourth degree of ♊, the bead rectified and brought to the horizon, ſhall there fall on 35 *gr.* 8 *m.* ſuch is the amplitude of the Sunnes riſing from the Eaſt, and of his ſetting from the Weſt; which amplitude is alwayes Northward when the Sunne is in the Northerne Signes, and when he is in the Southward Signes alwayes Southward.

2 *The day of the moneth, or the place of the Sunne being giuen, to finde the aſcenſionall difference.*

Let the bead rectified for the time, be brought to the horizon, ſo the degrees cut by the thread in the quadrant, ſhall ſhew the difference of aſcenſions.

As if the day giuen be the 15 of May, the Sunne being in the fourth degree of ♊, let the bead be rectified and brought to the horizon; ſo ſhall the thread in the quadrant ſhew the aſcenſionall difference to be 28 *gr.* and about 50 *m.*

Vpon the aſcenſionall difference depends this Corollarie.

To find the houre of the riſing and ſetting of the Sun, and thereby the length of the day and night.

The time of the Sunnes riſing may be gueſſed at by the 3 of the laſt *Cap.* but here by the aſcenſionall difference it may be better found, and that to a minute of time. For if the aſcenſionall difference be conuerted into time, allowing an houre for 15 *gr.* and 4 minutes of an houre for each degree, it ſheweth how long the Sunne riſeth before ſix of the clock in the Summer, and after ſix in the Winter.

As if the day giuen be the 15 of May, the Sunne being in the fourth of ♊, and his aſcenſionall difference found as before 28 *gr.* 50 *m*; this conuerted into time, maketh 1 *ho.* and ſomewhat more then 55 *m.* of an houre: wherefore the Sun at that time, in regard it was Summer, roſe 1 *ho.* and full 55 *m.* before 6 of the clocke; and ſo hauing the quantitie of the ſemidiurnall arke, the length of the day and night need not be vnknowne.

CHAP. VIII.

Of the fiue Starres.

I Might haue put in more Starres, but theſe may ſuffice for the finding of the houre of the night at all times of the yeare: and firſt I make choice of *Ala Pegaſi*, a ſtarre in the extremitie of the *wing of Pegaſus*, in regard it wants but 6 minutes of time of the beginning of ♈; but becauſe it is but of the ſecond magnitude, and not alwayes to be ſeene, I made choice of foure more, one for each quarter of the Ecliptique, as of *Oculus* ♉ the Buls eye, whoſe right aſcenſion conuerted into time, is 4 *ho.* 15 *m*; then of *Cor* ♌ the Lions heart, whoſe right aſcenſion is 9 *ho.* 48 *m*; next of *Arcturus*, whoſe right aſcenſion is 13 *H.* 58 *m*; and laſtly of *Aquila*, or the Vultures heart, whoſe right aſcenſion is 19 *H.* 33 *m.* Theſe fiue ſtarres haue all of them Northern declination; and if any others, ſome of theſe will be ſeene at all times of the yeare.

The vſe of them is,

The altitude of any of theſe fiue Starres being knowne, to find the houre of the night.

Firſt put the beade to the ſtarre which you intend to obſerue, take his altitude, and finde how many houres he is from the meridian by the fourth *Prop.* of the ſixt *Chap*; then out of the right aſcenſion of the ſtarre, take the right aſcenſion of the Sun conuerted into houres, and marke the difference; for this difference being added to the obſerued houre of the ſtarre from the meridian, ſhall ſhew how many houres the Sunne is gone from the meridian, which is in effect the houre of the night.

As if the **15** of May, the Sunne being in the fourth of ♊, I ſhould ſet the beade to *Arcturus*, and obſeruing his altitude ſhould find him to be in the Weſt about **52** *gr.* high, and the beade to fall on the houre-line of **2** afternoone, the houre would be **11** *ho.***50** *m.* paſt noone, or **10** *m.* ſhort of midnight.

For 62 *gr.* the right aſcenſion of the Sunne, conuerted into time, makes 4 *ho.* 8 *m.* which if we take out of **13** *ho.*58 *m.* the right aſcenſion of *Arcturus*, the difference will be 9 *ho.* 50 *m.* and this being added to **2** *ho.* the obſerued diſtance of *Arcturus* from the meridian, ſhewes the houre of the night to be **11** *ho.*50 *m.* Another example wil make all more plaine.

If the 9 of Iuly the Sunne being then in 26 *gr.* of ♋, I ſhould ſet the beade to *Oculus* ♉, and obſeruing his altitude ſhould find him to be in the Eaſt about **12** *gr.* high, and the bead to fall on the houre-line of 6 before noone, which is 18 *ho.* paſt the meridian, the houre of the night would be better then a quarter paſt **2** of the clocke in the morning.

For **118** *gr.* the right aſcenſion of the Sun, conuerted into time, makes 7 *ho.*5**2** *m*; this taken out of 4 *ho.*15 *m.* the right aſcenſion of *Oculus* ♉, adding a whole circle, (for otherwiſe there could be no ſubſtraction) the difference will be 20 *ho.* 23 *m.* and this being added to 18 *ho.* which was the obſerued diſtance of *Oculus* ♉ from the meridian, ſhewes that the Sun (abating 24 *ho.* for the whole circle) is 14 *gr.* 23 *m.* paſt the meridian, and therefore 23 *m.* paſt 2 of the clock in the morning.

CHAP. IX.

Of the Azimuth lines.

THoſe lines which are drawne betweene the equator and the tropiques, on that ſide of the quadrant which is neareſt vnto the ſights, and are numbred by 10. 20. 30. &c. doe repreſent the azimuths, the vttermoſt to the left hand repreſenteth the meridian, that which is numbred with 10 the tenth azimuth from the meridian, and that which is numbred with 20 the twentith, and ſo the reſt. Thoſe lines which are drawne from the equator to the left hand, doe ſhew the azimuth in the Summer; and thoſe other to the right hand, do ſhew the ſame in the Winter. The vſe of them is,

1 *The azimuth whereon the Sunne beareth from vs being knowne, to find the altitude of the Sun aboue the horizon.*

Firſt let the bead be ſet for the time, as in the former Chapter, then moue the thread vntill the bead fall on the azimuth; ſo the degrees which the thread cutteth in the quadrant, ſhall ſhew the altitude of the Sun at that time. Where you are to obſerue, that ſeeing the azimuths are drawne on the right ſide of the quadrant, you are alſo to begin to number the degrees of the Sunnes altitude from the right hand toward the left. As if the ſights had been ſet on the line *A B*, and you had turned your right hand towards the Sunne in obſeruing of his altitude, contrary to our practiſe in the former Chapter.

As if the time giuen were the 2 of Auguſt, when the Sun hath about 15 *gr.* of North declination, you may ſet the bead for the time, ſo you ſhall find the height at noone when the

Sunne is in the South, to be 53 *gr.* 30 *m.* when he is 10 *gr.* from the South 53 *gr.* 10 *m.* when 20 *gr.* then about 52 *gr.* 8 *m.* when 30 *gr.* then 50 *gr.* 20 *m.* when 40 *gr.* then 47 *gr.* 48 *m.* when 50 *gr.* then 44 *gr.* 12 *m.* when 60 *gr.* then 39 *gr.* 35 *m.* when 70 *gr.* then 33 *gr.* 50 *m.* when 80 *gr.* then 27 *gr.* when he is in the Eaſt or Weſt 90 *gr.* from the meridian, then is the height neare 19 *gr.* 20 *m*; when he comes to be 100 *gr.* then 11 *gr.* 15 *m.* when 110 *gr.* then 3 *gr.* 20 *m*; and before he commeth to the azimuth of 120 *gr.* he hath no altitude. For the Sunne hauing 15 *gr.* of North declination, will riſe and ſet at 114 *gr.* 34 *m.* from the meridian.

2. *The altitude of the Sunne being giuen, to find on what azimuth he beareth from vs.*

Let the beade be ſet for the time, and the altitude obſerued as before; then bring the thread to the complement of that altitude, ſo the bead ſhall ſhew the azimuth required.

As if the ſecond of Auguſt, hauing ſet the beade for the time, you ſhall find the altitude of the Sun to be 19 *gr.* 20 *m.* remoue the thread vnto 70 *gr.* 40 *m.* the complement of the altitude; or, which is all one, to 19 *gr.* 20 *m.* from the right hand toward the left, and the bead will fall on the line of 90 *gr.* from the meridian. And therefore the point whereon the Sunne beareth from vs, is one of theſe two, either due Eaſt or due Weſt. And which of theſe is the true point of the compaſſe, may be ſoone knowne by a ſecond obſeruation: for if the Sun riſe higher, it is the forenoone; if it be lower, it is the afternoone.

By knowing the azimuth or point of the compaſſe whereon the Sunne beareth from vs, it is eaſie to finde,

A meridian line; and thereby
The coaſting of the Countrey.
The ſite of a building.
The variation of the Compaſſe.

As if the ſecond of Auguſt in the afternoone, I ſhould find by the height of the Sunne that he beares from me 60 *gr.* from the meridian toward the Weſt; then there being 90 *gr.* belonging to each quarter, the Weſt will be 30 *gr.* to the right hand, the Eaſt is oppoſite to the Weſt, the North and South lie equally betweene them.

CHAP. X.

Of the Quadrat.

THe Quadrat hath two ſides diuided, the other two ſides next the Center may be ſuppoſed to be diuided, each of them into 100 equal parts: of the ſides diuided, that which is next the horizontall line containes the parts of right ſhadow, the other next the ſights, the parts of contrary ſhadow. The vſe of the Quadrat is,

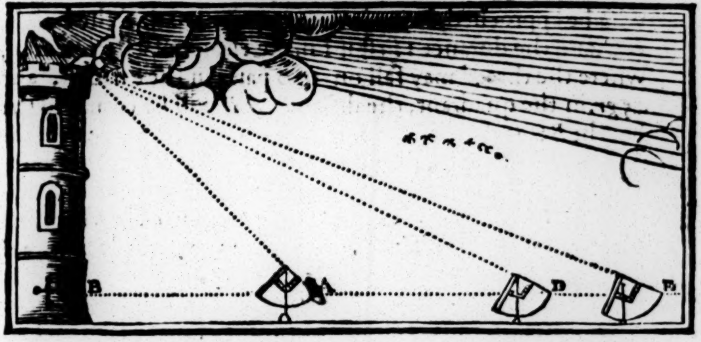
1 *Any point being giuen, to finde whether it be leuell with the eye.*

Lift vp the center of the quadrant, ſo as the thread with the plummet may play eaſily by the ſide of it; then looke through the ſights to the place giuen: for now if the thread ſhall fall on *A B* the horizontall line, then is the place giuen leuel with the eye: but if it ſhal fal within the ſaid line on any of the diuiſions, then it is higher; if without, then it is lower then the leuell of the eye.

2 *To find an height aboue the leuell of the eye, or a diſtance at one obſeruation.*

Looke through the ſights to the place, going nearer or farther from it, till the thread fall on 100 parts in the quadrat or 45 *gr.* in the quadrant, ſo ſhall the height of the place aboue the leuell of the eye, be equall to the diſtance betweene the place and the eye.

If the thread fall on 50 parts of a right ſhadow, the height is



is but halfe the diſtance: if it fall on 25, it is a quarter of the diſtance: if on 75, it is three quarters of the diſtance. For as oft as the thread falleth on the parts of right ſhadow,

As 100 to the parts on which the thread falleth:
So is the diſtance to the height required.

And on the contrary,

As the parts cut by the thread are to 100:
So the height vnto the diſtance.

But when the thread ſhall fall on the parts of contrary ſhadow: if it fall on 50 parts, the height is double vnto the diſtance; if on 25, it is foure times as the diſtance. For as oft as the thread falleth on the parts of contrary ſhadow,

As the parts cut by the thread are vnto 100:
So is the diſtance vnto the height.

And on the contrary,

As 100 are vnto the parts cut by the thread:
So is the height vnto the diſtance.

And what is here ſaid of the height and diſtance, the ſame may be vnderſtood of the height and ſhadow.

3 *To find a height or a diſtance at two obſeruations.*

As if the place which is to be meaſured might not otherwiſe

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wiſe be approched, & yet it were required to find the height *B C*, and the diſtance a firſt if I make choice of a ſtation at *A*, where the thread may fall on 100 parts in the quadrat, and 45 *gr.* in the quadrant, the diſtance *A B* will be equall to the height *BC*; then if I go farther in a direct line with the former diſtance, and make choice of a ſecond ſtation at *D*, where the thread may fall on 50 parts of right ſhadow, the diſtance *BD* would be double to the height *BC*: wherefore I may meaſure the difference betweene the two ſtations *A* and *D*, and this difference *A D* will be equall both to the diſtance *A B* and the height *BC*.

Or if I cannot make choice of ſuch ſtations, I take ſuch as I may, one at *D*, where the thread falleth at 50 parts of right ſhadow; the ſecond at *E*, where it falleth on 40 parts: and ſuppoſing the height *B C* to be 100, I find that

As 50 parts are vnto 100, the ſide of the quadrat:
So 100 the ſuppoſed height, vnto 200 the diſtance *BD*.
And as 40 parts, at the ſecond ſtation, vnto 100:
So 100 the ſuppoſed height, vnto 250 the diſtance *BE*.

Wherfore the difference between the ſtations *D* & *E* ſhould ſeeme to be 50; and then if in meaſuring of it, I ſhould find it to be either more or leſſe, the proportion will hold, as from the ſuppoſed difference to the meaſured difference, ſo from height to height, and from diſtance to diſtance.

As if the difference between the two ſtations *D* and *E* being meaſured, were found to be 30.

As 50 the ſuppoſed difference, vnto 30 the true difference:
So 100 the ſuppoſed height, vnto 60 the true height.
And 200 the ſuppoſed diſtance, vnto 120 the true diſtãce:
And 250 at the ſecond ſtation, vnto 150 the diſtance *B E*.

The like reaſon holdeth in all other examples of this kind: and if an Index with ſights were fitted to turne vpon the Center, it might then ſerue by the ſame reaſon for the finding of all other diſtances.

FINIS.

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